AN ALGORITHM FOR ACCURATELY ESTIMATING THE HARMONIC MAGNITUDES OF PERIODIC ARBITRARY SIGNALS USING ASYNCHRONOUS SAMPLING

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Abstract: A sampling algorithm that uses a high-resolution voltmeter for accurately measuring the magnitudes of the harmonics of low-frequency, arbitrary voltage signals is presented. The technique is an extension of a previously described approach. The uncertainties associated with the magnitude estimates relative to the fundamental depend on signal stability, harmonic content, and noise variance, and are less than 1.3×10^{-5} for signals with up to 64 harmonics. The differences between computed and measured values suggest that stable, digitally-synthesized signal generators can be used as calculable standards of harmonic distortion with an accuracy of less than 6 parts in 10^5 relative to the fundamental.

1. INTRODUCTION

The Swerlein´s algorithm (as it is now called in industry) was developed for the accurate measurement of RMS voltage at low frequencies using a digital voltmeter [1]. However, the discussed principles can also be extended to other digitizing instruments. The voltmeter is used in the DC voltage mode in which the voltage signal is directly applied to the input of a high-resolution integrating A/D converter (IADC). The algorithm has been exhaustively tested [2], [3]. An evaluation of uncertainty in measurement according to the ISO *GUM* [4] was recently published [5].

An extension of Swerlein's algorithm for accurately measuring the magnitudes of the harmonics of a low-frequency, low-distortion, voltage signal was already published [6]. It is shown in [7] that this extension allows one to obtain results with asynchronous sampling that numerically approach those obtained using synchronous sampling [8]. The contribution of this paper is to further extend the algorithm version based on discrete Fourier transforms described in [6] for accurately measuring the harmonic magnitudes of periodic nonsinusoidal signals. The algorithm fits the parameters of a truncated Fourier series to noisy discrete time readings. The sampling parameters are evaluated as in [1], thus avoiding the problems related to the frequency accuracy and to the choice of the best operating setting of the converter that were pointed out in [9].

2. MODEL

It is assumed that the noise level is low enough so that the fundamental frequency f_0 can be known from an internal DVM command or from the number of zero crossings of the signal. The fundamental frequency is used as an input to an algorithm that computes the sampling period t_{samp} and the number of samples N in a burst, so that $N{\cdot}t_{samp}$ approaches an integer number of signal periods and t_{samp} attends the sampling theorem for the specified number of signal harmonics [1]. A total of n bursts are taken. The internal level trigger of the DVM is used to start each burst, delayed by kt_D (k = 0, …, n–1) from a signal null-crossing. It is assumed that each burst can be modeled by

$$\mathbf{y}_k = \mathbf{W}_k \mathbf{x}\,, \tag{1}$$

where $\mathbf{y}_k = (y_{1k}, \ldots, y_{Nk})'$ is the data vector at the k-th burst, $\mathbf{W}_k$ is the known $N \times 2m$ matrix with (i, j)-th element $\cos 2\pi j f_0(t_i+kt_D)$ for j = 1, …, m and $\sin 2\pi(j-m)f_0(t_i+kt_D)$ for j = m+1, …, $2m$, $\mathbf{x}$ is the $2m$-vector of fitting parameters (uncorrected for the systematic effects), jf_0 is the j-th harmonic (j = 1, …, m) of the known constant fundamental frequency f_0, and m is the specified number of harmonics of the truncated Fourier series. The model assumes that the signal is stationary ($\mathbf{x}$ is the same for all bursts), the data set has a zero mean value (any nonzero average value has been subtracted from the data) and that the uncertainty associated with the time quantities can be disregarded.

The best estimate of **x**, assuming that the variance of the Gaussian uncorrelated noise superimposed on the samples is the same over all bursts, is [7]

$$\mathbf{x} = \left(\sum_{k=0}^{n-1} \mathbf{F}_k \right)^{-1} \sum_{k=0}^{n-1} \mathbf{W}_k' \mathbf{y}_k \tag{2}$$

where $\mathbf{F}_k = \mathbf{W}_k'\mathbf{W}_k$.

The error matrix $\Lambda_k = \mathbf{F}_k - (N/2)\mathbf{I}_{2m}$ for each value of kt_D where $\mathbf{I}_{2m}$ is the identity matrix of order $2m$, can be nullified if one increases the measurement time $N \cdot t_{samp}$. However, in order to keep the latter at a reasonable value, the algorithm tries to make $N \cdot t_{samp}$ nearly equal to an integer number of periods, so that $\mathbf{F}_k$ becomes nearly diagonal. The diagonal elements of Λ_k at each burst are bounded by the smaller of $1/4N$ or $q/4t_{samp}$ [1][7], where q is the time base resolution of the integrating A/D converter (IADC).

The algorithm designs the experiment so that the matrix resulting from the first summation in (2) is diagonalized. If one chooses t_D nearly equal to $1/nf_0$ and the average of $\mathbf{F}_k$ over all bursts is evaluated, the "frozen" error matrices Λ_k will be cancelled. For instance, for a 60-Hz signal with m = 42, it was verified that all elements of the matrix $\langle \Lambda_k \rangle$, where $\langle \cdot \rangle$ denotes the average over all bursts, were less than 4 parts in 10^8, with the majority being one order of magnitude less than that. (Since the frequencies in the diagonal elements of Λ_k are $2jf_0$ for $j = 1, \ldots, m$ and $2(j-m)f_0$ for $j = m+1, \ldots, 2m$, the sampling theorem implies a maximum value for t_D of $1/4mf_0$, i.e., a minimum number of $4m$ bursts, if the waveform is to be sampled over a period.). Thus, the average of $\mathbf{F}_k$ over all bursts approaches $(N/2)\mathbf{I}_{2m}$ and therefore the estimate (2) approaches the average of the discrete Fourier transforms over all bursts, i.e.,

$$\mathbf{x} = \frac{2}{nN} \sum_{k=0}^{n-1} \mathbf{W}_k' \mathbf{y}_k \tag{3}$$

The covariance matrix associated with the estimate (3) is a diagonal matrix of order $2m$ with diagonal element, for large $nN - 2m$, [7]

$$u^2(\mathbf{x}) = \frac{2}{nN(nN-2m)} \cdot \left\{ \sum_{k=0}^{n-1} \mathbf{y}_k' \mathbf{y}_k - \frac{nN}{2} \mathbf{x}'\mathbf{x} \right\} \tag{4}$$

As shown elsewhere [1][2],[5]-[9], the DVM input stages and nonideal sampling introduce several systematic errors that need to be corrected. After being applied to the DVM input terminals, the signal is conducted to a passive signal conditioner and an active amplifier. The frequency response correction of these input stages and the uncertainty associated with the estimate are, respectively, [7]

$$k_{\mathrm{bw}}(jf_0) = \left(1 + (jf_0/bw)^2\right)^{1/2}, \qquad j = 1, \ldots, m \tag{5a}$$

$$u(k_{\mathrm{bw}}(jf_0)) = \frac{(jf_c)^2}{\left[1 + (jf_c)^2\right]^{1/2}} \frac{u(bw)}{bw} \tag{5b}$$

where $f_c = f_0/bw$ and bw, $u(bw)$ are, respectively, the bandwidth of the input stages and the associated uncertainty both provided by the manufacturer [1].

Finally, the signal is applied to the IADC. The IADC frequency response correction and the uncertainty associated with the estimate are, respectively, [7]

$$k_{\mathrm{int}}(jf_0) = \left(\frac{\sin \pi jf_0 t_{\mathrm{aper}}}{\pi jf_0 t_{\mathrm{aper}}} \right)^{-1}, \qquad j = 1, \ldots, m \tag{6a}$$

$$u(k_{\mathrm{int}}(jf_0)) = \frac{\omega_j^2 t_{\mathrm{aper}} \cos \omega_j t_{\mathrm{aper}} - \omega_j \sin \omega_j t_{\mathrm{aper}}}{\sin^2 \omega_j t_{\mathrm{aper}}} \cdot u(t_{\mathrm{aper}}) \tag{6b}$$

where $\omega_j = \pi jf_0$, t_{aper} is the aperture time (see section III), and $u(t_{aper})$ is evaluated from the uncertainty associated with the IADC time base resolution [1][5].

The IADC uses an internal dc voltage reference that has to be periodically calibrated against an external dc reference standard. The correction estimate k_{dc} is obtained from the DVM dc voltage mode calibration certificate with an uncertainty $u(k_{dc})$. The manufacturer states the linearity error limits $[-c, c]$ for each voltage range and calibration period. The 24-h basic accuracy is chosen in this paper. The correction estimate k_L is unity with an uncertainty $c/\sqrt{3}$. The basic 24-h accuracy discussed above is based on an aperture time of 1 s or greater. For shorter apertures, the accuracy is reduced. The limits of the gain error $[-d, d]$ as a percentage of the reading are provided by the manufacturer. The correction estimate k_G is unity with an uncertainty $d/\sqrt{3}$. For the DVM used in this paper, the error limit d was modelled as [1]

$$d = \min(0.002/t_{\mathrm{aper}}, 30) \quad \mu V/V, \quad t_{\mathrm{aper}} \geq 100\ \mu s$$
$$d = 10 + 0.0002/t_{\mathrm{aper}} \quad \mu V/V, \quad 1\ \mu s < t_{\mathrm{aper}} < 100\ \mu s \tag{7}$$

The algorithm for calculating the sampling parameters is structured to guarantee immunity against aliasing only for the minimum number of harmonics m specified by the user [1]. The maximum sampling period should not violate the sampling theorem, i.e., $t_{\mathrm{samp}} < 1/2mf_0$. If it does, the algorithm increases the sampling frequency in order to ensure the alias occurrence exactly at mf_0. The algorithm takes this into account and estimates the aperture time as $t_{\mathrm{aper}} = t_{\mathrm{samp}} - \tau$, where τ (= 30 μs) is a small delay specified by the manufacturer [1] to prevent trigger-too-fast errors. Due to the above bandwidth requirement the actual aperture time is kept almost constant around 0.001 s up to $m = 1/2f_0t_{\mathrm{samp}}$ where it starts decreasing with increasing signal frequency as $t_{\mathrm{aper}} = 1/2mf_0 - \tau$. If m is very high, the aperture time will be very small, and this will cause the voltmeter accuracy to be degraded (the uncertainty contribution associated with the IADC gain error correction increases with decreasing aperture time). However, this contribution may be neglected if one is interested in the values of the harmonic magnitudes relative to the fundamental.

The IADC has two inputs: a low-speed input and a high-speed one [10]. The aperture time is used to select between the two. The threshold is 100 μs and is reached at mf_0 = 1/0.00026 Hz. When making measurements with 6½-digit resolution or above, the input voltage is applied to the low-speed input. For a 60-Hz signal, this amounts to choosing $m \leq 64$. This choice is reasonable when the signal does not present significant harmonic magnitudes for $m > 64$. The algorithm can operate with fundamental frequencies ranging from 2 Hz to 1 kHz. The uncertainty associated with the frequency response correction (5) is the dominant contribution at high frequencies. For a 1-kHz signal, the 100-μs threshold is reached at m = 3. The higher the fundamental frequency, the less distorted the signal should be in order to be accurately measured.

The measurement time of the algorithm increases roughly proportional to m for aperture times greater than 100 μs and roughly proportional to m^2 for aperture times below this threshold.

Since the uncertainty associated with the average error matrix $\langle \Lambda_k \rangle$ is negligible, the RMS magnitude of the j-th harmonic of a stationary signal, corrected for all known systematic effects, is [7]

$$V_j = k_{\mathrm{dc}} k_{\mathrm{L}} k_{\mathrm{G}} k_{\mathrm{bw}}(jf_0) k_{\mathrm{int}}(jf_0) \cdot \left[\left((\mathbf{x})_j^2 + (\mathbf{x})_{m+j}^2 \right) / 2 \right]^{1/2}$$
$$j = 1, \ldots, m \tag{8}$$

where the symbol $(\cdot)_j$ denotes the j-th element. The elements of $\mathbf{x}$ are uncorrelated. Assuming that the corrections are also uncorrelated, the uncertainty associated with the magnitude of the j-th harmonic is [7]

$$u(V_j) \approx \left\{ V_j^2 \cdot \left[u^2(k_{\mathrm{dc}}) + u^2(k_{\mathrm{L}}) + u^2(k_{\mathrm{G}}) + u^2(k_{\mathrm{bw}}(jf_0)) + u^2(k_{\mathrm{int}}(jf_0)) \right] + u^2(\mathbf{x})/2 \right\}^{1/2} \tag{9}$$

where the noise variance contribution is in general dominant for $j = 2, \ldots, m$ (for signals with small and monotonically decreasing harmonic magnitudes relative to the fundamental). The uncertainty associated with the magnitude d_j of the j-th harmonic ($j = 2, \ldots, m$) relative to the fundamental V_1 is

$$u(d_j) \approx \left\{ d_j^2 \left[u^2(k_{\mathrm{bw}}(jf_0)) + u^2(k_{\mathrm{bw}}(f_0)) + u^2(k_{\mathrm{int}}(jf_0)) + u^2(k_{\mathrm{int}}(f_0)) \right] + (1 + d_j^2) \cdot u^2(\mathbf{x}) / 2V_1^2 \right\}^{1/2} \tag{10}$$

where again the noise variance contribution is in general dominant.

The above analysis assumes that the actual frequency of the sampled waveform is stable and known (the time base accuracy is not important). The sensitivity of the output to frequency errors was evaluated by numerically inserting errors into the algorithm and checking the output for any variation. No difference was detected at the estimates for V_j when the errors were within $\pm 10^{-5}$. Besides, no difference was detected at the estimates for d_j when the errors were within $\pm 10^{-4}$. An uncertainty within these limits is easily attainable by the frequency measurement methods previously mentioned. In any case, the noise variance (4) increases with increasing frequency error, yielding more conservative uncertainty estimates.

3. RESULTS

Generators of calibrated harmonics have been reported in the literature [11][12]. However, the evaluation of the algorithm described above requires a standard of harmonic distortion with accuracy and stability at the level of a few parts in 10^6. Therefore, a stable, high-resolution DVM controlled by the algorithm was used instead to measure the harmonic magnitudes of periodic signals generated by a stable, digitally-synthesized, arbitrary signal generator (also used in [6]). The latter synthesizes the signals in a staircase or zero-order-hold approximation. The values stored in memory are equally-spaced samples of these signals with 2048 discrete steps per period with 12-bit amplitude resolution. The shape of each signal is specified mathematically.

The computation of the complex-exponential Fourier coefficients for piecewise linear waveform functions is straightforward [13]. The technique is to differentiate the function until the first occurrence of impulse functions. The Fourier coefficients of the resulting impulse train are determined and the result divided by $2\pi m f_0\sqrt{-1}$ to return to the Fourier coefficients of the original function. One should be careful to include all steps (even the step between the end of a period and the start of the next) of the staircase waveform function when computing the Fourier coefficients.

Three 60-Hz nonsinusoidal signals in the 10 V range were synthesized and separately applied to the DVM input: (a) a sinusoidal signal (staircase version), (b) an alternating parabolic signal, and (c) a half-wave rectified signal (with adequate dc component to provide the necessary signal null-crossing). The Fourier coefficients relative to the fundamental were numerically evaluated for each signal and compared with the algorithm output. The stability figures provided by the algorithm were the same as those reported in [1]. The staircase waveform functions and the results obtained are described below.

3.1 SINUSOIDAL SIGNAL (STAIRCASE)

The ideal waveform is

$$f(t) = \sin\left(\frac{2\pi}{T}t\right) \qquad (11)$$

and the synthesized waveform is

$$f(i) = \sin\left(\frac{2pi}{2047}\right), \qquad i = 0, \ldots, 2047 \qquad (12)$$

where p = 3.1415. The algorithm took about 40 s to evaluate the magnitudes of the first 8 (eight) harmonics. The reported total harmonic distortion (THD) was 0.0846%. The algorithm selected n = 32 bursts of N = 429 samples spaced by t_{samp} = 0.001049 s (with t_{aper} = 0.001019 s). The fundamental magnitude was measured with an uncertainty of 2.9 μV V^{-1}. The harmonic magnitudes relative to the fundamental were measured with an uncertainty of 2.4×10^{-6}. The differences between computed and measured values were less than 3.8×10^{-5} relative to the fundamental (Table I).

Table I *Computed and measured values for a sinusoidal signal (staircase version).*

Harm. Nº	Magnitude (%)		$u(d_i)$ (10^6)	Error (10^6)
	Comp.	Meas.(d_i)		
1	100.000	100.000	-	-
2	0.06123	0.06165	2.4	4
3	0.03443	0.03060	2.4	˜38
4	0.02449	0.02497	2.4	5
5	0.01913	0.02023	2.4	11
6	0.01574	0.01554	2.4	˜2
7	0.01339	0.01494	2.4	15
8	0.01166	0.01088	2.4	˜8

3.2 ALTERNATING PARABOLIC SIGNAL

The ideal waveform is

$$f(t) = \begin{cases} 4t(\pi - t)/\pi^2, & 0 < t < \pi \\ 4t(\pi + t)/\pi^2, & -\pi < t < 0 \end{cases} \qquad (13)$$

and the synthesized waveform is

$$f(i) = \begin{cases} 8 \cdot [(i-1023)/2047] \cdot [1 + 2(i-1023)/2047] \\ \qquad i = 0, \ldots, 1023 \\ 8 \cdot [(i-1024)/2047] \cdot [1 - 2(i-1024)/2047] \\ \qquad i = 1024, \ldots, 2047 \end{cases} \qquad (14)$$

The algorithm took about 1 min to evaluate the magnitudes of the first 13 harmonics. The reported THD was 3.7658%. The algorithm selected n = 52 bursts of N = 336 samples spaced by t_{samp} = 0.0006449 s (with t_{aper} = 0.0006149 s). The fundamental magnitude was measured with an uncertainty of 2.7 μV V^{-1}. The harmonic magnitudes relative to the fundamental were measured with an uncertainty of 1.6×10^{-6}. The differences between computed and measured values were less than 3.1×10^{-5} relative to the fundamental (Table II).

Table II *Computed and measured values for an alternating parabolic signal.*

Harm. Nº	Magnitude (%)		$u(d_j)$ (10^6)	Error (10^6)
	Comp.	Meas.(d_j)		
1	100.000	100.000	-	-
2	0.06019	0.06009	1.6	~2
3	3.66799	3.67116	1.6	31
4	0.03010	0.02957	1.6	~5
5	0.77685	0.77588	1.6	~9
6	0.02006	0.02028	1.6	2
7	0.27468	0.27328	1.6	~14
8	0.01505	0.01493	1.6	~1
9	0.12395	0.12514	1.6	12
10	0.01204	0.01277	1.6	7
11	0.06427	0.06615	1.6	19
12	0.01003	0.00966	1.6	~4
13	0.03630	0.03889	1.6	26

3.3 HALF-WAVE RECTIFIED SIGNAL

The ideal waveform is

$$f(t) = \begin{cases} -0{,}1 + \sin(t), & 0 < t < \pi \\ -0{,}1, & \pi < t < 2\pi \end{cases} \tag{15}$$

and the synthesized waveform is

$$f(i) = \begin{cases} -0.1 + \sin(2pi/2047), & i = 0,\ldots,1022 \\ -0.1, & i = 1023,\ldots,2047 \end{cases} \tag{16}$$

The algorithm took about 6.35 min to evaluate the magnitudes of the first 64 harmonics. The reported THD was 43.5645%. The algorithm selected n = 256 bursts of N = 168 samples spaced by t_{samp} = 0.0001303 s (with t_{aper} = 0.0001003 s). The fundamental magnitude was measured with an uncertainty of 13 μV V^{-1}. The harmonic magnitudes relative to the fundamental were measured with an uncertainty of less than 5.9×10^{-6}. The differences between computed and measured values were less than 6.0×10^{-5} relative to the fundamental (Table III).

4. DISCUSSION

It was verified that the harmonic magnitudes of low-noise arbitrary signals can be measured with an uncertainty of less than 13 μV V^{-1}. The maximum number of harmonics that can be accurately measured depends on the signal fundamental frequency, being 64 for a 60-Hz signal (observe that the aperture time for m = 64 is close to the threshold t_{samp} = 100 μs) and 3 for a 1-kHz signal.

The differences between computed and measured values suggest that stable, digitally-synthesized signal generators can be used as a calculable standard of harmonic distortion with an accuracy of less than 6 parts in 10^5 relative to the fundamental.

5. CONCLUSIONS

It was shown that an algorithm based on discrete Fourier transforms and Swerlein´s algorithm can be used to measure the magnitudes of the harmonics of low-noise arbitrary signals at low frequencies. Periodic nonsinusoidal signals were synthesized by a commercial source and measured by the algorithm. The differences between computed and measured values suggest that stable, digitally-synthesized signal generators can be used as calculable standards of harmonic distortion.

REFERENCES

[1] R. L. Swerlein., "A 10 ppm accurate digital ac measurement algorithm". In: *Proc. NCSL*, pp. 17-36, Albuquerque, USA, Aug. 1991.

[2] M. Kampik, H. Laiz, M. Klonz, "Comparison of three accurate methods to measure ac voltage at low frequencies", *IEEE Trans. Instrum. Meas.*, **49**, No. 2, pp. 429-433, Apr. 2000.

[3] R. Lapuh, A. Arnsek, I. Visocnik, Z. Svetik, "Evaluation of low frequency ac voltage measurement using integrating sampling technique", *IMTC Digest*, 1999.

[4] ISO/BIPM, *Guide to the Expression of Uncertainty in Measurement*, Geneva: International Organization for Standardization, 1995.

[5] G.A. Kyriazis, R.L. Swerlein, "Evaluation of uncertainty in ac voltage measurement using a digital voltmeter and Swerlein´s algorithm", *CPEM Digest*, Ottawa, Canada, June 16-21 2002. Extended version available from the authors.

[6] G.A. Kyriazis, "Extension of Swerlein's algorithm for ac voltage measurement in the frequency domain", *IEEE Trans. Instrum. Meas.*, **52**, no. 2, pp. 367-370, Apr. 2003.

[7] G.A. Kyriazis and M.L.R. Campos, "Bayesian inference of linear sine-fitting parameters from integrating digital voltmeter data", *Meas. Sci. Tech.*, **15**, pp. 337-346, Feb. 2004.

[8] W. Kürten Ihlenfeld, E. Mohns, H. Bachmair, G. Ramm and H. Moser, "Evaluation of the

synchronous generation and sampling technique", *IEEE Trans. Instrum. Meas.*, **52**, no. 2, pp. 371-374, Apr. 2003.

[9] U. Pogliano, "Precision Measurement of ac voltage below 20 Hz at IEN", *IEEE Trans. Instrum. Meas.*, **46**, no. 2, pp. 369-372, Apr. 1997.

[10] W.C. Goecke, "An 8½-digit integrating analog-to-digital converter with 16-bit, 100,000-sample-per-second performance", *Hewlett-Packard Journal*, Apr. 1989.

[11] R. Arseneau, P. Filipski, "Portable and stable source of ac voltage, current and power", *IEEE Trans. Instrum. Meas.*, **44**, no. 2, pp. 433-435, Apr. 1995.

[12] U. Pogliano, "Tracking generator of calibrated harmonics", *IEEE Trans. Instrum. Meas.*, **51**, no. 4, pp. 636-639, Aug. 2002.

[13] C.R. Paul, *Introduction to Electromagnetic Compatibility*, Wiley, pp. 352-359, 1992.

***Table III** Computed and measured values for a half-wave rectified signal.*

Har. Nº	Magnitude (%)		$u(d_i)$	Error	Har. Nº	Magnitude (%)		$u(d_i)$	Error
	Comp.	Meas.(d_i)	(10^6)	(10^6)		Comp.	Meas.(d_i)	(10^6)	(10^6)
1	100.000	100.000	-	-	33	0.00282	0,00314	5.4	3
2	42.4838	42.4782	5.9	˜56	34	0.11057	0,11013	5.4	˜4
3	0.03444	0.02846	5.4	˜60	35	0.00266	0,00292	5.4	3
4	8.49489	8.49689	5.4	20	36	0.09865	0,09816	5.4	˜5
5	0.01914	0.01927	5.4	13	37	0.00252	0,00264	5.4	1
6	3.64069	3.63908	5.4	˜16	38	0.08856	0,08755	5.4	˜10
7	0.01340	0.01168	5.4	˜17	39	0.00240	0,00129	5.4	˜11
8	2.02270	2.02127	5.4	˜14	40	0.07994	0,08037	5.4	4
9	0.01034	0.00746	5.4	˜29	41	0.00228	0,00222	5.4	˜1
10	1.28726	1.28748	5.4	2	42	0.07253	0,07250	5.4	0
11	0.00843	0.00526	5.4	˜32	43	0,00218	0,00180	5.4	˜4
12	0.89126	0.89422	5.4	30	44	0,06610	0,06714	5.4	10
13	0.00712	0.00795	5.4	8	45	0,00209	0,00208	5.4	0
14	0.65366	0.65330	5.4	˜4	46	0,06050	0,06171	5.4	12
15	0.00616	0.00522	5.4	˜9	47	0,00200	0,00327	5.4	13
16	0.49992	0.49963	5.4	˜3	48	0,05558	0,05571	5.4	1
17	0.00544	0.00334	5.4	˜21	49	0,00192	0,00317	5.4	13
18	0.39472	0.39656	5.4	18	50	0,05124	0,05017	5.4	˜11
19	0.00487	0.00429	5.4	6	51	0,00185	0,00180	5.4	˜1
20	0.31959	0.32059	5.4	10	52	0,04739	0,04703	5.4	˜4
21	0.00440	0.00289	5.4	˜15	53	0,00179	0,00172	5.4	˜1
22	0.26405	0.26739	5.4	33	54	0,04397	0,04460	5.4	6
23	0.00402	0.00592	5.4	19	55	0,00173	0,00384	5.4	21
24	0.22185	0.22297	5.4	11	56	0,04090	0,03797	5.4	˜29
25	0.00371	0.00557	5.4	19	57	0,00167	0,00067	5.4	˜10
26	0.18902	0.18892	5.4	˜1	58	0,03814	0,03845	5.4	3
27	0.00343	0.00421	5.4	8	59	0,00162	0,00111	5.4	˜5
28	0.16298	0.16295	5.4	0	60	0,03566	0,03573	5.4	1
29	0.00320	0.00385	5.4	7	61	0,00157	0,00094	5.4	˜6
30	0.14199	0.14217	5.4	2	62	0,03341	0,03368	5.4	3
31	0.00300	0.00398	5.4	10	63	0,00152	0,00103	5.4	˜5
32	0.12481	0.12417	5.4	˜6	64	0,03137	0,03219	5.4	8