

# Measurement of Systematic Geometric Errors in Coordinated Measuring Machines.

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The measurements carried out in the industry require, for their complexity and quantity, the use of Coordinated Measuring Machines (CMMs). This type of machines possesses inherent geometry errors due to its construction, the dominant causes of these errors are: the imperfections that presents the guides that moves the axes X, Y and Z of the CMM, the displacements of their transducers, the probe system and the adjustment to each other of the components of the machine. Additionally, these errors are influenced by the temperature variations.

For the case of Cartesian CMMs, twenty one errors of geometric nature plus the errors of the probe system, should be taken into account to improve the accuracy of the same ones. The big CMMs (>2000 mm) have additionally, the so called "errors of non rigid body".

The method that is presented was developed to measure and to correct the errors of CMMs of big dimensions of "rigid body" and of "non rigid body", for this are used unidimensional elements (ball beam, spherical beam or cylindrical beam). This method includes the measurement procedure that should be employed and a software that carries out the calculation of the geometrical errors.

## Introduction

There are several methods that have been used for the geometrical errors measurement of CMMs, between them the following ones can be enumerated: method of the ball plate[1] and classic methods using laser, electronic levels and reference patterns, however none embraces the CMMs of big dimensions completely. The method of the ball beam was developed to cover big CMMs as well as small ones. This new method is based partially on the same ideas that are used in the ball plate method, but it contains another form for the calculation of the roll errors [2], since with this method certain problems are avoided that are observed in the ball plate method.

## Description of the ball beam method (21 errors for CMM of "rigid body").

The method uses basically trigonometrical and geometrical principles, beside some classic methods already exist to calculate the 21 errors.

Firstly all the rotation errors : yaw and pitch, are evaluated by the error difference divided by the distances between the positions of the beams, and the lengths of the stylus, see figure 1.

Yaw (XRZ): Rotation of the Z axis that affects the position of the X axis.

$$XRZ = \frac{X2(X) - X1(X)}{a}; a = X2(Y) - X1(Y) \quad (1)$$

Pitch (XRY): Rotation of the Y axis that affects the position of the X axis.

$$XRY = -1 \cdot \left( \frac{X3(X) - X1(X)}{b} \right); b = X3(Z) - X1(Z) \quad (2)$$

Yaw (YRZ): Rotation of the Z axis that affects the position of the Y axis.

$$YRZ = -1 \cdot \left( \frac{(Y3(Y) - Y4(Y)) + (Y1(Y) - Y2(Y))}{2a} \right);$$

$$a = Y3(P) - Y4(P) \quad (3)$$

Pitch (YRX): Rotation of the X axis that affects the position of the Y axis.

$$YRX = \frac{(Y3(Y) + Y4(Y)) - (Y1(Y) + Y2(Y))}{2b};$$

$$b = Y3(Z) - Y1(Z) \quad (4)$$

Yaw (ZRY): Rotation of the Y axis that affects the position of the Z axis.

$$ZRY = \frac{Z1(Z) - Z2(Z)}{b}; b = Z1(P) - Z2(P) \quad (5)$$

Pitch (ZRX): Rotation of the X axis that affects the position of the Z axis.

$$ZRX = \frac{Z3(Z) - \left( \frac{Z1(Z) + Z2(Z)}{2} \right)}{a}; a = Z3(P) \quad (6)$$

Where: X, Y and Z are the Cartesian coordinates of each one of the balls of the beam and P it is the length of the stylus in a certain position. Example: Y3(Y) represents the value of the coordinate Y in the position Y3, Z3(P) represents the length of the stylus in the position Z3, they are not functions but positions.

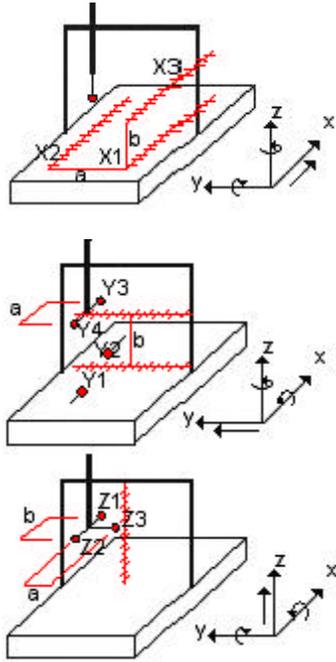


Figure 1. Positions of the ball beam to evaluate the yaw, pitch and position errors.

The position errors are directly evaluated from the length errors in the parallel positions to the axes and then referred to the axes of the coordinate system using the corresponding errors of rotation, with the distance of an axis of the coordinated system to the line of measurements of the beam.

Position (XTX): traslation of the X axis that affects the position of the X axis.

$$XTX = X_{cal} - \{X1(X) - (XRZ \times X1(Y)) - (XRY \times X1(Z))\} \quad (7)$$

Position (YTY): traslation of the Y axis that affects the position of the Y axis.

$$YTY = Y_{cal} - \left\{ \frac{(Y1(Y) + YRX \times Y1(Z)) + (Y3(Y) + YRX \times Y3(Z))}{4} + \frac{(Y2(Y) + YRX \times Y2(Z)) + (Y4(Y) + YRX \times Y4(Z))}{4} \right\} + \left\{ \frac{-YRZ \times Y1(P) - YRZ \times Y2(P) - YRZ \times Y3(P) - YRZ \times Y4(P)}{4} \right\} \quad (8)$$

Position (ZTZ): traslation of the Z axis that affects the position of the Z axis.

$$ZTZ = Z_{cal} - \left\{ \frac{(Z1(Z) + ZRY \times Z1(P)) + (Z2(Z) + ZRY \times Z2(P))}{2} \right\} \quad (9)$$

The roll errors in X and Y are evaluated from the differences of straightness derived from measurements carried out in parallel positions of the beam respect to the axes of the CMM, dividing the result between the distance of the beams. This calculation is carried out only after correcting the orientation of the ball beam nominally parallel. This correction is carried out with the measurements of the ball beam in diagonal position. These diagonals (three balls measured in each diagonal position) contains the flatness deviation information of the geometry of the CMM in the four balls of the ends that form the diagonals [2]. The two parallel beams to the axes that are located in the respective plane of the CMM (X1 and X2, X1 and X3, Y1,Y2 and Y3,Y4), are aligned mathematically, in such a way that the end ball of each beam, follow the flatness error obtained from the diagonals, see figure 2 and 3. Note that the end ball of each beam should coincide in an approximate way with the positions of the end balls from the diagonals.

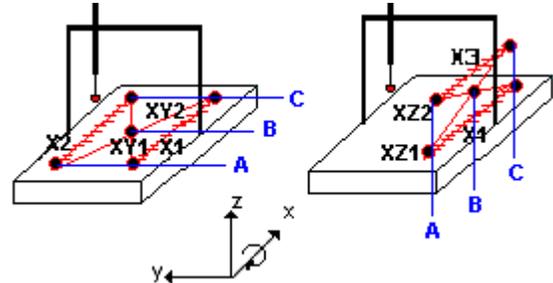


Figure 2. Positions of the ball beam to evaluate the roll errors and straightness of the X axis.

Correction of the coordinate Z of the balls measured in the position X2.

$$Z_{corr} = -1 \cdot \left\{ \frac{(XY1C(Z) - XY2C(Z)) - (XY2A(Z) - XY1A(Z))}{2 \cdot (XY1B(Z) - XY2B(Z))} \right\} \quad (10)$$

$$\left\{ \frac{(X2C(Z) - X1C(Z)) - (X2A(Z) - X1A(Z))}{2} \right\}$$

Correction of the coordinate Y of the balls measured in the position X3.

$$Y_{corr} = -1 \cdot \left\{ \begin{array}{l} (XZ1C(Y) - XZ2C(Y)) - \\ (XZ2A(Y) - XZ1A(Y)) - \\ 2 \cdot (XZ1B(Y) - XZ2B(Y)) \end{array} \right\} - \left\{ \begin{array}{l} (X3C(Y) - X1C(Y)) - \\ (X3A(Y) - X1A(Y)) \end{array} \right\} \quad (11)$$

Roll (XRX): Rotation of the X axis that affects to the same axis.

$$XRX = -1 \cdot \left\{ \frac{X2(Z_{corr}) - X1(Z) + X1(Y) - X3(Y_{corr})}{2} \right\} \quad (12)$$

Correction of the coordinated X of the balls measured in the positions Y3 and Y4.

$$X_{corr} = -1 \cdot \left\{ \begin{array}{l} (YZ1C(X) - YZ2C(X)) - \\ (YZ2A(X) - YZ1A(X)) - \\ 2 \cdot (YZ1B(X) - YZ2B(X)) \end{array} \right\} - \left\{ \begin{array}{l} (Y_{3,4}C(X) - Y_{1,2}C(X)) - \\ (Y_{3,4}A(X) - Y_{1,2}A(X)) \end{array} \right\} \quad (13)$$

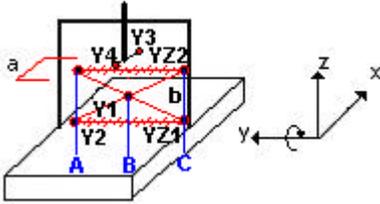


Figure 3. Positions of the ball beam to evaluate the roll errors and straightness of the Y axis.

Roll (YRY): Rotation of the Y axis that affects to the same axis.

$$YRY = -1 \cdot \left\{ \frac{(Y3(X_{corr}) + Y4(X_{corr})) - (Y1(X) + Y2(X))}{2b} \right\} \quad (14)$$

The roll error in the Z axis of the CMM, is calculated from the difference of straightness when the beam is measured in a parallel position to the Z axis, using two different lengths of stylus. See figure 1.

Correction of the coordinate Y, for the positions Z1 and Z2.

$$Z1(Y) = Z1(Y) - \{XRX(Z1(X) - Z1(P)) \times Z1(Z)\} \quad (15)$$

$$Z2(Y) = Z2(Y) - \{XRX(Z2(X) - Z2(P)) \times Z2(Z)\} \quad (16)$$

Roll (ZRZ): Rotation of the Z axis that affects to the same axis.

$$ZRZ = \frac{Z2(Y) - Z1(Y)}{Z1(P) - Z2(P)} \quad (17)$$

The errors of straightness are directly evaluated from the parallel beams to the axes, correcting the errors of straightness, because of this, the errors are valid in the coordinate axes (the correction is the roll error multiplied with the distance from the beam to the axis of the respective coordinate).

Straightness (XTZ): Straightness of the X axis in the direction of the Z axis.

$$XTZ = Z_{cal} - \{X1(Z) + (XRX(X) \times X1(Y_0))\} \quad (18)$$

Straightness (XTY): Straightness of the X axis in the direction of the Y axis.

$$XTY = Y_{cal} - \{X1(Y) - (XRX(X) \times X1(Z_0))\} \quad (19)$$

Straightness (YTX): Straightness of the Y axis in the direction of the X axis.

$$YTX = X_{cal} - \left\{ \frac{Y1(X) + Y2(X)}{2} + (YRY(Y) \times Y1(Z_0)) \right\} \quad (20)$$

Straightness (Y TZ): Straightness of the Y axis in the direction of the Z axis.

$$Y TZ = Z_{cal} - \left\{ \frac{Y1(Z) + Y2(Z)}{2} + (YRY(Y) \times Y1(P)) \right\} + (YRY(Y) \times Y2(P)) \quad (21)$$

Straightness (ZTX): Straightness of the Z axis in the direction of the X axis.

$$ZTX = X_{cal} - \left\{ \frac{Z1(X) + Z2(X)}{2} \right\} \quad (22)$$

Straightness (ZTY): Straightness of the Z axis in the direction of the Y axis.

$$ZTY = Y_{cal} - \left\{ \frac{(Z1(Y) - ZRZ(Z) \times Z1(P)) + (Z2(Y) - ZRZ(Z) \times Z2(P)) + Z3(Y)}{3} \right\} \quad (23)$$

Finally the perpendicularity errors are evaluated from the difference of lengths of the diagonals measurements on each plane of the coordinate system (see figures 2 and 3). However, these lengths are only evaluated after correcting the coordinates of the respective balls with all the calculated errors, applying in a correct way the stylus vectors used during the measurement of the diagonals.

Perpendicularity (XWY): perpendicularity between the X axis and the Y axis.

$$XWY = \frac{(LXY2_{corr} - LXY1_{corr}) \times LX Y1_{corr}}{2 \times U(XY1) \times V(XY1)} \quad (24)$$

Perpendicularity (XWZ): perpendicularity between the X axis and the Z axis.

$$XWZ = \frac{(LXZ2_{corr} - LXZ1_{corr}) \times LX Z1_{corr}}{2 \times U(XZ1) \times V(XZ1)} \quad (25)$$

Perpendicularity (YWZ): perpendicularity between the Y axis and the Z axis.

$$YWZ = \frac{(LYZ1_{corr} - LYZ2_{corr}) \times LY Z1_{corr}}{2 \times U(YZ1) \times V(YZ1)} \quad (26)$$

Where: L represents the length of the measured diagonals with the end balls (length of the beam), U and V are the lengths of each diagonal projected to each axes. Example: LXY2corr is the length of the diagonal XY2 corrected by all those errors except those of perpendicularity and V(XZ1) is the length of the diagonal XZ1, projected on the Z axis.

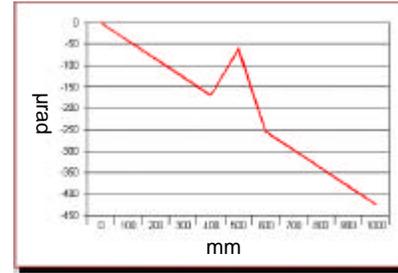
### CALMAC, version 1.0.

This program which initials make reference to CMM calibration with Beams was developed to calculate the 21 errors of CMM of "rigid body" and the three errors of CMM of "elastic body". This version includes measurement temperature correction, robust interpolation, union of beams for the case when the beam length doesn't cover the total volume of the CMM and the obtention of the correction format developed by the PTB of Germany.

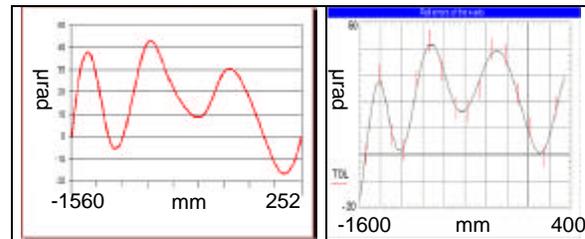
### Tests of validation for CALMAC.

Two tests were carried out: one of them was based on synthetic data, studying the capacity to analyze each error by separate, and a test in which the results of a real measurements with the ball beam method were compared with the measurements carried out with the

method of the ball plate [1] (making use of the program KALKOM 4.0) with the same CMM.



Figures 5. Roll Error in the Y axis with a lineal content and a " local pulse". The results went identical to the results that in theory should be obtained.



Figures 6: Comparison of results between the ball plate method, plate of 960 mm x 960 mm (right) and a ball beam of 3 m (left). The evaluation was carried out with KALKOM 4.0 from the PTB and with CALMAC 1.0. Here the roll errors of the X axis are shown.

### Conclusions.

This method is of great utility not only to evaluate and to correct CMM geometrical errors with the objects method, but also has the flexibility of being able to be used to evaluate these errors making use of the classic methods that use laser interferometer, or the relatively new methods as the laser tracker to evaluate and to correct errors. Especially the program Calmac 1.0, can be adapted to the use of these instruments.

### References.

- [1] Trapet, E., Wäldele, F.: Kalibrierung von Koordinatermessgeräten mit Kugel und Lochplatten. Anleitung für DKD-Laboratorien, PTB Braunschweig, April 1993.
- [2] Arriba, L. (TRIMEK, Spain), Trapet, E. (TRIMEK, Spain), Bartcher, M. (PTB, Germany), Franke, M. (PTB Germany), Balsamo, A. (IMGC, Italy), Costelli, G. (DEA Broen and Sharpe, Italy), Torre, S.(DEA, Italy), Kitzsteiner, F. (Zeiss, Germany), San Martin, F. (GPE, Spain): Artefacts and methods to establish traceability of large CMMs Final Report, MESTRAL Project PTB, June,2001.