CRYOGENIC CURRENT COMPARATOR FOR THE MEASUREMENT OF ELECTRICAL RESISTANCE IN THE RANGE OF 10 kΩ TO 1 GΩ

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1. INTRODUCTION

The development of measurement systems of electrical resistance based on the principle of current comparison in the late 1960s [1] has provided some of the best ways to achieve very low measurement uncertainties in the range of 1 Ω to 10 kΩ. Examples of this principle are seen in the commercial versions of current comparator resistance bridges operating at room temperature. Superconducting materials and their application as magnetic shields and in the development of low temperature instrumentation have allowed the development of cryogenic current comparators (CCC) that have greatly surpassed the measurement capabilities of current comparators operated at room temperature [2, 3]. This paper presents a new CCC, developed in a collaboration between the National Institute of Standards and Technolgy (NIST) of the USA, the National Measurement Institute (NMI) of Australia, the Instituto Nacional de Tecnología Industrial (INTI) of Argentina and the Centro Nacional de Metrología (CENAM) of Mexico. The CCC we describe is designed to measure resistance values in the range of 10 kΩ to 1 GΩ [4].

2. PRINCIPLE OF OPERATION

The operating principle of a CCC is based on two fundamental aspects of physics, the first one being Ampere's circuital law and the other the Meissner effect. To describe the operation of a CCC one can start by imagining that there are two conductors that are located inside a long tube made of superconducting material, as shown in Figure 1. In one of these conductors circulates a current I_1 in a certain direction, while the other conductor carries a current I_2 in the opposite direction to the current I_1.

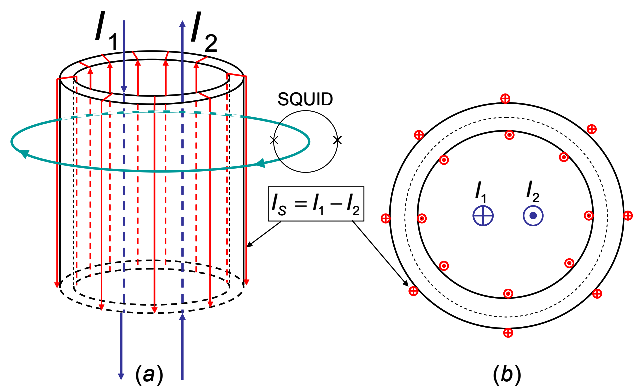

Fig. 1

Moreover, due to the Meissner effect it is known that it is not possible (in type I superconducting materials) for the magnetic field lines generated by the currents that are inside the tube to penetrate into the wall of the tube. As a consequence of the above a surface current I_S is generated on the surface of the superconductor material, which flows in one direction inside the tube and on the outside flows in the opposite direction to preserve the condition imposed by the Meissner effect. If we now apply Ampere's circuital law in a closed path that is located inside the superconductor material, then we can establish that

$$\oint_C H \cdot dl = \sum I = I_1 - I_2 - I_S \tag{1}$$

The integral on the left side of this expression is taken around the closed path *C,* which is represented by the dotted line in Fig 1b. Since the magnetic field is zero everywhere along the path, the current I_S is in reality the difference between the currents I_1 and I_2, and on the outside of the tube a magnetic field exists, that is not affected by the placement of current-carrying wires inside the tube. The external magnetic field is sensed by a SQUID magnetometer (Superconducting Quantum

Interference Device), which is an instrument that offers the best sensitivity to detect very small magnetic fields.

In the construction of a CCC, the application is focused on comparing the resistance values of two resistors of widely different value. This can be achieved if the total current in one direction I_1 is multiplied by an integer number by passing the wire through the tube N_1 times. Similarly the wire that carries the current I_2 can be multiplied by another integer N_2, and then the equation (1) is as follows:

$$\oint_c H \cdot dl = \sum I = N_1 I_1 - N_2 I_2 - I_S = 0 \qquad (2)$$

The multi-turn windings can form a toroid that is enclosed with a superconducting overlapping foil, which is electrically insulated at the overlap so that it appears as snake swallowing its tail [5]. The magnetic flux, produced by the current I_S, is coupled to the SQUID via a flux transformer located at the central window of the toroid. Fig. 2 illustrates the final construction of the CCC.

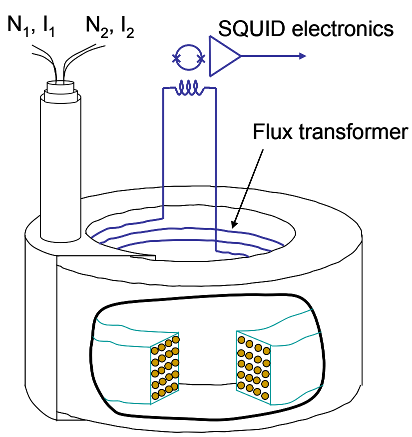

Fig. 2

3. MEASUREMENT SYSTEM

Based on the theory of operation of the CCC, described above, it is possible to address the explanation of the measurement system design for the comparison of resistors in the range of 10 kΩ to 1 GΩ. The high resistance CCC (HRCCC) is equipped with winding ratios to determine resistance values of resistors of 1 MΩ and 10 MΩ directly against the quantized Hall resistance (QHR), which is the international standard for electrical resistance, and also to compare resistance values in ratios of 1 to 1, 10 to 1, and 100 to 1 in the range of 10 kΩ to 1 GΩ. Figure 3 shows the schematic design of the HRCCC.

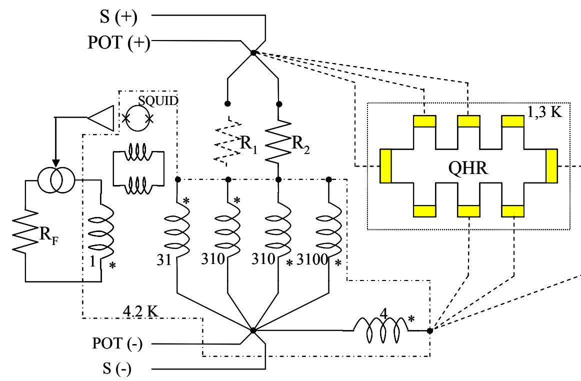

Fig. 3

Not shown in Fig. 3 is a voltage source that supplies current between points S (+) and S (-); this source has the ability to reverse polarity during the measurement process. The measurement voltage V is applied at superconducting “star” connection points located inside the CCC cryostat. Parallel current paths (arms) begin and end at these points; one arm is formed by resistor R_1 in series with a winding of N_1 turns, while the other arm is made up of the resistor R_2 in series with a winding of N_2 turns. The CCC has a winding of 31 turns, two windings of 310 turns, and a winding of 3100 turns. Accordingly, by the choice of these windings measurements can be made in ratios 1 to 1 (310:310), 10 to 1 (310:31 or 3100:310), and 100 to 1 (3100:31).

When determining the value of a resistor R_1 of 1 MΩ or 10 MΩ by comparing directly against the QHR, one of the arms is constituted by the resistor R_1 in series with a winding of 310 turns or 3100 turns, respectively, while the other arm of the system is composed of the i = 2 plateau value of the QHR (12 906.403 5 Ω) in series with a winding of 4 turns. For this case, the Delahaye scheme of multiple connections [6] should be used. When a QHR standard is used for two-terminal measurements with this technique, errors due to lead resistance can be eliminated as it has been shown in [7].

Recalling from the theory of operation of the CCC that the difference between the currents I_1 and I_2 is represented by the screening current I_S flowing on the surface of the superconductor material, the magnetic field produced by this current is first detected by the SQUID magnetometer and used to generate a feedback current (I_F) passing through a third winding N_F of one turn in order to maintain the zero balance condition in the SQUID. The difference in this feedback current for positive and negative source polarities is $2I_F$ and this value is determined by measuring the voltage V_F in a resistor R_F of 200 kΩ. Accordingly it is possible to determine the condition of balance in the CCC as

$$N_1 I_1 = I_2 N_2 + I_F N_F \tag{3}$$

If this expression is modified and defined in terms of voltage and resistance, the final balance condition of the system is determined to be

$$\frac{V}{R_1 + r_1} N_1 = \frac{V}{R_2 + r_2} N_2 + \frac{V_F}{R_F} N_F \tag{4}$$

Where

- V is the voltage measured between points POT (+) and POT (-); the nominal voltages for this case can be 1.1 V, 5 V and 10 V
- N_1 is the number of turns of winding N_1
- R_1 is the resistance value of resistor R_1
- r_1 is the resistance of the winding N_1
- N_2 is the number of turns of winding N_2
- R_2 is the resistance value of resistor R_2
- r_2 is the resistance of the winding N_2
- R_F is the resistance value of resistor R_S
- V_F is the voltage drop across the resistor R_S
- N_F is the number of turns of winding N_S

The resistance of the windings can be avoided if all connections between the resistors are superconducting, as described in [8]; however this is not possible if the resistors are at room temperature. In fact, it is advantageous for there to be resistance in the CCC windings because this can minimize noise amplification near the resonance frequencies of the largest windings, which improves the sensitivity of the system. In the case of determining the resistance of a resistor of 1 MΩ or 10 MΩ directly against the QHR, the 4 turns winding is made of superconducting wire and the balance equation for this case is given by:

$$\frac{V}{R_1 + r_1} N_1 = \frac{V}{R_H} N_2 + \frac{V_F}{R_F} N_F \tag{5}$$

Where the nominal voltages V for this case typically can be 0.5 V, 0.7 V or 1.1 V. R_H is the QHR value (12 906.4035 Ω).

4. HIGH RESISTANCE CCC PERFORMANCE

During the collaboration between the four participating institutions four measurement systems were developed, which have been optimized, compared to each other, as well as compared to other measuring systems. In addition to this, several laboratories have conducted performance tests in their respective institutions. One of the first tests to be carried out was to assess the agreement between measurements when using different ratios and different voltages. In Fig. 4 is shown comparison data obtained when a 10 MΩ resistor was measured at two different test voltages; 10 V and 1.1 V, using as the reference a 1 MΩ resistor. Also shown are the values obtained on the same 10 MΩ resistor, but when using the QHR as a reference at 1.1 V test voltage. The results are in good agreement; also it is possible to observe that the dispersion of each set varies as expected with the level of current and with the noise that is produced by thermal excitations in the reference resistors [9]. The same experiment was repeated with a different resistor and in different time periods; the measurements obtained are shown in Fig. 5.

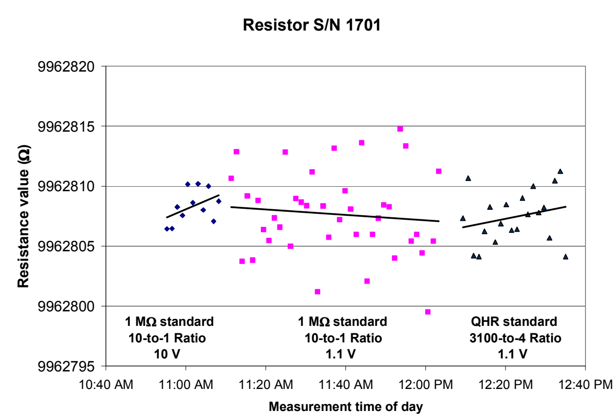

Fig. 4

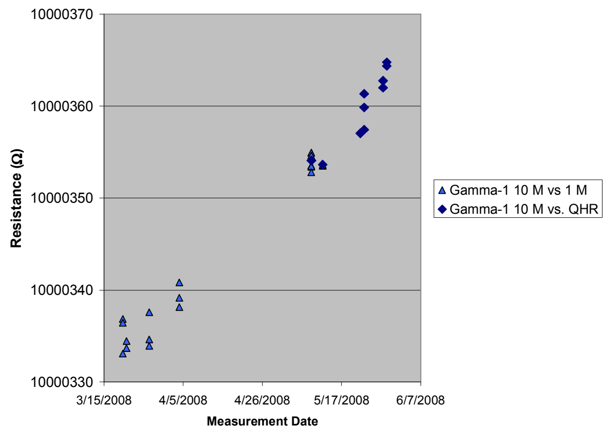

Fig. 5

Another test was the comparison of the measurements of a 1 MΩ resistor, using the HRCCC with the values obtained from 2004 to 2007 using a single ratio CCC developed by Elmquist *et al*. [10]. The results of such comparison can be seen in Fig. 6, where the results of both measurement systems are shown to be in good agreement.

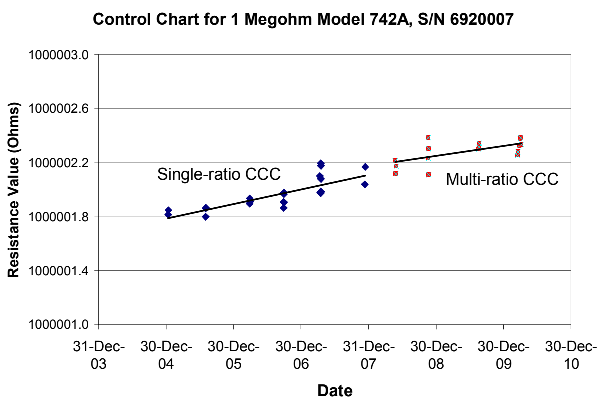

Fig. 6

At the resistance value of 1 GΩ measurements using the HRCCC have been compared with the active arm bridge (AAB) measurement technique [11]. The measurements with the HRCCC were carried out at three different source voltages (1 V, 5 V, and 10 V), all the measurements are shown in Fig. 7 and are in good agreement with those obtained with the AAB at 10 V.

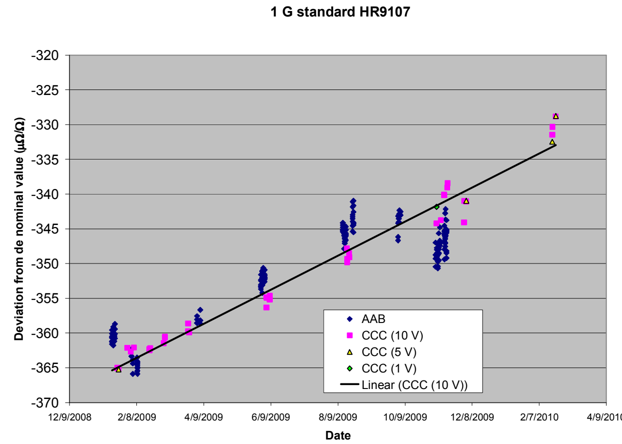

Fig. 7

While the measurement uncertainty that can be achieved with the HRCCC is, in most of the measurement ranges, mainly limited by the SQUID 1/f noise, there are other components that should be taken in to account. The most significant of these are the Johnson noise, leakage currents, winding resistance measurements, and the measurement of the feedback current. A combined standard uncertainty on the order of 0.1 μΩ/Ω for resistors of 100 MΩ and 1 μΩ/Ω for resistors of 1 GΩ with 10 V test voltage are expected for the HRCCC. A detailed discussion about the measurement uncertainties of the high resistance CCC is given in [9].

CONCLUSIONS

A new type of measurement system for dc resistance based on comparison of currents, the high resistance CCC, is presented in this work. This technique has been applied to resistors in the range of 10 kΩ to 1 GΩ, and it has been reproduced in a systematic manner without detriment to the performance, which shows that it is a reliable metrology tool for the measurement range intended.

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