

# REFERENCE PLANE POSITION AND ANGULAR RESPONSIVITY OF SPECTRORADIOMETER DIFFUSERS

Erkki Ikonen<sup>1,2</sup>, Pasi Manninen<sup>1</sup>, Jari Hovila<sup>1</sup>, and Petri Kärhä<sup>1</sup>

<sup>1</sup>Metrology Research Institute, Helsinki University of Technology (TKK)

P.O. Box 3000, FI-02015 TKK, Finland

<sup>2</sup>Centre for Metrology and Accreditation (MIKES),

P.O. Box 9, FI-02151 Espoo, Finland

Tel. +358-94512283, Fax +358-94512222, e-mail: erkki.ikonen@tkk.fi

**Abstract:** The effective reference plane position of the spectroradiometer measuring head determines the variation of the measured irradiance signal at different distances from the radiation source. Incomplete knowledge of the true reference plane position may result in significant measurement errors, if calibration and use of the spectroradiometer take place at different distances from the source. We describe a method for determining the effective measurement plane of spectroradiometer diffusers, based on distance dependence of the measured signal. The angular responsivity of the diffuser needs to be taken into account in the data analysis, especially if there are large deviations from the ideal cosinusoidal responsivity. A clear correlation was found between the measured shifts of the reference plane positions and deviations from ideal angular responsivity which may, in general, help to identify diffusers with large shifts of reference plane positions.

## 1. INTRODUCTION

The measuring heads of spectroradiometers are often equipped with diffusers to allow uniform collection of radiation from all directions. Ideal diffusers should follow a  $\cos \theta$  curve when the measurement direction is deviated by angle  $\theta$  from the symmetry axis of the measuring head. In the calibration of the spectroradiometer, the outermost point of the diffuser is typically used to define the reference plane when measuring the distance from the radiation source. However, the distance dependence of the signal does not necessarily obey the inverse-square-law with respect to this plane, because operation of the outermost surface of the diffuser as the receiving aperture is not self-evident. We have recently shown that the effective reference plane of the photometer [1] and spectroradiometer [2] diffusers may be shifted by several millimeters from the assumed position, potentially leading to measurement errors of several percents if the true reference plane position is not properly taken into account.

In this report we describe in detail the method for determination of the reference plane position of the diffuser. Here are presented for the first time the effects of rectangular radiation source shape and deviation of the angular responsivity of the detector from the  $\cos \theta$  curve. The derived equations are applied to a re-analysis of the data of Ref. [2],

showing that large deviations from the ideal cosinusoidal angular responsivity may have a significant effect on the diffuser's reference plane position. However, for diffusers with reasonably good angular responsivity, the earlier uncertainty estimates of the determined reference plane positions remain valid.

## 2. DESCRIPTION OF THE METHOD

Consider a circular source (radius  $r_S$ ) and a circular detector (radius  $r_D$ ), located parallel with each other at a distance  $d$  on the optical axis of the measurement system. For a homogeneous Lambertian source and a detector with uniform spatial and cosinusoidal angular responsivity,  $R(\theta) = R_0 \cos \theta$ , the distance dependence of the detector signal at a certain wavelength is given by

$$S(d) = \frac{g I_e A R_0}{d^2 + r_S^2 + r_D^2}, \quad (1)$$

where  $I_e$  is the axial spectral radiant intensity of the source,  $A$  is the area of the detector, and  $g$  is a multiplication factor [3] which depends only on the geometry of the system. For the purposes of this work, the multiplication factor is taken as  $g = 1$ , since  $g - 1$  is approximately proportional to  $(r_S r_D / d^2)^2$  and  $d$  is much larger than the parameters  $r_S$  and  $r_D$ .

In practical measurements, a distance  $d_0$  is often determined between selected auxiliary planes of the lamp holder and the detector's front surface. If the detector has a well-defined circular aperture as the front surface, the distance offset of the source  $\Delta d_S$  can be determined from the  $d_0$  dependence of the measured signal by fitting Eq. (1) to the data with  $d = d_0 + \Delta d_S$ . The fitted parameters are  $\Delta d_S$  and the scaling factor  $S_0 = I_e AR_0$ , whereas the parameters  $r_S$  and  $r_D$  are fixed to appropriate values. Such a measurement gives the shift  $\Delta d_S$  of the true reference plane of the lamp relative to the defined auxiliary plane at the lamp holder. The lamp irradiances at measurement distances determined relative to the true reference plane position obey the modified inverse-square-law of Eq. (1).

The next step is to determine the true reference plane of the spectroradiometer's measuring head equipped with a diffuser. The outermost point of the diffuser is often used to define the auxiliary plane at the detector for distance measurements. When  $\Delta d_S$  for the lamp is known, the  $d_0$  dependence of the signal is measured and Eq. (1) is used in the least-squares fitting of the data with  $d = d_0 + \Delta d_S + \Delta d_D$ , where  $\Delta d_D$  is the offset of the true reference plane of the detector from the auxiliary plane. As before, the fitted parameters are  $\Delta d_D$  and  $S_0$ , while other parameters are fixed to the known values. The data analysis specifies the reference plane position, for which the modified inverse-square-law is obeyed in distance dependence of the detector signal.

Equation (1) is an exact result valid for any distance between the circular source and circular detector with ideal properties. In practice, the angular responsivity of the detector with diffuser may deviate from the ideal cosinusoidal curve and many lamp sources have more or less rectangular, rather than circular, shape. The angular responsivity of a nonideal, spatially uniform detector within a range of a few degrees around the diffuser symmetry axis can be approximated by a parabolic shape

$$R(\theta) \approx R_0(1 - \alpha\theta^2/2) \approx R_0 \cos^\alpha \theta, \quad (2)$$

where the latter approximation is written in a form, which quantifies the deviation from the cosine responsivity. The value  $\alpha = 1$  in Eq. (2) corresponds to ideal angular responsivity of the detector.

The influence of parameter  $\alpha$  and the rectangular shape of a homogeneous source on Eq. (1) can be estimated by considering energy transfer between

infinitesimal source and detector elements and integrating the result over the full emitting and receiving surfaces. The first order approximation of the distance dependence of the measured signal is then still given by Eq. (1), but the radius parameters of the detector and source are modified to

$$r_D = r_0(3 + \alpha)^{1/2} / 2 \quad (3)$$

and

$$r_S = r_{\text{eff}} [6\pi(3 + \alpha)(x_0 y_0 + y_0 x_0)]^{1/2} / 12, \quad (4)$$

where  $r_0$  is the radius of the circular detector,  $x_0$  ( $y_0$ ) is the width (height) of the rectangular source, and  $r_{\text{eff}}$  is defined by  $\pi r_{\text{eff}}^2 = x_0 y_0$ . Terms of order  $(r_{\text{eff}}/d)^4$ ,  $(r_0/d)^4$ , and  $(r_{\text{eff}} r_0/d^2)^2$  are neglected in this calculation. The error thus introduced is of the same order of magnitude as made in neglecting the multiplication factor  $g$  in Eq. (1). For  $\alpha = 1$  and  $x_0 = y_0$ , the value of parameter  $r_S$  is approximately  $1.023 r_{\text{eff}}$ , which indicates that a square source is well described by a circular source of the same area.

An improved determination of the reference plane positions of the source and detector can be made using Eqs. (3) and (4) in the modified inverse-square-law, instead of equations  $r_D = r_0$  and  $r_S = r_{\text{eff}}$  which were used, for example, in the data analysis of Ref. [2].

### 3. EXPERIMENTAL

For the purposes of the new data analysis we briefly describe how measurements of Ref. [2] were carried out. The distance dependence of detector signals was measured using a stable light source, an optical rail with a high-accuracy magnetic distance-measurement system, a reference detector, and the studied spectroradiometer diffusers as measuring heads. The light source was a 1-kW halogen lamp of type BN-9101 from Gigahertz Optik. Its light emitting volume has approximately rectangular cross section with a width of  $x_0 = 5$  mm and height  $y_0 = 18$  mm. A standard photometer of type PRC TH15 from PRC Krochmann GmbH was used as the reference detector with a well-known circular aperture plane of 8-mm diameter. We consider the results of three planar diffusers labeled as #1, #2, and #3 which are of Bentham types D7, D3, and D5, respectively. Diffusers #1 and #3 are made of Teflon and diffuser #2 has been manufactured from water free quartz. The diameters  $2r_0$  of diffusers #1, #2, and #3 are 10 mm, 25 mm, and 23 mm, and their thicknesses are 1.8 mm, 3.8 mm, and 0.6 mm, respectively.

The determination of the distance offset for the lamp and for the various diffusers was carried out during the same lamp burn to ensure that the lamp filament position stayed constant during the measurements. Distances  $d_0$  were between 0.3 m and 2 m and they were determined relative to the front surface of the lamp housing and the aperture plane of the reference detector or the front surface of the diffuser.

We replaced the monochromator and the photomultiplier tube of the spectroradiometer with filtered trap detectors to avoid noise problems related to low signal level of the spectroradiometer at large measurement distances. The light collected by the diffuser was guided by an optical quartz fiber to the detector through an optional filter which selected the desired wavelength bands. The filters used were a UV filter of type UG11, a  $V(\lambda)$  filter, and a 700-nm interference filter. The UV filter has a wide pass-band of 100 nm in the UVA region around 350 nm (denoted as UVA band in the following). Other measuring bands were in the neighborhood of 570 nm (Green), 700 nm (Red), and 860 nm (NIR). These effective wavelengths were obtained by using the  $V(\lambda)$  filter, the 700 nm interference filter, and without any filter, respectively. Some measurements were carried out also with the actual spectroradiometer to confirm the results obtained with diffusers coupled to filtered broadband detectors.

The angular responsivities of the spectroradiometer diffusers were measured with a step size of  $1^\circ$  using a computer-controlled turntable. The same wavelength bands as described above were used in the measurements. The diffuser was mounted on the turntable in such a way that the rotation axis was perpendicular to the symmetry axis of the diffuser and crossed it on the front surface of the diffuser. The diffuser was illuminated at a distance of 0.7 m from the lamp. A baffle with an opening diameter of 40 mm was situated between the lamp and the diffuser at a distance of 0.2 m from the lamp.

#### 4. RESULTS

##### 4.1. Angular responsivities

The angular responsivity results of Ref. [2] are reproduced in Figs. 1, 2, and 3 for diffusers #1, #2, and #3. In the figures, crosses, triangles, circles, and squares indicate the angular responsivity in the UVA, Green, Red, and NIR wavelength bands, respectively. It is seen that the angular responsivity functions vary from shapes close to ideal (Fig. 1) to

strongly peaked curves (Fig. 3). For diffusers #1 and #2 the wavelength dependence of angular responsivity is weak, in contrast to the long-wavelength behavior of diffuser #3.

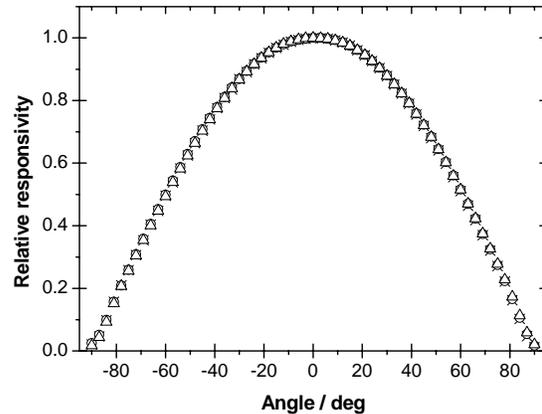


Fig. 1. Measured angular responsivities of diffuser #1 at three wavelength bands.

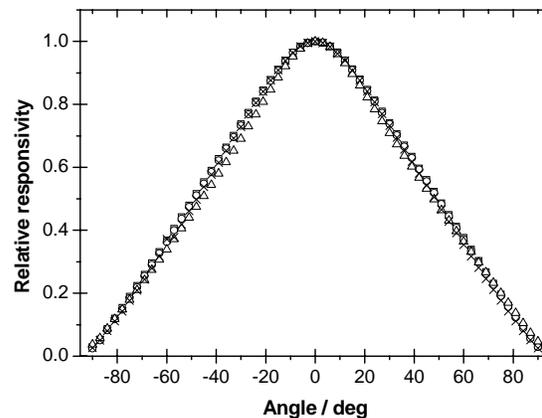


Fig. 2. Measured angular responsivities of diffuser #2 at four wavelength bands.

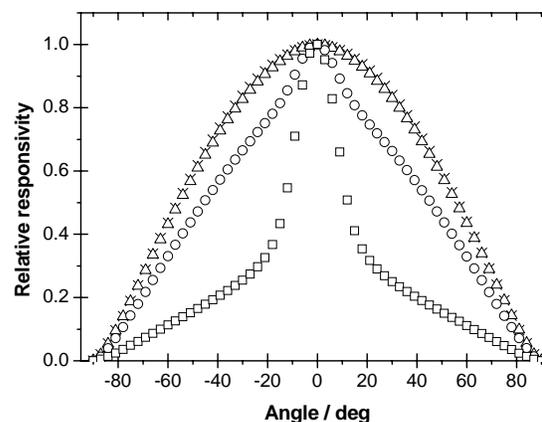


Fig. 3. Measured angular responsivities of diffuser #3 at four wavelength bands.

For measurement of the distance offset, it is necessary to determine the value of parameter  $\alpha$  in Eq. (2) within a narrow angular range around the optical axis. Table 1 shows these  $\alpha$  values for each of the diffusers as determined from the data of Figs. 1, 2, and 3 within the angular range of  $\pm 4^\circ$ . This range covers all directions of the detected direct radiation at the closest distance of 0.3 m between the lamp and diffuser.

Table 1. Values of parameter  $\alpha$  [see Eq. (2)] as determined close to the symmetry axis of the diffusers.

Diffuser	$\alpha$			
	UVA	Green	Red	NIR
#1	1.0	1.0	1.0	1.0 <sup>*)</sup>
#2	3.3	3.8	2.9	2.9
#3	1.8	1.9	9.7	27

<sup>\*)</sup>extrapolated using data at other wavelength bands

Parameter  $\alpha$  can have a large value if the angular responsivity curve is sharply peaked as in the case of Red and NIR wavelength bands of diffuser #3. In the latter case the values of the radius parameters in Eq. (1) become  $r_D = 2.7r_0$  and  $r_S = 3.9r_{eff}$  using Eqs. (3) and (4), whereas for diffuser #1 with cosinusoidal angular responsivity they are  $r_D = r_0$  and  $r_S = 1.4r_{eff}$ .

**4.2. Distance offsets**

The method described in Sec. 2 was used to determine the distance offsets  $\Delta d_S$  and  $\Delta d_D$ . Geometrical parameter values of the lamp and diffusers given in Sec. 3 were used in Eqs. (3) and (4) as well as the  $\alpha$  values from Table 1. For the reference detector, an ideal angular responsivity ( $\alpha = 1$ ) was assumed. The obtained distance offsets of the diffusers at the different wavelength bands are presented in Table 2. Relative standard deviations between the data and the fitted curves were less than 0.3 %.

In the earlier analysis [2] of the distance dependence data, the angular responsivity of the measuring head and the rectangular shape of the radiation source were not taken into account. As compared with those results, the largest changes due to the more rigorous analysis method of the present work are about 1 mm and 2 mm in the case

of Red and NIR wavelength bands of diffuser #3, respectively. The rest of the changes are less than 0.3 mm and thus within the standard uncertainty limits estimated in Ref. [2].

Table 2. Measured distance offsets of the diffusers.

Diffuser	Offset $\Delta d_D$ (mm)			
	UVA	Green	Red	NIR
#1	-0.3	0.1	0.3	0.1
#2	1.9	2.2	3.3	2.2
#3	0.1	0.9	2.2	5.7

Comparing Fig. 1 and Table 2 it is seen that the angular responsivity of diffuser #1 is close to the ideal cosinusoidal shape and the reference plane position is on the front surface of the diffuser throughout the measured wavelength bands. Diffuser #2 has almost triangular angular responsivity at all wavelength bands and a relatively large, almost constant distance offset. The angular responsivity of diffuser #3 strongly depends on the wavelength band. The diffuser has a very small distance offset in the UVA region, but it increases with wavelength. In the NIR region, the 0.6 mm thick diffuser #3 appears to be transparent and the reference plane position is located close to the entrance surface of the optical fiber connecting the diffuser and the spectroradiometer.

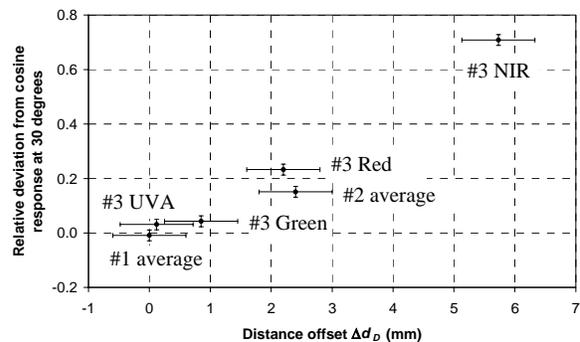


Fig. 4. Comparison of distance offsets of the diffusers with their angular responsivities. For diffusers #1 and #2, the wavelength dependence of the angular responsivity is weak and average of the distance offsets of Table 2 is used.

Figure 4 shows graphically the relation between the distance offset of the diffusers and the relative deviation of the measured angular responsivity curve from the ideal cosine curve at 30°. The figure

demonstrates a clear correlation between these parameters.

## 5. CONCLUDING REMARKS

The method for determining the reference plane position of measuring heads equipped with diffusers was improved. The present analysis takes into account the effects of nonideal angular responsivity of the detector and the rectangular shape of the source, which were neglected in the earlier analysis method. With these improvements, the standard uncertainty of the measured reference plane positions remains at the value of 0.3 mm.

The shift of the reference plane position from the front surface of the diffuser may cause a significant systematic error, if it is not properly taken into account. For example, a distance offset of 3 mm in the diffuser causes a relative measurement error of 1.6 % when the spectroradiometer is calibrated at a distance of 370 mm from the reference lamp and used to measure radiation from a source at large distance. An uncertainty of 0.3 mm in the reference plane position is sufficient for high-accuracy measurements, since it introduces an uncertainty of less than 0.2 % in the measured irradiances with the shortest normally used calibration distance of 370 mm.

Old measurement data may contain uncorrected effects of the diffuser distance offsets. Fortunately, correction of these results is straightforward, provided that the distance offset of the measuring head can be determined afterwards. If the angular responsivities of planar diffusers of the old measurements are known, the tentative correlation observed between the angular responsivity and distance offset may help in identifying those cases where the corrections would be most significant.

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## REFERENCES

- [1] Hovila J., Mustonen M., Kärhä P., and Ikonen E., *Appl. Opt.* **44**, 5894-5898 (2005).
- [2] Manninen P., Hovila J., Seppälä L., Kärhä P., Ylianttila L., and Ikonen E., *Metrologia* **43**, S120-S124 (2006).
- [3] Walsh J. W. T., "The Principles of photometry," in *Photometry*, (Dover, New York, 1965), pp. 120-173.