

# THE FRENCH WATT BALANCE PROJECT

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**Abstract:** The kilogram is the only base unit of the Système International d'unités (S.I.) still defined by a material artifact. Regarding the stability of the international prototype, its definition is not satisfactory. On the long term, it would be better to move to a definition based on a fundamental constants. To reach this goal, one of the most promising way seems to be the watt balance. Its principle consists in comparing a mechanical power to an electromagnetic power. This comparison results of a measurement performed in two steps: a static measurement during which the Laplace force taking place on a coil driven by a DC current and submitted to an induction field is compared to the weight of a standard mass, and a dynamic measurement where the voltage induced at the terminals of the same coil is determined when it is moved in the same field at a known velocity. The measurement of electrical quantities by comparison to the Josephson effect and the quantum Hall effect allows then to link the mass unit to the Planck constant. Despite the principle of the experiment remains simple and direct, obtaining sufficiently low uncertainty ( $1 \times 10^{-8}$ ) implies that devices relevant of various fields of physics must be implemented at their best level. In this paper, the main characteristics of the French watt balance project are presented.

## 1. INTRODUCTION

The International system of units (SI) includes seven base units. Among them, the kilogram is the last one still defined by a material artifact: the P<sub>th</sub>r international prototype. Since 1880, successive comparisons with other artifacts or national prototypes have shown relative drifts of  $3.10^{-8}$  with a scattering of the order  $1.10^{-7}$  [1].

It must be noted that at the present time, there is no way to characterize with a sufficient accuracy the evolution of the international prototype, in regard to any invariant quantity of nature.

Moreover, several units depending on the kilogram (A, mol,...) may suffer from this inconsistency.

Consequently, the 20<sup>th</sup> and 21<sup>th</sup> Conférence Générale des Poids et Mesures issued recommendations encouraging that "national laboratories continue their efforts to refine the experiments that link the unit of mass to fundamental or atomic constants with a view to a future redefinition of the kilogram"[2].

Among the possible ways (counting ions, levitating body, silicon crystal), the laboratories of LNE, like other laboratories (NPL [3], NIST [4], METAS [5] and BIPM [6]) have decided to develop a watt balance type experiment in order to contribute to the international effort. The realization of the different parts of the experiment started in 2002, resulting from the effort of several NMI or academic research laboratories [7].

Different choices and the main parts of the experiment are described hereunder.

## 2. PRINCIPLE

The principle of the experiment, suggested by B. Kibble in 1976 [8], is based on the one of electrodynamicometers formerly used to materialise the ampère. It consists in a comparison of mechanical and electrical powers allowing to link the mass to electrical quantities in a two phases experiment. The first (static phase), in its principle, corresponds to the direct determination of the ampère mentioned above. The second (dynamic phase), is an indirect determination of the so called geometric parameter of the experiment.

### 2.1 Static phase

A wire of length  $l$ , driven by a current  $i$ , is placed in an induction field  $B$ , in such a way that the resulting Laplace force be vertical.

The force on the wire suspended to a mass comparator is counterbalanced by the weight of a mass  $m$  submitted to the acceleration of gravity  $g$

$$F = mg = Bli \quad (1)$$

## 2.2 Dynamic phase

During the dynamic phase, the same wire is moved in the same magnetic field at a known velocity  $v$ .

The flux variation induces an emf  $\varepsilon$  at the terminals of the wire:

$$\varepsilon = Bl \frac{dz}{dt} = Blv \quad (2)$$

The dynamic phase constitutes an indirect determination of the geometric parameter  $Bl$ , with an uncertainty mainly due to voltage and velocity measurements. The only requirement is the invariance of the induction  $B$  and the length  $l$  during the experiment.

Combining the two equations leads to the equality of a mechanical and an electrical power:

$$mgv = \varepsilon i \quad (3)$$

The current  $i$  is measured by the voltage drop  $V$  it creates at the terminals of a standard resistor  $R$ . Voltages and resistances can be determined by comparison to standards based on the Josephson effect and the quantum Hall effect. Then a relation between  $m$ , the Josephson constant  $K_J$  and the von Klitzing constant  $R_K$  appears:

$$mK_J^2 R_K = \frac{A}{gv} \quad (4)$$

where  $A$  is relative to the measurement of voltages and resistances.

Provided it is believed that  $K_J$  and  $R_K$  are equal to their theoretical values (respectively  $2e/h$  and  $h/e^2$ ), the ratio of the Planck's constant to the mass may be measured:

$$\frac{h}{m} = \frac{4gv}{A} \quad (5)$$

The knowledge of  $m$  allows to determine  $h$ . At the opposite, the invariance of  $h$  is a reference from which the stability of the kilogram can be monitored at any time.

In regard to the variations of the international prototype, a relative uncertainty of  $1.10^{-8}$  must be obtained. This implies that the uncertainty components on each measured quantity remain of the order  $1.10^{-9}$  in relative.

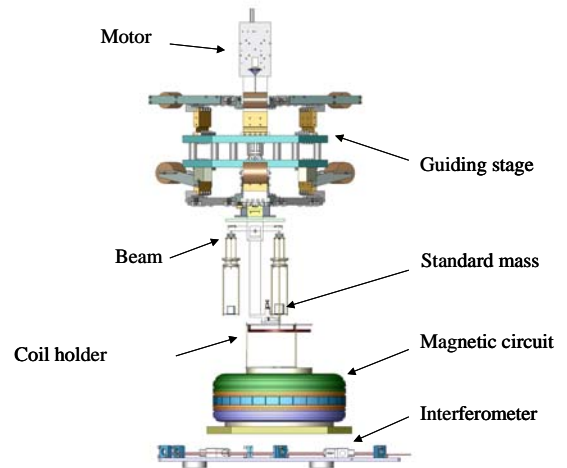
## 3. EXPERIMENTAL DEVICE

### 3.1 Main characteristics

Prior to the development of the watt balance, a great importance has been attached to environmental conditions. The new building of LNE located in Trappes (South-West of Paris), has been specially adapted to the watt balance requirements. The watt balance rooms are shielded. Temperature and humidity are controlled and a clean room will host the balance itself. They include two 2 meters thick concrete slabs of 33 m<sup>2</sup> each, anchored in stable layers of the ground to ensure a reduction of the vibration level and a good horizontal stability with time.

The principle of the experimental device presently in development is represented in fig 1. We have chosen to move jointly the force comparator, the standard mass and the coil by the mean of a step motor. This assembly is suspended under a flexure strips guiding stage, allowing a total rectilinear translation of 80 mm, a part of which will be traversed by the moving coil during the dynamic phase.

The coil (266 mm diameter, 600 turns) is immersed in the air gap of an annular magnetic circuit providing an induction of about 1 T



**Fig. 1: experimental device**

These dimensions are sufficient to provide during the static phase a force of 2.5 N with a 5 mA current through the coil, corresponding to the use of a

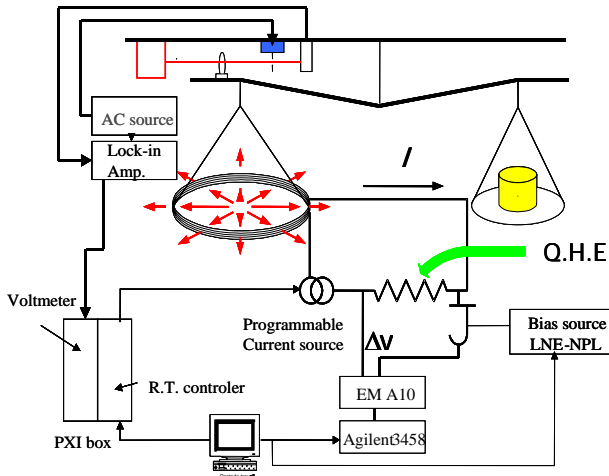
standard mass of 500 g. During the dynamic phase, an heterodyne interferometer, acting on a piezoelectric actuator, controls the velocity of the coil at about 2 mm/s.

### 3.2 Operating procedure

The  $h/m$  determination is a succession of static and dynamic phases in alternation in such a way to reduce the influence of eventual drifts of the field generated by the magnetic circuit.

#### Static phase

The measurement is taken in two stages. A tar is adjusted in such a way that a mass difference of  $m/2$  exists between the two arms of the force comparator. At first, the standard mass is deposited on the balance pan and the current in the coil  $i$  is adjusted to reach equilibrium. Then, the mass is removed and the current is reversed to  $-i$ . This procedure makes it possible to be freed from thermal drifts of the circuit.



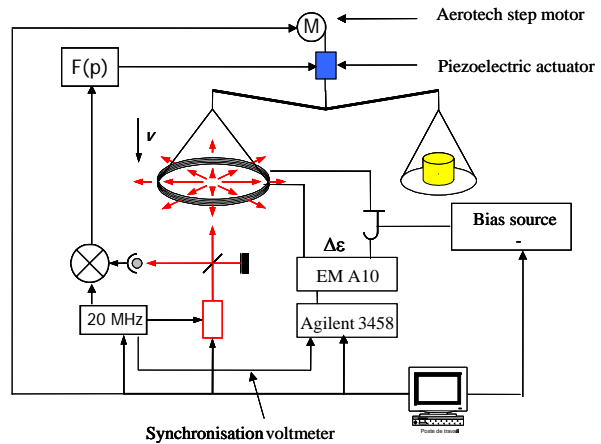
**Fig. 2: static phase (schematic diagram)**

During the comparison, the beam position is maintained to horizontal by the current in the coil. It is detected by an optical device associated to a lock-in amplifier and a real time control unit driving a programmable current source.

The current  $i$  is then measured through the comparison of the voltage drop it produces at the terminals of a calibrated resistance to a reference voltage issued from the Josephson set-up.

#### Dynamic phase

During the dynamic phase, the coil describes several displacements in both up and down directions resulting in a reverse in polarization of the induced voltage. During each trajectory, the coil is accelerated from 0 mm/s to 2 mm/s, then stabilized by the interferometer and stopped after a 40 mm displacement at nominal velocity.



**Fig. 3: dynamic phase (schematic diagram)**

During acceleration and deceleration and in order to avoid saturating the preamplifier, the Josephson voltage is controlled in real time in such a way that its difference with the unknown voltage at the terminals of the coil remains always lower than 1 mV. This technique requires heavy developments, but it makes it possible to be freed from a protection switch, possible source of instabilities.

During the displacement, the interferometer generates signals used to trigger the voltmeter on the frequency counter measuring the Doppler frequency at known positions of the trajectory.

The adjustment of a polynomial function on the values of the ratio of the induced voltage to the velocity,  $v/V$ , will be used to determine the geometric parameter  $BI$  and interpolate its value at the point where the coil will be positioned during the static phase.

### 3.3 Present status

Presently, the various parts of the experimental set up are in development. They are briefly described hereunder.

## Electrical standard

In both static and dynamic phases, the 1 V emf at the terminals of the coil and of the standard resistor is compared to a reference voltage delivered by a Josephson set-up based on a SINIS programmable array kindly provided by PTB. The stability and ease of use of the device make it adapted to the watt balance application. This array, associated to a detector composed of a low noise preamplifier and a digital multimeter has been compared to other arrays with agreement within  $1.10^{-10}$  at the 1 V level [9]. The dynamic phase of the experiment needing rapid changes of the reference voltage, a rapid bias source and a dedicated real time controller have been developed in collaboration with NPL and PTB [10].

The weighing current delivered to the coil during the static phase is issued from a low noise programmable current source driven by the mean of a real time controller. The current source is composed of two DAC with 16 bits resolution generating coarse and fine voltages that are added and converted to a variable current in the range  $\pm 5$  mA. Characterization of the source in closed loop operation shows that the relative Allan deviation of  $1.10^{-9}$  on the current may be reached in some hundred seconds.

The value of this current is measured through the voltage drop it produces at the terminals of a 200  $\Omega$  resistor calibrated against the quantum Hall effect.

This resistor is constituted of two commercial TGAM low temperature coefficient resistors in series placed in a thermostabilised air bath with a stability better than  $\pm 0.1$  K.

## Magnetic circuit

The main element of the 550 mm diameter magnetic circuit is a ring composed of 64 sections of SmCo magnets enclosed between two soft material poles [11]. The profile of the airgap is defined in such a way that the magnetic field remains constant within  $1.10^{-4}$  T in the portion where the coil is moved during the dynamic phase. Within this specification, the field also presents a minimum at the middle point in such a way to reduce the positioning error during the static phase. This implies that the dimensions of the magnetic circuit are defined within  $\mu\text{m}$  accuracy, and that every deformation due to mounting (magnetic force, magnetostriction, fitting ...) are modeled and anticipated.

## Guiding stage

The accuracy of the measurement strongly depends on the alignment of the coil during both static and dynamic phases. In particular, any horizontal displacement that should result in unwanted induced voltages at the terminals of the coil must be avoided during the dynamic phase. Moreover, it must be guaranteed that the coil passes exactly at the place it occupies during the static phase.

To satisfy these requirements, a guiding stage based on flexure strips has been developed, avoiding vibrations and allowing to compensate the efforts in such a way that only a small power is needed to move the coil on an effective 40 mm trajectory, part of a total 80 mm stroke [12].

The guiding system is constituted of two stages, each one having three symmetrical legs composed of two flexible elements, linked together by a sliding nut supporting the force comparator. The choice of a symmetrical structure leads to an hyperstatic guidance needing a very accurate machining. Thanks to this hyperstatism, the integrity of the system is preserved in the event of rupture of an element.

In order to reduce as much as possible the stage driving force, compensation masses are fixed to the rigid parts of the guiding stage. They compensate the total mass suspended to the sliding nut and the stiffness of flexible elements.

Complete measurements of the departure from the nominal rectilinear motion show that straightness errors in the two horizontal directions are about 0,9  $\mu\text{m}$  and 0,4  $\mu\text{m}$  respectively, for a 72 mm displacement, while the maximum driving force over the same range has been reduced from 175 N to 25 N.

## Interferometer

During the dynamic phase, the sliding nut is moved by the mean of a commercial linear stage constituting the first part of a two stage loop controlling the coil's velocity. The second stage is a piezoelectric actuator located as near the coil as possible. The two actuators are driven by the mean of a Michelson heterodyne laser interferometer associated to a home made control electronics including a clock at 640 MHz [13].

The electronic circuit acts on the laser source in such a way that the optic phase (i.e. the position of a mirror fixed to the moving object) is controlled by reference to a digitally variable electronic phase.

Phase steps of  $2\pi/32$  are generated, corresponding to optical phase steps of  $\lambda/128$  (double pass interferometer), where  $\lambda = c/v_{\text{ref}}$  is the wavelength of the laser source. The frequency  $v_{\text{ref}}$  is calibrated by comparison to one of the national references. The phase reference being driven by an external frequency standard, the relative uncertainty on the velocity remains of the order of  $1.10^{-9}$ .

### Force comparator

The main element of the force comparator is a symmetrical beam, specifically elaborated for the experiment, whose pivots are constituted of flexure strips.

Clamped flexure strips allow to adapt the sensitivity to the total mass suspended to the beam, including the 500 g standard mass. The eventuality of using masses of 250 g or 1 kg is then preserved in view of determining possible systematic effects.

### Gravimeter

Based on the competences of the LNE-SYRTE in atomic interferometry and inertial sensors, the choice has been made to develop a cold atoms gravimeter, instead of using a classical free-fall gravimeter [14].

Once determined, the  $g$  value must be transferred from the place where it is measured to the center of gravity of the standard mass. For this, the gravimetric cartography of the laboratory and  $g$  gradients are determined with a Scintrex CG5 relative gravimeter. First measurements show that the precautions taken during the construction of the laboratory allow to get a relative difference in  $g$  lower than some  $10^{-9}$  between the central points of the concrete slabs of the gravimetry room and the watt balance room.

Finally, transfer difference determination will be completed by modeling the influence of all the masses surrounding the standard mass.

## 4 CONCLUSION

If it is still premature to give an extensive uncertainty budget on our experiment, at least the main sources may be identified. They concern the references (mass, voltage, resistance, velocity, gravimetry) that may be controlled within some  $10^{-9}$  at the exception of the mass, for which it is required to develop standard masses in nonmagnetic alloys to reach this level.

An important source of uncertainty is due to the alignment of the experiment. The origin is in

unwanted components of the velocity (horizontal and angular velocities), unwanted forces (horizontal forces and torques) and interferometer alignment.

This requires special procedures to align the whole system. Simple calculations show that, in order to reach the  $1.10^{-8}$  level, all alignments and unwanted displacements must stay within some  $\mu\text{rad}$  and some  $\mu\text{m}$ .

Type A uncertainties result from two types of contributions. The first one is due to the individual behavior of each element of the experiment (current source, detector, magnetic field...) while the second will come from their combination. From this last point of view, mechanical vibrations and fluctuations of the ambient magnetic field will certainly dominate.

Despite the fact that the above mentioned uncertainties are at the limit of what is possible today, the watt balance seems to be an adequate way to reach a relative uncertainty of  $1.10^{-8}$ .

The emergence of recent results [15] has strongly stimulated the reflection on the possible change of definition of the kilogram, based on a fundamental constant. Among the conditions that must be fulfilled before moving to new definitions, obtaining coherent results from a sufficiently high number of independent experiments is of prime importance.

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