Traceable Mass Determination and Uncertainty Calculation

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ABSTRACT

Numerous basic conditions are specified for weights and their mass comparisons in the OIML R 111-1 International Recommendation and in the ASTM E617-13 Standard Specification for Laboratory Weights and Precision Mass Standards. Based on the maximum permissible errors for the nominal values of weights of different classes, these guidelines also specify the geometric shapes and material characteristics for these weights besides the requirements for performing and evaluating mass comparisons. For instance, these requirements include limits for magnetic properties, surface quality and density of the material used for weights.

As the majority of mass comparisons are performed under atmospheric conditions, weights experience buoyant force, depending on their volume and the air density, where such buoyancy is directed opposite to the downward force exerted by these weights. This systematic effect can be eliminated if the air density and the densities of the materials of the test and reference weights are known. In mass metrology, two methods are commonly used for highly accurate determination of air density. In the first method, the air density is determined by measuring weights of a known mass and calculating the significant density difference. The second method analytically defines air density as a function of various influence quantities based on an updated formula [4]. Air density can be calculated if the temperature, barometric pressure and humidity as the key influence quantities are known. The uncertainties in measuring these climate data result in an air density specification that has a component of uncertainty. In order to determine the uncertainty of an air buoyancy correction, the densities of the test and reference weight materials and the uncertainties of these weights must also be known besides the air density and its uncertainty. Under these aspects, requirements can be placed on the uncertainty of climate data measurements and on the uncertainty of the densities of materials.

Keywords:

air bouyancy correction
uncertainty calculation
application software

1. INTRODUCTION

All models of the Cubis series of mass comparators are standard-equipped with a module for measuring the ambient conditions of air temperature, barometric pressure and relative humidity in order to determine the air density. This module is located within the weighing chamber of mass comparators with a draft shield to determine the environmental conditions actually prevailing at the time of measurement. On Cubis mass comparators without a draft shield, the climate data module is attached on an external stand and/or holder so that the relevant ambient conditions are measured in this case as well.

Figure 1: Cubis MCM2004 / Climate Data Module

The plug-in connector of this module permits easy removal for calibration at regular intervals. The calibration data and associated characteristic curves for correction are stored in the internal memory of the climate data module. As a result, synchronous and metrologically traceable climate data and their measurement uncertainties are available for mass comparisons.

The following standard uncertainties are specified for a calibrated climate module:

Temperature: $u_t = 0.15\ \mathrm{K}$, in measuring range $t = (18 \ldots 24)°\mathrm{C}$

Pressure: $u_p = 1\ \mathrm{hPa}$, in measuring range $p = (800 \ldots 1100)\ \mathrm{hPa}$

Humidity: $u_{hr} = 1\ \%$, in measuring range $hr = (30 \ldots 70)\ \%$

Therefore, this module enables exact determination of the air density synchronous to measurement of mass in order correct for errors resulting from air buoyancy in a mostly automatic way.

In addition, Cubis mass comparators have built-in application software for mass determination. This software guides the user in a clearly structured and exactly timed way throughout the individual steps of mass comparison. A record is automatically generated that shows all relevant information on mass comparison in addition to the result of this comparison. The mass comparator provides a complete uncertainty budget of each mass comparison performed, along with the associated climate data, besides showing the conventional mass of the test weight used.

The following describes the essential fundamentals for determining an uncertainty budget during calibration of weights, as well as the permissible uncertainty contributions of the climate data quantities to an air buoyancy correction. In this description, it is assumed that the conventional mass of the test weight and the uncertainty of the conventional mass are to be determined by calibration of a test weight to a reference weight of higher accuracy.

2. DETERMING THE UNCERTAINTY IN COMPLIANCE WITH OIML R111-1/ ASTM E617-13

The expanded uncertainty of the conventional mass of a test weight $U(m_{\mathrm{ct}})$ is yielded by the expansion factor, or coverage factor, k and the combined standard uncertainty $u_{\mathrm{c}}(m_{\mathrm{ct}})$ according to [1, Equation (abbreviated as "Eq.") C.6.5-3] as follows:

$$U(m_{\mathrm{ct}}) = k \times u_{\mathrm{c}}(m_{\mathrm{ct}}) \tag{1}$$

In this equation, the coverage factor determines the confidence internal. As a rule, a confidence interval of 95% is used, which corresponds to a coverage factor of $k = 2$. The expanded uncertainty of each weight must satisfy the condition below (cf. [1, Eq. 5.2-1]):

$$U(m_{\mathrm{ct}}) \leq \frac{1}{3}\mathrm{MPE} \tag{2}$$

The maximum permissible errors, MPE, dependent on mass and weight class are indicated in [1, Table 1]. Based on the values of this table, the maximum permissible relative errors plotted in Figure 2 were calculated for the different weight classes. The highest requirements are placed on weight class E1 for weights with a nominal mass m_0 greater than 100 g. The maximum permissible relative error for this class is as follows:

$$\frac{\mathrm{MPE}}{m_0} = 0.5 \cdot 10^{-6} \tag{3}$$

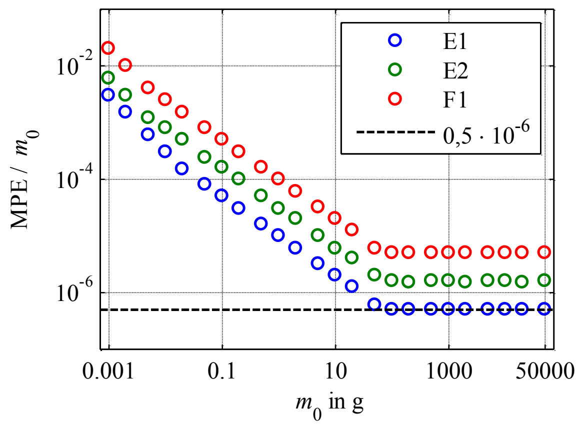

Figure 2: MPE of OIML-compliant Weights from 1 mg to 50 kg

The combined standard uncertainty of the test weight is comprised of the standard uncertainty of the weighing process $u_{\mathrm{w}}(\overline{\Delta m_{\mathrm{c}}})$, the standard uncertainty of the reference weight $u(m_{\mathrm{cr}})$, the standard uncertainty of air buoyancy correction u_{b} and the combined standard uncertainty of the weighing instrument u_{ba}. According to [1, Eq. C.6.5-1], the combined standard uncertainty is calculated using the following formula:

$$u_{\mathrm{c}}(m_{\mathrm{ct}}) = \sqrt{u_{\mathrm{w}}^2(\overline{\Delta m_{\mathrm{c}}}) + u^2(m_{\mathrm{cr}}) + u_{\mathrm{b}}^2 + u_{\mathrm{ba}}^2} \tag{4}$$

The uncertainty of a weighing process is determined using the standard deviation of the individual mass comparisons $s(\Delta m_{\mathrm{c}})$ and the number of weighing cycles n [1, Eq. C.6.1-1]:

$$u_{\mathrm{w}}(\overline{\Delta m_c}) = s(\Delta m_{ci})/\sqrt{n} \tag{5}$$

The standard uncertainty of a reference weight is yielded by the following [1, Eq. C.6.2-1]:

$$u(m_{\mathrm{cr}}) = \sqrt{(U/k)^2 + u_{\mathrm{inst}}^2(m_{\mathrm{cr}})}, \tag{6}$$

where U represents the expanded uncertainty of the reference weight given on the calibration certificate and $u_{\text{inst}}(m_{\text{cr}})$ is the standard uncertainty due to instability of the reference weight. This can be determined based on the calibration history of the particular weight.

According to OIML R111-1, the standard uncertainty of air buoyancy correction is calculated as follows [1, Eq. C.6.3-1]:

$$u_{\text{b}}^2 = \left[m_{\text{cr}} \frac{\rho_{\text{r}} - \rho_{\text{t}}}{\rho_{\text{r}}\, \rho_{\text{t}}}\, u(\rho_{\text{a}})\right]^2 + [m_{\text{cr}}(\rho_{\text{a}} - \rho_0)]^2 \frac{u^2(\rho_{\text{t}})}{\rho_{\text{t}}^4} + m_{\text{cr}}^2(\rho_{\text{a}} - \rho_0)[(\rho_{\text{a}} - \rho_0) - 2(\rho_{\text{al}} - \rho_0)] \frac{u^2(\rho_{\text{r}})}{\rho_{\text{r}}^4} \tag{7}$$

Assuming that $\rho_{\text{al}} = \rho_0$, [cf 3, Eq. 34.67], this equation is simplified to the following:

$$u_{\text{b}} = m_{\text{cr}}\sqrt{[(\rho_{\text{r}} - \rho_{\text{t}})/(\rho_{\text{r}} \cdot \rho_{\text{t}})\, u(\rho_{\text{a}})]^2 + (\rho_{\text{a}} - \rho_0)^2(u(\rho_{\text{t}})^2/\rho_{\text{t}}^4 + u(\rho_{\text{r}})^2/\rho_{\text{r}}^4)} \tag{8}$$

Here, ρ_{r} and ρ_{t} are the densities of the materials of the reference weight and the test weight, respectively; ρ_{a} is the current air density; and ρ_0 is the reference value of the air density, where $\rho_0 = 1.2\ \text{kg m}^{-3}$. Were the reference weight to be calibrated using the same weighing instrument to a reference weight of higher accuracy, there would be a correlation. In this case, the air density of the last calibration of the reference weight ρ_{al} would have to be considered in calculating the air buoyancy correction. As this is generally not the case, it is assumed that $\rho_{\text{al}} = \rho_0$ based on [3, Eq. 34.67].

The combined standard uncertainty of a balance is obtained according to [1, Eq. C.6.4-5]:

$$u_{\text{ba}} = \sqrt{u_{\text{s}}^2 + u_{\text{d}}^2 + u_{\text{E}}^2 + u_{\text{ma}}^2} \tag{9}$$

This result is comprised of the standard uncertainty of the sensitivity of the mass comparator u_{s}, the standard uncertainty of the display resolution u_{d}, the standard uncertainty due to eccentricity (off-center loading error) u_{E} and the standard uncertainty due to magnetism (magnetic properties and influences) u_{ma}. The standard uncertainty of the sensitivity is obtained according to [1, Eq. C.6.4-1]:

$$u_{\text{s}} = |\overline{\Delta m_{\text{c}}}|\sqrt{u^2(m_{\text{s}})/m_{\text{s}}^2 + u^2(\Delta I_{\text{s}})/\Delta I_s^2} \tag{10}$$

A possible, universally valid simplification is given in [3, Sections 3.4.6.2–3.4.6.3], where

$$u_{\text{s}} \approx |\overline{\Delta m_{\text{c}}}|\, u(m_{\text{s}})/m_s \approx 5 \cdot 10^{-4}|\overline{\Delta m_{\text{c}}}| \tag{11}$$

We have found that indicating the relative uncertainty as $5 \cdot 10^{-4}$ is a value that works in practice as it can be reliably obtained during regular calibration of the balance or mass comparator.

The standard uncertainty of the display resolution is obtained according to [1, Eq. C.6.4-2]:

$$u_{\text{d}} = d\sqrt{2}/(2\sqrt{3}) \tag{12}$$

In conventional measurements, the standard uncertainty due to eccentricity is already included in the standard uncertainty of the weighing process $u_{\text{w}}(\overline{\Delta m_{\text{c}}})$. As a result, the following is yielded with reference to [1, Section C.6.4.4.1]:

$$u_{\mathrm{E}} = 0 \tag{13}$$

The uncertainty component due to magnetism can likewise be neglected to the extent that weights compliant to standards, such as OIML R111-1, are used (cf. [1, Section C.6.4.5]).

$$u_{\mathrm{ma}} = 0 \tag{14}$$

3. DETERMINING THE AIR DENSITY

The air density is an essential influence quantity in highly accurate mass comparisons. The reason is that systematic buoyancy effects can be corrected if the air density is known. To correct for the effect of air density, OIML R111-1 specifies the following equation [1, Eq. 10.2-1]:

$$m_{\mathrm{ct}} = m_{\mathrm{cr}}\,(1 + C) + \overline{\Delta m_{\mathrm{c}}} \tag{15}$$

with [Eq. 10.2-2]:

$$C = (\rho_{\mathrm{a}} - \rho_0)\frac{\rho_{\mathrm{t}} - \rho_{\mathrm{r}}}{\rho_{\mathrm{r}}\,\rho_{\mathrm{t}}} \tag{16}$$

However, the uncertainty of air density results in an uncertainty component in the difference in mass determined. The air density can be calculated according to the CIPM-2007 equation [4] from the climate quantities of temperature, pressure, humidity and from the molecular constituents of air, with the latter usually the average mole fraction of carbon dioxide in air. Assuming that the exact mole fraction of carbon dioxide is known and equal to $0.4 \cdot 10^{-3}$, we indicate a combined relative uncertainty (excluding the contributions of the climate quantities of temperature, barometric pressure and humidity) of $u_{\mathrm{F}}(\rho_{\mathrm{a}})/\rho_{\mathrm{a}} = 22 \cdot 10^{-6}$ for the CIPM-2007 equation. As an alternative to this CIPM-2007-formula, the air density can be calculated using a simplified equation [1, Eq. E.3-1].

$$\rho_{\mathrm{a}} = \frac{0.34848\,p - 0.009\,hr \cdot exp(0.061\,t)}{273.15 + t}, \text{where } hr \text{ is in \% and } t \text{ in °C} \tag{17}$$

The relative standard uncertainty of this simplified formula is as follows (cf. [1, Section E.3]):

$$u_{\mathrm{F}}(\rho_{\mathrm{a}})/\rho_{\mathrm{a}} = 2 \cdot 10^{-4} \tag{18}$$

If we calculate the air density from the quantities of temperature, barometric pressure and humidity, the standard uncertainty of air density is obtained as follows: [1, Eq. C.6.3-3]:

$$u(\rho_{\mathrm{a}}) = \sqrt{u_{\mathrm{F}}^2 + \left(\partial\rho_{\mathrm{a}}/\partial p\ u_p\right)^2 + (\partial\rho_{\mathrm{a}}/\partial t\ u_t)^2 + (\partial\rho_{\mathrm{a}}/\partial hr\ u_{hr})^2} \tag{19}$$

The following values are given for the sensitivity coefficients under normal conditions (cf. [4, Section 2.2]):

$$\begin{gathered} \partial\rho_{\mathrm{a}}/\partial p \approx +1 \cdot 10^{-5}\ \mathrm{Pa}^{-1} \\ \partial\rho_{\mathrm{a}}/\partial t \approx -4 \cdot 10^{-3}\ \mathrm{K}^{-1} \\ \partial\rho_{\mathrm{a}}/\partial hr \approx -9 \cdot 10^{-3}, \text{ where } 0 \le hr \le 1 \end{gathered} \tag{20}$$

4. BUDGETING THE UNCERTAINTY COMPONENTS

It is necessary to budget the uncertainty components in order to generally verify whether a mass comparator is suitable for various accuracy classes. The following requirements have proven to be useful in practice:

$$u_{\mathrm{w}}(\overline{\Delta m_{\mathrm{c}}}) \leq \ 4/5\, u_{\mathrm{c}}(m_{\mathrm{ct}}) \text{ and} \tag{21}$$

$$u(m_{\mathrm{cr}}) = u_{\mathrm{b}} = u_{\mathrm{ba}} \leq \ 1/3\, u_{\mathrm{c}}(m_{\mathrm{ct}}) \tag{22}$$

If these individual requirements are met, the combined uncertainty of a mass comparator will always be lower than the uncertainty limit prescribed.

The required repeatability $s(\Delta m_{\mathrm{c}})$ of a mass comparator can likewise be derived from the required standard uncertainty of the weighing process $u_{\mathrm{w}}(\overline{\Delta m_{\mathrm{c}}})$.

$$u_{\mathrm{w}}(\overline{\Delta m_{\mathrm{c}}}) = \ s(\Delta m_{\mathrm{c}})/\sqrt{n} \leq 4/5\, u_{\mathrm{c}}(m_{\mathrm{ct}}) = \ 2/5\, U(m_{\mathrm{ct}}) = 2/15\,\mathrm{MPE}; \text{ i.e.,} \quad s(\Delta m_{\mathrm{c}}) \leq 2/15\,\mathrm{MPE}\sqrt{n} \tag{23}$$

The required number of weighing cycles n is specified by OIML and ASTM for the respective accuracy class of weights. Table 1 shows the minimum number of weighing cycles as a function of accuracy class along with the required repeatability for ABA cycles.

Table 1: Number of ABA Weighing Cycles

OIML Class	E1	E2	F1	F2	M1	M2	M3
ASTM Class	0	1 | 2	3 | 4	5	6	7	F
n Cycles	5	3	2	1	1	1	1
$s_{\max}(\Delta m_{\mathrm{c}}) = 2/15\,\mathrm{MPE}\sqrt{n}$	0.30MPE	0.23MPE	0.19MPE	0.13MPE	0.13MPE	0.13MPE	0.13MPE

5. UNCERTAINTY OF AIR BOUYANCY CORRECTION

In order to ensure that the total uncertainty budget for a mass comparison does not exceed the maximum permissible errors, a requirement has to be placed on the uncertainty of air buoyancy correction. With reference to the budget selected according to Equation (22), the uncertainty of an air buoyancy correction may be no more than 1/3 maximum of the permissible combined standard uncertainty of the conventional mass.

$$u_{\mathrm{b}} \leq \frac{1}{3} u_{\mathrm{c}}(m_{\mathrm{ct}}) \tag{24}$$

Based on the equation above, it follows that the required uncertainty of air buoyancy correction in relation to the maximum permissible error ($k = 2$) is obtained by:

$$u_{\mathrm{b}} \ \leq \frac{\mathrm{MPE}}{3 \cdot 2 \cdot 3} = \frac{\mathrm{MPE}}{18} \tag{25}$$

Therefore, the maximum permissible uncertainty of air buoyancy correction for class E1 weights with a nominal value greater than 100 g is yielded by:

$$u_{\mathrm{b}} \leq \frac{0.5 \cdot 10^{-6}}{18}\, m_0 = 2.7\overline{7} \cdot 10^{-8}\, m_0 \tag{26}$$

The standard uncertainty of the air buoyancy correction is given in Equation (8), which can be broken down into three individual formulas.

$$u_{\mathrm{b}}^2 / {m_{\mathrm{cr}}}^2 = u_{\mathrm{b1}}^2 + u_{\mathrm{b2}}^2 + u_{\mathrm{b3}}^2 \quad \text{, where} \tag{27}$$
$$u_{\mathrm{b1}} = \frac{\rho_{\mathrm{r}} - \rho_{\mathrm{t}}}{\rho_{\mathrm{r}}\, \rho_{\mathrm{t}}}\, u(\rho_{\mathrm{a}})$$
$$u_{\mathrm{b2}} = (\rho_{\mathrm{a}} - \rho_0)\, \frac{u(\rho_{\mathrm{t}})}{\rho_{\mathrm{t}}^2}$$
$$u_{\mathrm{b3}} = (\rho_{\mathrm{a}} - \rho_0)\, \frac{u(\rho_{\mathrm{r}})}{\rho_{\mathrm{r}}^2}$$

The term u_{b1} depends on the density difference between the reference weight and the test weight and on the uncertainty of the air density measured. Both terms u_{b2} and u_{b3} are each yielded by the product of the deviation of air density from the reference value $\rho_0 = 1.2\ \mathrm{kg/m^3}$ and the uncertainty of the density of the test weight and of the reference weight.

Taking the problematic nature of this calculation into general consideration, we assumed that all three terms would yield the same uncertainty contribution to the air buoyancy correction; i.e.

$$u_{\mathrm{b1}} = u_{\mathrm{b2}} = u_{\mathrm{b3}} = u_{\mathrm{b}}/\sqrt{3} \leq 2.7\overline{7} \cdot 10^{-8}/\sqrt{3} = 1.6 \cdot 10^{-8} \tag{28}$$

This assumption enables us to derive the requirements on the uncertainty of the air density measured and, therefore, on the uncertainty of measurement of the climate quantities of temperature, barometric pressure and humidity.

6. REQUIREMENTS ON CLIMATE DATA MEASUREMENT

The starting point for considering these conditions is the requirement on term u_{b1}:

$$u_{\mathrm{b1}} = (\rho_{\mathrm{r}} - \rho_{\mathrm{t}})/(\rho_{\mathrm{r}}\, \rho_{\mathrm{t}})\ u(\rho_{\mathrm{a}}) \leq 1.6 \cdot 10^{-8} \tag{29}$$

Hence, the following is obtained as the required uncertainty of measurement for air density:

$$u(\rho_{\mathrm{a}}) \leq 1.6 \cdot 10^{-8} \rho_{\mathrm{r}}\, \rho_{\mathrm{t}}/(\rho_{\mathrm{r}} - \rho_{\mathrm{t}}) \tag{30}$$

The permissible ranges for the density of the materials of weights are provided in [1, p. 17, Table 5]. For weights of accuracy class E1 with a nominal mass of at least 100 g, the density must be within the range of $7934\ \mathrm{kg/m^3} \leq \rho \leq 8067\ \mathrm{kg/m^3}$. If we plug in such maximum permissible errors for measurement of the density into Equation (30), the following requirements will be yielded for the uncertainty of air density:

$$u(\rho_{\mathrm{a}}) \leq 7.7 \cdot 10^{-3}\ \mathrm{kg/m^3} \tag{31}$$

and for the relative uncertainty:

$$u(\rho_{\mathrm{a}})/\rho_{\mathrm{a}} \leq 7.7 \cdot 10^{-3}/\rho_{\mathrm{a}} \leq 6.4 \cdot 10^{-3}, \text{ where } \rho_{\mathrm{a}} = \rho_0 \tag{32}$$

The maximum permissible errors on measurement of the density are higher for weights smaller than 100 g. However, this error on measurement of the density is established as such: "… that a deviation of 10% from the specified air density (1.2 kg m^{-3}) does not produce an error exceeding one-quarter of the absolute value of the maximum permissible error given in Table 1." [1, p. 17]. The ratio of the maximum permissible relative error of the weights to their maximum permissible errors on measurement of their density is therefore approximately constant.

The relative uncertainty of $2 \cdot 10^{-4}$ in simplified Equation (17) for calculating the air density is lower by a factor of 32 than the required uncertainty for air buoyancy correction. Hence, the uncertainty contribution of this equation is negligible. The relative uncertainty of air density, along with the corresponding sensitivities, is obtained using the three essential influence quantities for determining the air density, temperature, barometric pressure and relative humidity:

$$\frac{u^2(\rho_\mathrm{a})}{\rho_\mathrm{a}^2} = (-4 \cdot 10^{-3}\ \mathrm{K}^{-1} \cdot u(t))^2 + (1 \cdot 10^{-5}\ \mathrm{Pa}^{-1} \cdot u(p))^2 + (-9 \cdot 10^{-3} \cdot u(h))^2 \le (6.4 \cdot 10^{-3})^2 \tag{33}$$

Assuming that all three climate quantities have the same uncertainty contribution, the following is yielded:

$$\begin{gathered} u(\rho_\mathrm{a})/(\sqrt{3} \cdot \rho_\mathrm{a}) = 4 \cdot 10^{-3}\ \mathrm{K}^{-1} \cdot u(t) \le 6.4 \cdot 10^{-3}/\sqrt{3} = 3.7 \cdot 10^{-3} \text{ , or} \\ \boldsymbol{u(t) \le 0.92\ \mathrm{K}} \\ u(\rho_\mathrm{a})/\left(\sqrt{3} \cdot \rho_\mathrm{a}\right) = 1 \cdot 10^{-5}\ \mathrm{Pa}^{-1} \cdot u(p) \le 6.4 \cdot 10^{-3}/\sqrt{3} = 3.7 \cdot 10^{-3} \text{ , or} \\ \boldsymbol{u(p) \le 3.7\ \mathrm{hPa}} \\ u(\rho_\mathrm{a})/(\sqrt{3} \cdot \rho_\mathrm{a}) = 9 \cdot 10^{-3} \cdot u(h) \le 6.4 \cdot 10^{-3}/\sqrt{3} = 3.7 \cdot 10^{-3} \text{ , or} \\ \boldsymbol{u(h) \le 0.41} \end{gathered} \tag{34}$$

The requirements calculated for the uncertainty of climate data measurement can be met using conventional climate data sensors. The uncertainty of the relative humidity of 0.41 means that the relative humidity may be measured using a standard uncertainty of 41%. Under this aspect, it would not be mandatory to measure the humidity.

7. INFLUENCE OF HEIGHT ABOVE SEA LEVEL

E1 laboratories place high requirements on air conditioning. Such requirements usually cover devices for controlling the temperature and relative humidity. However, the barometric pressure is not controlled. The average barometric pressure essentially depends on the height of a laboratory above sea level. Therefore, temperature and humidity play a subordinate role for determining the mean value of air density. Figure 3 indicates the correlation between air density and height above sea level [1, Eq. E.3-2].

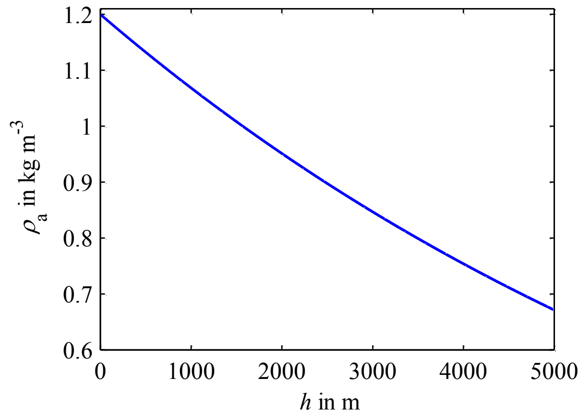

Figure 3: Air Density as a Function of the Height above Sea Level

The deviation of the air density from the reference value of 1.2 kg/m^3, in conjunction with the uncertainty of the density for the test weight and the reference weight, yields an uncertainty component for air buoyancy correction (Eq. 8). This correlation is clearly indicated by the two terms u_{b2} and u_{b3} in Equation (27). A maximum permissible error of $1.6 \cdot 10^{-8}$ is required for the relative uncertainty of air buoyancy correction according to Equation (28) for the three uncertainty components $u_{b1},\ u_{b2}$ and u_{b3}. For instance, if we plug in a value for the density uncertainty of $u(\rho_r) = u(\rho_t) = 5\ \mathrm{kg/m^3}$, (see also OIML R 111-1, Table B5 Estimated typical uncertainties on page 43), the curve depicted in Figure 4 is yielded for the terms u_{b2} and u_{b3} as a function of the height above sea level. The dotted line represents the required maximum permissible error for the uncertainty of the air buoyancy correction.

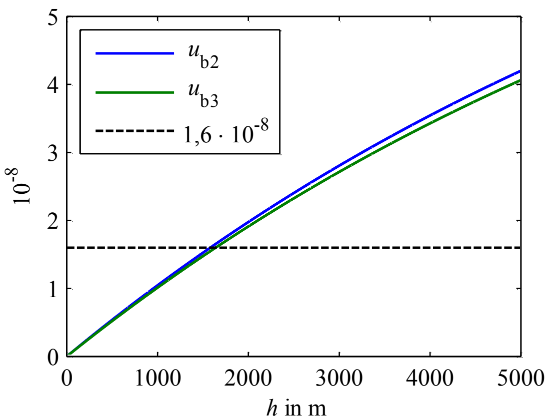

Figure 4: Uncertainty Contribution $\boldsymbol{u_{b2}}$ and $\boldsymbol{u_{b3}}$ as a Function of the Height above Sea Level

It can be seen that independently of the uncertainty of climate data measurements, the required maximum permissible error for the uncertainty of air buoyancy correction is exceeded at an elevation of approximately 1,600 m (5,259 ft.). On the other hand, an unambiguous requirement can be derived for the uncertainty of test weight and reference weight density measurements as a function of height above sea level. This correlation is shown in Figure 5. The calculation of this density is based on a density of $8{,}000\ \mathrm{kg/m^3}$ for the material of a weight.

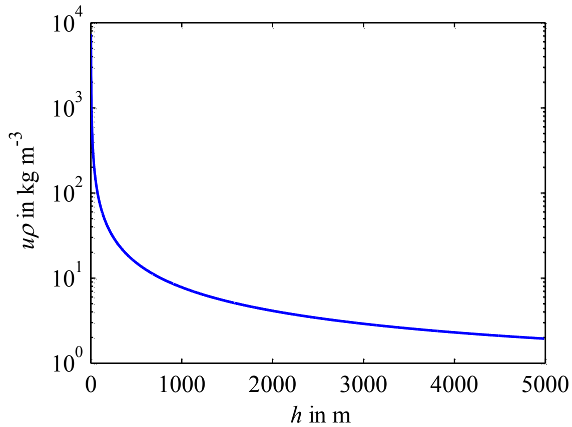

Figure 5: Required Uncertainty of the Density of the Weights as a Function of the Height above Sea Level

To meet the requirement placed on the uncertainty of air buoyancy correction at higher elevations, the uncertainty of the density measurement for the test weight and the reference weight has to be reduced. For example, to fulfill the requirement on this uncertainty at elevations of up to 5,000 m (~16,404 ft.), the standard uncertainty of the density measurement for both the test weight and the reference weight must be less than 2 $\mathrm{kg/m^3}$.

8. CONCLUSION

This article examines the essential aspects of calculating the uncertainty budget in compliance with international standards for the measurement of the conventional mass of weights. Special focus was given to the role of air density and its associated uncertainty. If air density is calculated from the climate quantities of air temperature, barometric pressure and relative humidity, uncertainty components will result from climate data measurements and from the equation for calculating the air density. By weighting the uncertainty components as prescribed, it is possible to specify the maximum permissible errors for the uncertainty of measurement of climate data. To determine the uncertainty of air buoyancy correction, the uncertainties of the density of the test weight and of the reference weight as well as the deviation of the air density from the reference value of 1.2 kg/m^{-3} must be considered besides the uncertainty of the air density itself. An uncertainty component of an air buoyancy correction will result from the height of a mass standards laboratory above sea level, regardless of the uncertainty of climate data measurements as the average air density essentially depends on elevation. On the other hand, the required uncertainties of the density measurements of test and reference weights can be derived as a function of elevation.

Cubis MC mass comparators permit exact determination of the conventional mass of a test weight, providing a detailed record of the associated uncertainty budget, which includes the air buoyancy correction and its uncertainty calculation under the limiting conditions described.

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