ESTABLISHMENT OF THE MEXICAN NATIONAL STANDARD FOR RF AND MICROWAVE SCATTERING (*s*-) PARAMETERS

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Abstract: Scattering (*s*-) parameters are quantities involved in RF and high frequency measurements, normally performed in fields such as telecommunications. A description of the establishment of the mexican national standard for providing traceability to these measurements is presented in this work.

1. INTRODUCTION

Scattering (*s*-) parameters are closely related to other quantities like impedance and attenuation, which are widely used in RF and microwave applications. Most of the high frequency applications in our country are located in the frequency range from 100 kHz to 18 GHz. In few cases, higher limits such as 26,5 GHz and 50 GHz are reached. Also, most widely used connector types are N and GPC-7.

In order to provide traceability toward domestic standards, and facilitate the participation of CENAM in international comparisons, it was determined to establish *s*-parameters standards for the frequency range from 50 MHz to 18 GHz and for type-N connectors [1]. The benefits of these new standards include reliable measurement services for other CENAM's laboratories such as microwave power, electromagnetic compatibility, antennas, and of course, services for other calibration, test and industry laboratories. Coherence among RF and microwave measurements performed by different actors in our country is a matter of prime importance.

2. DESCRIPTION OF THE STANDARD

For the purpose of establishing this standard, a 6 pc set of precisely machined beadless coaxial air-lines was purchased. Lines are manufactured with a core of copper beryllium alloy and finished with gold plating. Connector type is N. Line nominal lengths are 15 cm; 10 cm; 7,5 cm; 6 cm; 5 cm and 3 cm. Nominal outer conductor (O.C.) diameter is 7 mm and nominal inner conductor (I.C.) is 3,04 mm. A picture of the set is shown in Figure 1.

Electrical parameters of these precision air-lines can be calculated from their dimensional characteristics and the physical properties of manufacturing materials, namely, conductivity and permeability of conductors, permittivity and permeability of dielectrics.

Figure 1. *Set of the precision coaxial air-lines.*

According to well known equations, air-line electrical properties like characteristic impedance Z_{00}, impedance Z_0, skin penetration δ, and propagation constant γ, are all related to coaxial air-lines dimensions and physical constants. Reflection coefficient Γ, as well as *s*-parameters s_{11} and s_{21} can be obtained from those. This is shown by equations (1) to (7).

$$Z_{00} = \frac{1}{2\pi}\sqrt{\frac{\mu_d}{\varepsilon_d}}\ln(b/a) \quad (1)$$

$$Z_0 = Z_{00}\left[1+\frac{\delta(1/a+1/b)}{4\ln(b/a)}(1-j)\right] \quad (2)$$

$$\delta = \frac{1}{\sqrt{\pi\mu_c\sigma_c f}} \quad (3)$$

$$\gamma = jZ_0(2\pi f)\left[\frac{2\pi\ \varepsilon_d}{\ln(b/a)}\right] \quad (4)$$

$$\Gamma = \frac{Z_0 - Z_{00}}{Z_0 + Z_{00}} \quad (5)$$

$$s_{11} = \frac{\Gamma\left[1 - \exp(-j2\gamma l)\right]}{1\text{-}\Gamma^2 \exp(-j2\gamma l)} \quad (6)$$

$$s_{21} = \frac{\left(1-\Gamma^2\right)\exp(-j\gamma l)}{1\text{-}\Gamma^2 \exp(-j2\gamma l)} \quad (7)$$

where b and a are the air-line outer and inner conductor diameters, respectively, l the air-line length, μ_d and ε_d are the permeability and permittivity of the dielectric, μ_c and σ_c are the permeability and conductivity of conductors, f the operation frequency and $j = \sqrt{-1}$.

The coaxial precision air-lines were sent to the dimensional metrology laboratories of CENAM for a full characterization of their inner and outer conductor length and diameters, pin-depth, as well as conductor profiles and roundness. Some of the dimensional calibration results are shown in Table 1.

***Table 1**. Dimensional characteristics of coaxial standards. Uncertainty figures are k=2 @ 95,45 % confidence level.*

Std ID	Length l (m)	Outer conductor diameter b (m)	Inner conductor diameter a (m)
AL-T15	0,1498042± 0,0000010	0,0070000± 0,0000008	0,0030402± 0,0000008
AL-T10	0,0998946± 0,0000009	0,0070013± 0,0000008	0,0030424± 0,0000008
AL-T7.5	0,0749080± 0,0000009	0,0070029± 0,0000008	0,0030417± 0,0000008
AL-T6	0,0599255± 0,0000009	0,0070004± 0,0000008	0,0030407± 0,0000008
AL-T5	0,0499503± 0,0000009	0,0070011± 0,0000008	0,0030439± 0,0000008
AL-T3	0,0299798± 0,0000008	0,0070002± 0,0000008	0,0030419± 0,0000008

3. ELECTRICAL PROPERTIES OF THE STANDARD AND UNCERTAINTY EVALUATION

All electrical characteristics of the air-lines were calculated from their dimensional properties by using equations (1) to (7).

Work was also extensively performed regarding the uncertainty evaluation of the standard. Some electrical quantities like characteristic impedance Z_{00} are scalar but those like impedance, reflection coefficient and *s*-parameters are vectors. Results for combined uncertainty of Z_{00}, estimated according to the Guide [3] are shown in Table 2.

***Table 2.** Characteristic Impedance and estimated combined uncertainty for the coaxial air-lines.*

Coaxial line	**Characterístic impedance ± uncertainty, (Ω)**
AL-T15	49,991 ± 0,017
AL-T10	49,959 ± 0,017
AL-T7.5	49,987 ± 0,017
AL-T6	49,985 ± 0,017
AL-T5	49,928 ± 0,017
AL-T3	49,959 ± 0,017

For vector quantities, a vector-like treatment and the general directions given by the Guide [2, 3] were followed. Regarding high frequency quantities, in the past, it was commonly preferred to treat these complex quantities by expressing them in magnitude and phase, then analyzing and separating error sources that have to do with each one of those. However, an approach which allows considering all involved quantities and their effects over both real and imaginary parts, including their correlation, is more complete and should not only provide for a better estimation of the measurement uncertainty, but also turn out in a clearer statement and reporting.

For the reflection coefficient Γ we first separate the real and imaginary parts, and we get (8).

$$\Gamma = \frac{Z_R^2 + Z_I^2 - Z_{00}^2}{\left(Z_R + Z_{00}\right)^2 + Z_I^2} + j\frac{2Z_I Z_{00}}{\left(Z_R + Z_{00}\right)^2 + Z_I^2} = \Gamma_R + j\Gamma_I \quad (8)$$

Defining a vector containing input quantities, we have (9):

$$\underline{X_1} = \left(Z_R, Z_I, Z_{00}\right) \quad (9)$$

while a vector containing the output quantities is

$$\underline{Y_1} = (\Gamma_R, \Gamma_I) \quad (10)$$

and there exists a relationship between them as given by (11):

$$\underline{Y_1} = \underline{f_1}(\underline{X_1}) \quad (11)$$

The J_Γ matrix of the Jacobian of transformation $\underline{f_1}$ contains the sensitivity coefficients and it is given as shown by (12):

$$J_\Gamma = \begin{pmatrix} \frac{\partial \Gamma_R}{\partial Z_R} & \frac{\partial \Gamma_R}{\partial Z_I} & \frac{\partial \Gamma_R}{\partial Z_{00}} \\ \frac{\partial \Gamma_I}{\partial Z_R} & \frac{\partial \Gamma_I}{\partial Z_I} & \frac{\partial \Gamma_I}{\partial Z_{00}} \end{pmatrix} \quad (12)$$

In its matrix form, the uncertainty propagation law establishes that

$$V(\underline{Y_1}) = J_\Gamma V(\underline{X_1}) J_\Gamma^T \quad (13)$$

where $V(\underline{X_1})$ y $V(\underline{Y_1})$ are the covariance matrices for the input and output vectors, respectively, J_Γ is the matrix of the Jacobian of $\underline{f_1}$ transformation and J_Γ^T is the transposed matrix. The matrix of input vector $V(\underline{X_1})$ allows expressing the uncertainty of the input quantities, including their correlation.

From this, we have that:

$$V(\underline{X_1}) = \begin{pmatrix} u^2(Z_R) & u(Z_R, Z_I) & 0 \\ u(Z_I, Z_R) & u^2(Z_I) & 0 \\ 0 & 0 & u^2(Z_{00}) \end{pmatrix} \quad (14)$$

while the covariance matrix for the output vector is:

$$V(\underline{Y_1}) = J_\Gamma V(\underline{X_1}) J_\Gamma^T = \begin{pmatrix} u^2(\Gamma_R) & u(\Gamma_R, \Gamma_I) \\ u(\Gamma_I, \Gamma_R) & u^2(\Gamma_I) \end{pmatrix} \quad (15)$$

As it can be clearly seen in equation (15), terms on diagonal represent the uncertainty of real and imaginary parts of reflection coefficient and the off-diagonal terms are the covariance; correlation can be calculated from this information.

Table 3 shows calculated reflection coefficient at some frequencies for AL-T15 air-line. Uncertainty figures represent calculated combined uncertainties. A similar treatment was followed for all the complex quantities. Table 4 and 5 show calculated *s*-parameters for AL-T15 line and their estimated uncertainties.

Table 3. *Real and imaginary parts of reflection coefficient and their estimated uncertainty for the AL-T15 air-line standard at some frequencies.*

f (GHz)	$\text{Re}[\Gamma] = \Gamma_R$	$\text{Im}[\Gamma] = \Gamma_I$
2,5	0,00026 ± 0,0002	-0,00026 ± 0,0001
5,5	0,00018 ± 0,0002	-0,00018 ± 0,0001
10,5	0,00013 ± 0,0002	-0,00013 ± 0,0001
15,5	0,00011 ± 0,0002	-0,00011 ± 0,0001
17,5	0,00010 ± 0,0002	-0,00010 ± 0,0001

Table 4. *Real and imaginary parts of s_{11} parameter and their estimated uncertainty for the AL-T15 air-line standard at some frequencies.*

f (GHz)	$\text{Re}[s_{11}] = s_{11R}$	$\text{Im}[s_{11}] = s_{11I}$
2,5	0,00052 ± 0,0005	-0,00053 ± 0,0001
5,5	0,00035 ± 0,0005	-0,00036 ± 0,0001
10,5	0,00025 ± 0,0005	-0,00026 ± 0,0001
15,5	0,00021 ± 0,0005	-0,00021 ± 0,0001
17,5	0,00020 ± 0,0005	-0,00020 ± 0,0001

Table 5. *Real and imaginary parts of s_{21} parameter and their estimated uncertainty for the AL-T15 air-line standard at some frequencies.*

f (GHz)	$\text{Re}[s_{21}] = s_{21R}$	$\text{Im}[s_{21}] = s_{21I}$
2,5	-0,00551 ± 0,0024	-0,99998 ± 0,0001
5,5	0,00623 ± 0,0085	0,99998 ± 0,0001
10,5	-0,00546 ± 0,0161	-0,99999 ± 0,0001
15,5	0,00365 ± 0,0237	0,99999 ± 0,0001
17,5	0,00276 ± 0,0267	1,00000 ± 0,0001

(25)

4. DISEMINATION OF THE REFERENCE VALUES OF THE STANDARD

Having well-characterized standards, allows the calibration of impedance and *s*-parameter measuring instruments like vector network analyzers. This is performed by means of network analyzer circuital modeling and error-correction techniques. Several of these methods are known [4]. Some like TRL (Thru-Reflect-Line) and LRL (Line-

Reflect-Line) relay completely on the definition of the line standards; in these cases the traceability of measurements can be clearly established upward the *s*-parameters standard and then to the national standard of length.

Software code was written to perform measurements of standards and devices under test; raw data are taken remotely with a computer and stored for further processing in a spreadsheet program. Correction routines are intended for network analyzer instrumentation error corrections; a model containing main instrumentation errors has been defined and solved. Resultant equations are implemented in the spreadsheet program in such a way that errors can be taken into account and mathematically removed from device under test measurements. Figure 2 shows an example of the s_{11} parameter measurement result of a microwave attenuator performed using a network analyzer calibrated using the AL-T15 line.

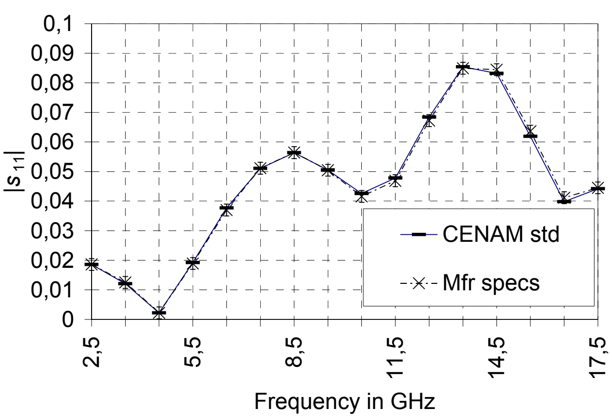

Figure 2. *$|s_{11}|$ for a type N 20 dB coaxial fixed attenuator measured using the AL-T15 air-line as calibration standard*.

5. CONCLUSIONS

The establishment of the reflection coefficient and *s*-parameters standard was described in this paper. This standard consists of a set of type N precisely machined beadless coaxial air-lines. Electrical parameters of standards were calculated from calibrated dimensional characteristics traceable to the national standard of length. Because *s*-parameters are complex quantities, their uncertainty evaluation was performed by following a vector like treatment and the general directions given in the Guide. The standard is being used for the calibration of impedance and *s*-parameters measurement instrumentation such us RF and microwave network analyzers and other similar equipment.

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