THE EVALUATION OF THE UNCERTAINTY OF POSITIONAL DEVIATIONS OF CNC MACHINE TOOLS

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Abstract The assessment of the measurement uncertainty is an indispensable task in all calibration procedures. By international accord, the evaluation is to be done in accordance with the ISO Guide to the Expression of Uncertainty in Measurement (GUM). To calibrate the positional deviations of computer numerically controlled (CNC) machine tools, calibration laboratories will usually follow the guidelines in ISO 230-2 International Standard. However, that standard does not address uncertainty. In this paper we present an uncertainty evaluation scheme that is firmly grounded in the GUM, and can therefore be of use as a guide to develop appropriate uncertainty calculations in this and similar types of calibrations.

1. INTRODUCTION

The ISO 230-2 International Standard [1] is accepted and used worldwide to determine the accuracy and repeatability of positioning of computer numerically controlled (CNC) machine tool axes. The test method involves repeated measurements at predefined positions of the axis under test. The results are used to determine various parameters intended to quantify the performance of the machine. Some of these parameters depend only on the values measured at a given position; other parameters characterize the global behavior of the axis. The local parameters are the mean positional deviations, the reversal values and the repeatabilities. The global parameters are the reversal value, the mean reversal value, the repeatability, the systematic positional deviation, the mean positional deviation and the accuracy.

The repeatability and accuracy depend upon a further local parameter that ISO 230-2 defines as the standard uncertainty of positioning, *s*. For example, the unidirectional repeatability at position *i* is defined as $4s_i$. The intention is clearly to provide a range derived from an expanded uncertainty using a coverage factor of 2. The latter terminology is taken from the ISO Guide to the Expression of Uncertainty in Measurement (GUM) [2], which ISO 230-2 declares to follow. However, the GUM defines the parameter *s* as the “sample standard deviation”; under no circumstances it should be taken as an estimator of standard uncertainty. It follows that there is a conflict in terminology between these two documents.

Moreover, since the test code specified by ISO 230-2 can be interpreted as a calibration certificate of the machine, the uncertainty of measurement should be reported together with the measurement results [3]. ISO 230-2 does not address this crucial point.

It should also be mentioned that the ISO 230-2 procedure is often used for verifying whether the measured positional deviations of the machine are no greater than a certain specified limit. In that case, the acceptance decision should be based on the rules set forth in the ISO 14253-1 International Standard [4], for which the uncertainty of measurement should be taken into account. Further discussion on these matters can be found in [5].

The purpose of this paper is, therefore, to propose a procedure by which to evaluate the uncertainty associated with the measured positional deviations of machine tools.

2. THE ISO 230-2 TEST METHOD

In the standard method the machine is programmed to locate its moving part at a series of predefined target positions along the axis under test. The standard cycle consists of *n* unidirectional approaches in both the positive (increasing) and negative (decreasing) directions to *m* target positions. Usually $n = 5$, while the value of *m* depends on the length of the axis. The symbols ↑ and ↓ are used to represent each of these directions of movement.

The measured quantities are the positional deviations. For point *i* in unidirectional approach *j*, these are defined as the differences between the actual positions p_{ij} and the target positions p_i:

$$x_{ij}\,| = p_{ij}\,| - p_i \qquad (1)$$

where the symbol $|$ stands for either ↑ or ↓. In this expression the index j runs from 1 to n and the index i runs from 1 to m,

Several parameters are then derived from the positional deviations. For example, their mean values:

$$\overline{x_i}\,| = (1/n) \textstyle\sum_j x_{ij}\,| \qquad (2)$$

and the experimental variances:

$$s_i^2\,| = \textstyle\sum_j (x_{ij}\,| - \overline{x_i}\,|)^2 / (n - 1). \qquad (3)$$

It is then clear that the main contribution of ISO 230-2 is the standardization of the various parameters that characterize the behavior of machine tools. However, it does not address the calculation of the measurement uncertainty: it does not consider resolution, misalignment, and calibration of the measuring equipment. In addition, its terminology does not comply with that in the GUM.

3. A PROPOSAL TO EVALUATE THE UNCERTAINTY IN THE TEST OF CNC MACHINES

In order to evaluate measurement uncertainties, the measurands must be defined through appropriate measurement models. When the models are linear or weakly nonlinear, they allow to express the standard uncertainties of the output quantities in terms of the standard uncertainties of the input quantities by using the so-called law of propagation of uncertainties, LPU [2,6]. This law states that for a quantity modeled as

$$z = f(x,y) \qquad (4)$$

the standard uncertainty is obtained as the square root of

$$u^2[z] = c_x^2 u^2[x] + c_y^2 u^2[y] + 2\, c_x\, c_y\, r_{xy}\, u[x]\, u[y] \qquad (5)$$

where $c_x = \partial f/\partial x$, $c_y = \partial f/\partial y$ and r_{xy} is the correlation coefficient between the input quantities x and y. If these are independent, the correlation coefficient is zero.

3.1 Basic model

In the calibration of machine tools, the measurands are the positional deviations. These are modeled as

$$x_i\,| = p_i\,| - p_i \qquad (6)$$

where $p_i\,|$ is the corrected indication of the measurement system at position i and p_i is the corrected target position.

The models for the quantities $p_i\,|$ and p_i should in turn include all conceivable random and systematic effects that influence them. It is reasonable to assume that measurements in testing machine tools are in general affected or limited by the characteristics of the test equipment and also by the conditions at which the test is carried out. The latter include temperature, alignment and resolution. Therefore, we write:

$$p_i\,| = f_t\, f_a\, p_{mi}\,| + c_s \qquad (7)$$

and

$$p_i = p_{ni} + c_r \qquad (8)$$

where

$$p_{mi}\,| = \textstyle\sum_j p_{ij}\,| \;/\; n \qquad (9)$$

is the arithmetic mean of the actual positions $p_{ij}\,|$ in the n unidirectional approaches to the nominal target position p_{ni}, f_t is a correction factor to compensate for thermal expansion of the positioning scale, f_a is a correction factor to compensate for possible misalignment of the measurement system, c_s is a correction associated with the measurement system, and c_r is a correction associated with the resolution of the axis scale.

Replacing (7) and (8) in (6), using the LPU, assuming that all input quantities are independent, and recognizing that no uncertainty should be attached to the given nominal positions, we obtain

$$u^2[x_i\,|] = (f_t\, f_a\, u[p_{mi}\,|])^2 + (f_a\, p_{mi}\,|\; u[f_t])^2 + (f_t\, p_{mi}\,|\; u[f_a])^2 + u^2[c_r] + u^2[c_s]. \qquad (10)$$

Finally, the model for the mean bidirectional positional deviation at position i is

$\overline{x_i} = ½ (x_i↑ + x_i↓)$. (11)

The uncertainty associated with this quantity is then

$u^2[\overline{x_i}] = ¼ (u^2[x_i↑] + u^2[x_i↓] + 2\, u[x_i↑]\, u[x_i↓])$. (12)

This last expression assumes a correlation coefficient between $x_i↑$ and $x_i↓$ equal to one. This is reasonable, since all influence quantities that are likely to affect $x_i↑$, such as barometric pressure, ambient temperature and humidity, are likely to affect $x_i↓$ in the same sense.

3.2 Type A uncertainties

According to the GUM, the squares of the standard uncertainties associated with the unidirectional position means $p_{mi}|$ should be evaluated as the sample variances divided by the number n of unidirectional measurements. Thus, we set

$u^2[p_{mi}|] = S_i|^2 / n$ (13)

where

$S_i|^2 = \sum_j (p_{ij}| - p_{mi}|)^2 / (n - 1)$. (14)

Note the difference between the sample variances $S_i|^2$ and $s_i|^2$ in equation (3). The former uses the actual positions $p_{ij}|$ while the latter uses the deviations $x_{ij}|$ If the measuring instrument indicates directly the deviations, the actual positions must be obtained as $p_{ij}| = x_{ij}| + p_{ni}$.

3.3 Type B uncertainties

The type A evaluation of standard uncertainty applies only to quantities that are measured directly several times under repeatability conditions. The uncertainty of input quantities that are measured only once, that are evaluated from models that involve further quantities, or that are imported from other sources, should be evaluated by type B means. In many cases this involves obtaining the uncertainty as the standard deviation of the probability density function (pdf) that is assumed to apply to the quantity involved.

Consider first the nominal differential expansion (NDE) correction. The model for this correction is derived from the equation that defines the linear coefficient of thermal expansion α. This equation is

$L - L_o = L_o\, \alpha\, (T - T_o)$ (15)

where, in this case, L_o is the position p_{oi} at temperature T_o = 20 °C and L is the measured position $p_{mi}|$ at the temperature of the test. Therefore, the thermal correction factor is

$f_t = (1 + \alpha\, \Delta T)^{-1}$ (16)

where $\Delta T = T - 20$ °C. Application of the LPU then gives

$u^2[f_t] = (\Delta T^2\, u^2[\alpha] + \alpha^2\, u^2[\Delta T])\, f_t^4$. (17)

The temperature T should be measured at at least two places along the scale and at various times, for example, at the beginning and at the end of the cycle. Doing this allows to establish a range of values for ΔT. This range should be increased by including the expanded uncertainty of the temperature measurement device. In this way one obtains a range w_T over which a uniform pdf is a reasonable assumption. Thus, from Equation 3.19 in [6] we set

$u^2[\Delta T] = w_T^2 / 12$. (18)

The coefficient α is normally not measured; its value is imported from the manufacturer of the machine or from tabulated handbook values. The standard uncertainty $u[\alpha]$ should then be evaluated from a uniform pdf whose width, w_α, has to be assigned depending on how well one knows the material and on how well that material's coefficient is known. In other words, w_α corresponds to twice the maximum possible error in the assignment of the value of α. With this assumption we get

$u^2[\alpha] = w_\alpha^2 / 12$. (19)

Consider next the alignment correction factor f_a. This is modeled as

$f_a = \cos\theta$ (20)

where θ is the angle between the actual measurement direction and the axis direction. Application of the LPU gives:

$u[f_a] = \sin\theta$. (21)

However, normally the best estimate for the misalignment angle will be $\theta = 0$, yielding $f_a = 1$ and $u[f_a] = 0$ irrespective of the uncertainty associated with θ. This is an unreasonable result, and is due to the strong non-linearity of $\cos\theta$ in the vicinity of $\theta = 0$. Therefore, we recommend to estimate the uncertainty of f_a from:

$$u^2[f_a] = ½\,(f_a \cos\theta_{max} + 1) - f_a^{\,2} \qquad (22)$$

where θ_{max} is an upper bound for the maximum deviation angle, and where we must use:

$$f_a = \sin\theta_{max} / \theta_{max}. \qquad (23)$$

The angle θ_{max} has to be established based on the particulars of the measurement system. Since normally this angle should be small, $\cos\theta_{max} \approx 1$ and $u[f_a] \approx 0$. Equations (22) and (23) are derived in [7].

The evaluation of the resolution correction is straightforward. Its value is equal to zero and its standard uncertainty is obtained from a uniform pdf of width equal to the resolution r of the axis scale. Thus:

$$u^2[c_r] = r^2/12. \qquad (24)$$

The measurement system correction c_s depends on the instrument that is used to perform the measurements. An appropriate model for this correction is:

$$c_s = c_{sr} + c_c + c_e \qquad (25)$$

where c_{sr} is a resolution correction, c_c is a calibration correction and c_e is a correction arising from use of the instrument at non standard environmental conditions. From this model we get:

$$u^2[c_s] = u^2[c_{sr}] + u^2[c_c] + u^2[c_e]. \qquad (26)$$

The value of c_{sr} is zero, and its uncertainty is obtained as $u^2[c_{sr}] = r_s^{\,2} / 12$, where r_s is the resolution of the measurement system. Since this resolution should normally be much smaller than r, the uncertainty $u[c_{sr}]$ can be neglected.

The value and uncertainty of c_c should be obtained from the calibration certificate of the instrument, thus providing the traceability for the calibration of the machine.

Finally, if the instrument performs environmental corrections automatically, the value of the correction c_e may be taken as zero. However, the uncertainty $u[c_e]$ is normally not given in the calibration certificate of the measuring system. A reasonable assumption is to take this uncertainty as being proportional to the nominal position p_{ni} and to the temperature difference $\Delta T_e = T_e - T_o$, where T_e is the ambient temperature. In other words, we may write $u[c_{ei}] = K\ p_{ni}\ \Delta T_e$, where the value for the proportionality constant K should be established based on the characteristics of the measurement system.

4. EXAMPLE

We calculate the uncertainty associated with the mean bidirectional deviations reported in the example given in ISO 230-2. The example refers to the test of a linear axis at $m = 11$ positions. Figure 1, showing the final results, was obtained as follows.

The test report given in ISO 230-2 mentions that NDE correction was performed, but it does not state the temperature difference used. It indicates a minimum temperature of 21.8 °C at the start of the test and a maximum temperature of 23.1 °C at its end. Therefore we assumed an average temperature T = 22.45 °C and a temperature difference ΔT = 2.45 °C. For the uncertainty of ΔT we took w_T = (23.1 − 21.8) °C = 1.3 °C. Since this temperature spread is likely to be much greater than the uncertainty of the thermometer, we ignored the latter. This gave $u[\Delta T]$ = 0.38 °C.

The thermal expansion coefficient is reported as being $\alpha = 11\times10^{-6}$ °C^{-1}. For the associated uncertainty we used $w_\alpha = 0.2\alpha$, that is, we assumed a maximum error of 10 % of the nominal value to each side. This gave $u[\alpha] = 0.635\times10^{-6}$ °C^{-1}.

For the maximum misalignment angle we used $\theta_{max} = 0.1°$, giving $u\,[f_a] = 0.45\times10^{-6}$. For consistency, in (10) we used the value f_a = 0.99999949 obtained from (23). However, to evaluate the deviations $x_i\!\uparrow$ and $x_i\!\downarrow$ we took $f_a = 1$ in order not to modify the measured values given in the example of ISO 230-2.

From the nominal positions in the example we assumed that the scale resolution was $r = 1$ μm. This gave $u[c_r] = 0.289$ μm.

The example does not give details on the measurement system used to obtain the data. We assumed that a laser interferometer was used with $r_s = 0.01$ μm resolution, giving $u[c_{sr}] = 0.0029$ μm. We assumed also that the calibration certificate of the interferometer states a negligible correction c_c at standard conditions with a typical relative expanded uncertainty $U[c_c] = 2\times10^{-7}$ and a coverage factor $k = 2$. We then took $u[c_{ci}] = p_{ni}\times10^{-7}$, giving $u[c_c] = 0.175$ μm at nominal position 1750.92 mm.

Finally, the ambient temperature is reported as being 20.6 °C at the start of the test and 20.9 °C at the end. These temperatures provide only an indication of the environmental conditions during the test, but are not used for NDE correction. In our model we used $c_e = 0$, $T_e = 20.75$ °C, $\Delta T_e = 0.75$ °C and $K = 0.05$ °C^{-1}. These values gave a maximum uncertainty $u[c_e] = 0.066$ μm at nominal position 1750.92 mm.

5. CONCLUSIONS

ISO 230-2 standardizes the various parameters that characterize the behavior of machine tools. However, it does not address the calculation of uncertainty, which limits its use as reference for issuing calibration certificates and for verifying the machine based on the test method. In this paper we have presented a detailed procedure to evaluate the uncertainty in the calibration of the positional deviations of linear axes of numerically controlled machine tools. The method complies strictly with the GUM´s recommendations: it is derived from a model for the measurand, whose uncertainty is obtained through application of the law of propagation of uncertainties together with type A and B evaluation methods.

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REFERENCES

[1] ISO 230-2. Test Code for Machine Tools – Part 2: Determination of Accuracy and Repeatability of Positioning Numerically Controlled Axes. Geneva: International Organization for Standardization, 1997.
[2] ISO Guide to the Expression of Uncertainty in Measurement. Geneva: International Organization for Standardization, 1993 and 1995.
[3] EA-4/02 Expression of the Uncertainty of Measurement in Calibration. European Co-operation for Accreditation, 1999.
[4] ISO 14253-1. Geometrical product specification (GPS) - Inspection by measurement of workpieces and measuring instruments - Part 1: decision rules for proving conformance or non-conformance with specifications. Geneva: International Organization for Standardization, 1998.
[5] Lira I. “A Bayesian approach to the consumer´s and producer´s risks in measurement”. Metrologia, 36, 1999, 397-402.
[6] Lira I. Evaluating the Uncertainty of Measurement: Fundamentals and Practical Guidance. Bristol: Institute of Physics Publishing, 2002.
[7] Lira I. and Cargill G. “Uncertainty analysis of positional deviations of CNC machine tools”. *Precision Engineering*, 28, 2004, 232-239.

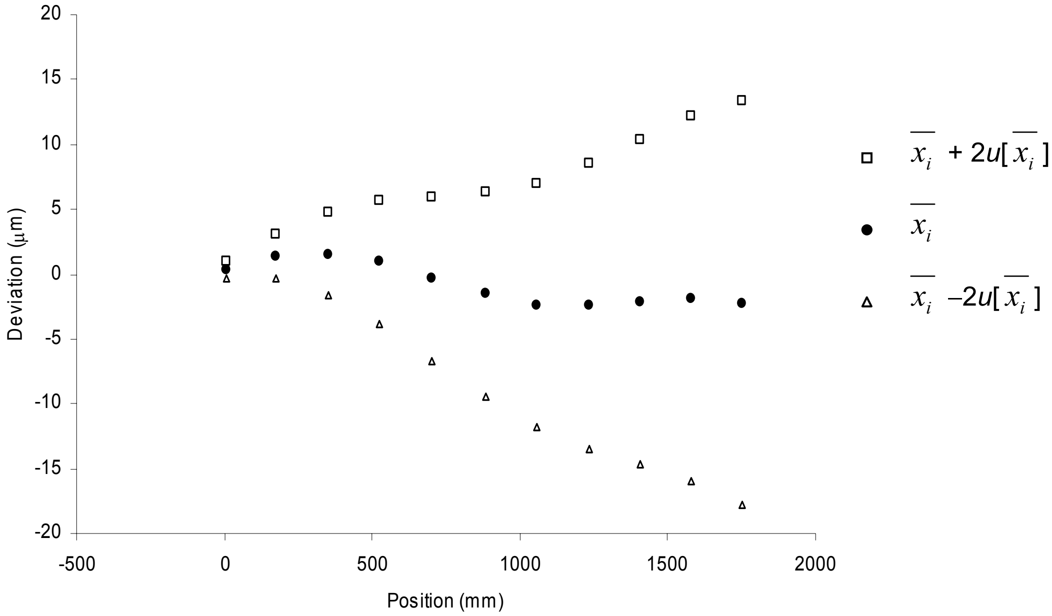

Figure 1. Mean bidirectional positional deviations and expanded uncertainties for the example of ISO 230-2.