

WATT BALANCES AND THE FUTURE OF THE KILOGRAM

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Abstract: The kilogram is the only base unit of the SI system still defined by a material artefact. A number of laboratories are working on experiments to replace the current definition by one based on fundamental constants. The paper gives an overview over these techniques with special emphasis on watt balances. The BIPM, being the custodian of the international prototype of the kilogram, has recently started to develop a watt balance whose distinctive features are described. The paper concludes with an overview of the existing experimental results and their implications for the evolution of the kilogram definition and of the SI as a whole.

1. INTRODUCTION

The kilogram is the last base unit of the SI system still defined by a material artefact, the international prototype of the kilogram, kept at the BIPM.

The international prototype has served mass metrology very well over more than 100 years [1]. Nevertheless, an artefact standard for the mass unit has a number of disadvantages. It is only available at a single place and the mass unit cannot be realized elsewhere. It can be damaged or destroyed. An artefact standard can accumulate contamination and needs to be cleaned with incompletely known effects. Since the creation of the BIPM in 1875, three comparisons have been carried out between the international prototype and its six official copies. The masses of the official copies seem to increase relative to the international prototype at a rate of about 5 parts in 10^8 over 100 years. There is no reason to expect that the mass of the international prototype is more stable than those of its official copies. The question of stability can only be definitely answered by an experiment linking the international prototype to an unalterable fundamental constant of nature, with an uncertainty of the order of 1 part in 10^8 .

2. LINKING THE KILOGRAM TO FUNDAMENTAL CONSTANTS

The kilogram can be linked to different fundamental constants or atomic masses. The most obvious approach is to compare the mass of a known and very large number of atoms of the same type with a macroscopic mass. This establishes the kilogram as a known multiple of an atomic mass, which is

equivalent to a determination of the Avogadro constant N_A if the molar mass of the atoms is known. The difficulty lies in collecting a sufficiently large, and exactly known, number of atoms for the weighing operation. Two different approaches are currently pursued. The PTB is developing an experiment in which a weighable mass is accumulated from gold or bismuth ions delivered by an ion beam [2]. The number of ions is determined from an integration of the beam current over time. The uncertainties are currently of the order of 1 part in 10^3 . A significant improvement is expected from work focusing on increasing the ion beam current. An alternative approach is to determine the number of silicon atoms in a nearly perfect single-crystal silicon sphere. The number of atoms is deduced from the volume of the unit cell and the volume of the whole sphere. This project is carried out as an international collaboration of several institutes. The uncertainty of this technique, named X-ray crystal density method (XRCD), is currently limited by the determination of the isotopic composition of the sphere to about 3 parts in 10^7 [3]. Future experiments are planned with a sphere made of at least 99.99 % ^{28}Si , and uncertainties of several parts in 10^8 are expected.

Another class of electro-mechanical techniques links the kilogram to the Planck constant h . The most successful experiment of this type is the watt balance. The principle is explained in the next section. Five research groups are working on this technique, and in 2005 the NIST announced that they had reached an uncertainty of 5 parts in 10^8 . Further progress seems possible. Therefore watt balances appear nowadays as the most promising approach for a redefinition of the kilogram.

3. WATT BALANCES

3.1 The principle

A watt balance compares a measurement of electrical and mechanical power. The electrical quantities are measured based on macroscopic quantum effects, the Josephson effect and the quantum Hall effect [4]. These two effects link the electrical quantities to fundamental constants and allow one to establish a relation between a macroscopic mass, measured in the unit kilogram, and the Planck constant h .

A watt balance experiment consists of two parts, the “weighing experiment” and the “moving experiment”. In the weighing experiment (Fig. 1) the gravitational force on a mass m (its weight) is balanced by the Lorentz force on a coil with wire length L , carrying a current I , in a magnetic field with flux density B . If equilibrium is obtained the equation

$$m g = I L B \quad (1)$$

holds.

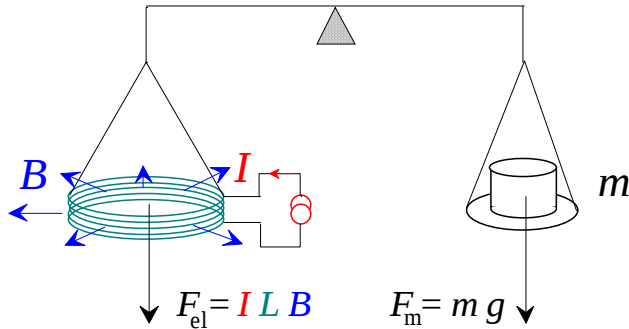


Figure 1: Weighing experiment: An equilibrium between gravitational force and Lorentz force is established.

In the moving experiment, the same coil is moved at constant velocity v through the same magnetic field as before (Fig. 2). An induced voltage ε appears across the coil which is given by

$$\varepsilon = v L B \quad (2)$$

Both equations can be combined by eliminating the quantities describing the coil, L , and the magnetic field, B , resulting in

$$m g v = U I \quad (3)$$

This equation demonstrates that electrical and mechanical power are directly compared. It is,

however, important to notice that these powers are virtual. They are not realized as a real power, because the current I and the voltage U are measured in different phases of the experiment. The same is true for the mechanical power. This has the important consequence that power losses during the movement, for example due to mechanical friction, do not appear directly in the measurement equation.

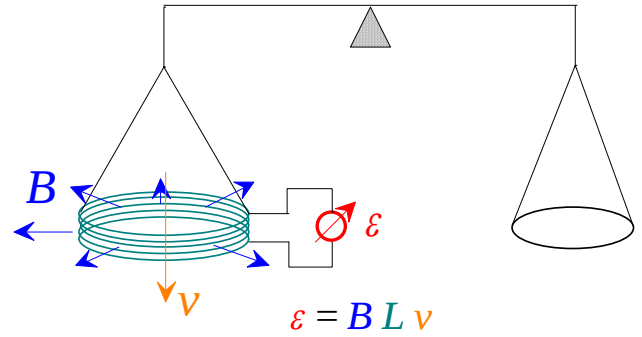


Figure 2: Moving experiment: A voltage is induced by moving the coil with constant velocity through the magnetic field.

To bring in the Planck constant, the electrical quantities are measured using the conventional electrical unit system based on the Josephson and the quantum Hall effect [4]. The Josephson effect allows one to realize a reproducible voltage over a junction between two superconductors irradiated with microwaves. The quantum Hall effect allows one to realize a reproducible resistance using a two-dimensional electron gas in a strong magnetic field. This leads to the following equation:

$$m = h \frac{K_{J-90}^2 R_{K-90}}{4} \frac{U_{90} I_{90}}{g v} \quad (4)$$

K_{J-90} and R_{K-90} are the conventional values for the Josephson constant and the von Klitzing constant. U_{90} and I_{90} are, respectively, the voltage and the current measured in the conventional electrical unit system.

This equation establishes the relationship between a mass m and the Planck constant h . At first, the experiment would be carried out to determine h , based on the current definition of the kilogram. If the uncertainty has been reduced to several parts in 10^8 and the metrology community feels sufficiently confident about the results, the best value obtained can be used to define the value of the Planck constant (without uncertainty), this being equivalent to redefining the kilogram.

3.2 The existing experiments

In the following, the main characteristics of the existing experiments are described briefly; more details are given in [5]. The BIPM watt balance is described in more detail in section 4.

The NPL (UK) began to construct the first watt balance soon after the original proposal by B. Kibble [6] in 1975. The balance is constructed as a classical beam balance and the magnetic field is produced by a permanent magnet. Results obtained with the first version of the apparatus had a standard uncertainty of 2 parts in 10^7 and were published in 1990 [7]. The current second version will be replaced by the Mark III balance in the near future.

The NIST (USA) started its experiment only a few years later with an electromagnet, which was subsequently replaced by a superconducting magnet. Instead of a balance beam a rotating wheel is used which restricts the movement of the coil to the vertical axis. In 1998, results with an uncertainty of 9 parts in 10^8 were published [8]. Further improvements allowed to reduce the uncertainty to 5 parts in 10^8 in 2005 [9]. Both results are in good agreement. A further reduction of the uncertainty is expected in the near future.

METAS (Switzerland) started its watt balance project in 1997. The most obvious difference from the other experiments is the small size of the apparatus as a consequence of using a test mass of only 0.1 kg instead of 1 kg [5]. During the moving experiment, the test mass is decoupled from the mass comparator, to reduce hysteric problems.

The LNE (France) started to develop its watt balance in 1999. This will be a medium-size apparatus working with a mass of 0.5 kg [10]. One of the design criteria was to avoid an unbalance of the mass comparator during the moving mode. This will be achieved by moving the balance together with the coil. Many of the components of the experiment have been prepared and will be assembled in 2006.

4. THE BIPM WATT BALANCE

4.1 Design

For the validity of the basic equation (3), it is essential that the magnetic flux density B is the same in both parts of the experiment. The temperature coefficient of the remanent magnetization of SmCo magnets, used for the

permanent magnets in the existing watt balances, is about 0.035 %/K. Therefore small temperature changes of the magnet lead to significant changes of the magnetic flux density. If both measurement phases are carried out sequentially, at a time interval of typically one hour, the magnetic flux density can therefore be significantly different.

At the BIPM we plan to carry out both parts of the experiment simultaneously. A constant current will be injected in the coil to create a magnetic force to balance the weight, while the coil is moving at constant velocity through the magnetic field. The induced voltage related to the movement will be superimposed on the voltage across the coil due to the current flow. Both types of voltages need to be separated with an uncertainty of the order of 1 part in 10^8 . Most critically in this respect is the large temperature coefficient of the electrical resistance of the coil, 0.4 %/K. Resistance changes due to temperature changes of the wire during the time for a measurement (of the order of 1 minute) would mask changes of the induced voltage. We therefore plan to use ultimately a superconducting coil in which the resistive voltage drop would not exist. In this "cryogenic watt balance" the permanent magnet will also be held at cryogenic temperature.

We have started to assemble a room temperature experiment to gain experience on the general feasibility of simultaneous force and velocity measurement, and to find out how much the uncertainty of such an experiment can be reduced. Only after this stage will we start to work on a cryogenic system.

4.2 Current Status

The assembly of the BIPM watt balance started in May 2005. We have started to build the balance suspension, which includes an electrostatic motor, moving the coil vertically (Fig. 3). The motor consists of a fixed, grounded central electrode and a mobile unit of two high voltage electrodes, above and below the central one. The electrostatic force between the grounded electrode and the high voltage electrodes leads to a vertical movement of the latter. This movement is translated to a vertical movement of the coil by 3 beams connected with flexure strips. The displacement range is about 20 mm. The force varies with the square root of the voltages and with the inverse square root of the distance between grounded and high-voltage electrodes. The voltage dependence can be linearized by applying the same bias voltage to both electrodes and by

superimposing opposite voltage changes on both electrodes. We are currently working on a digital control system to control the vertical position and the velocity of the coil. The vertical displacement should not introduce any rotations or horizontal displacements of the coil. These are monitored by using position-sensitive detectors together with laser diodes.

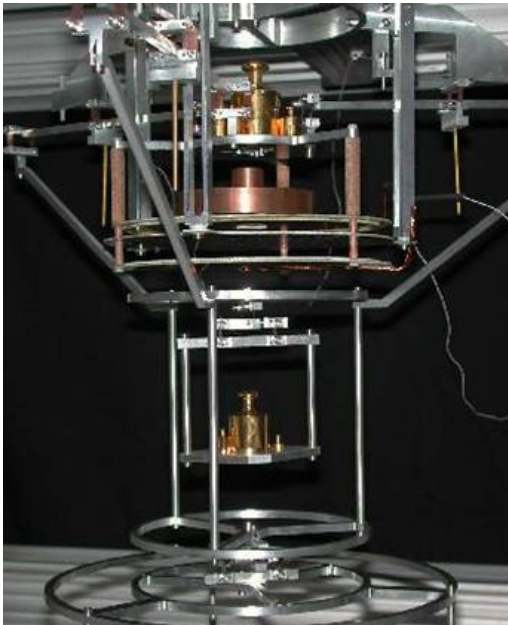


Figure 3: Upper part of the suspension, showing the electrostatic motor and – close to the center - the weight. The upper weight serves as counterweight to reduce the force on the motor.

We have developed a design for the magnetic circuit (Fig. 4). The most important criteria are a sufficiently high flux density, a high uniformity of the flux density within the gap, and a good screening against external magnetic perturbations. The flux source will be two discs of $\text{Sm}_2\text{Co}_{17}$ with opposite magnetization. They will be inserted in a closed magnetic circuit made of high permeability FeNi-alloy of austenitic structure, which does not become brittle at cryogenic temperature. A horizontal and radial field is obtained in the horizontal symmetry plane. The gap will be 13 mm wide with a diameter of 250 mm. The flux density in the gap, at the position of the coil is about 0.7 T. Since the gap is completely surrounded by material with high magnetic permeability, it should be efficiently screened against external magnetic fields. It is an important requirement that the flux density in the gap is constant in the vertical direction, so that the force on the moving coil does not change more than about 100 ppm. The system is symmetrical about the

horizontal symmetry plane so that no linear slope of the field profile is expected. The inner and outer pole are 84 mm and 80 mm high. The small difference in height increases the uniformity of the field considerably.

The flux density in the gap depends critically on its width, that is on the machining and assembly of the pole pieces.

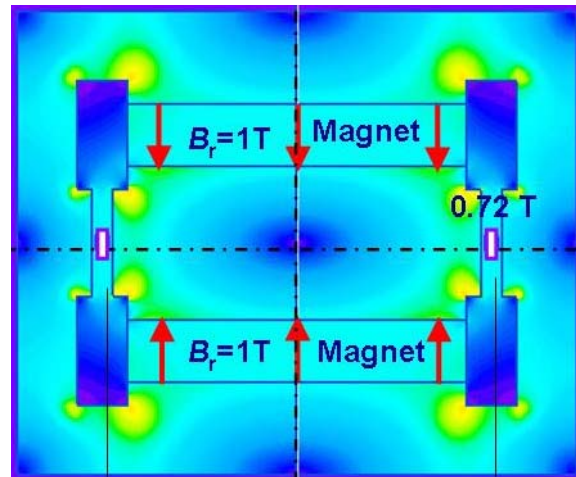


Figure 4: The magnetic circuit, consisting of two discs of $\text{Sm}_2\text{Co}_{17}$ and a closed FeNi yoke. The flux density in the central plane, where the gap is located, is horizontal and radial.

The required uniformity demands that the width of the gap is constant within $6\text{ }\mu\text{m}$. We are currently looking for a company capable of achieving this. The upper part of the magnet needs to be removable to give access to the coil. For the coil suspension, holes are needed in the upper part of the yoke.

We have developed a first version of a current source which delivers a current of 1 mA with a short term stability of about 2 parts in 10^7 . A 10 V monolithic voltage reference is connected in series with a 10 k Ω resistor, in which flows the current to be stabilized, at the input terminals of an operational amplifier. The output of the amplifier provides this current through a buffer amplifier.

To measure the coil velocity, we have purchased a commercial heterodyne interferometer, which is currently under test.

5. THE EVOLUTION OF THE SI SYSTEM

As described above, several institutes are working to establish a link between the international prototype of the kilogram and fundamental constants, as a first step towards a new definition of the kilogram.

An overview of the results of experimental determinations of the Planck constant h is given in Figure 5 (data taken from [11], which also includes references to the different experiments). The results NIM-95 and NPL-79 are obtained from measurements of the gyromagnetic ratio of the shielded proton in a high magnetic field. NIST-80 is a measurement of the Faraday constant. PTB-91 and NML-89 are determinations of the Josephson constant K_J using a voltage balance and a liquid-mercury electrometer, respectively. N/P/I-03 is the determination of the Avogadro constant N_A by the Working Group on the Avogadro constant using the X-ray crystal density (XRCD) method. The results NPL-90, NIST-98 and NIST-05, having the smallest uncertainties, are obtained with watt balances.

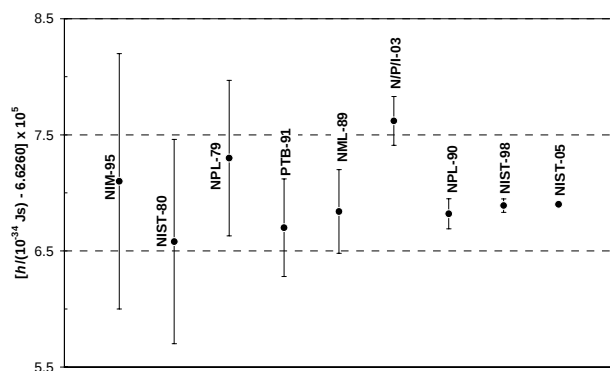


Figure 5: Experimental results for the Planck constant h , sorted according to the standard uncertainty.

The most obvious feature of Figure 5 is the discrepancy of about 1 ppm between the watt balance results and that of the XRCD work. The origin of this difference is unclear. One of the greatest difficulties in the XRCD technique is the determination of the isotopic composition of the Si-sphere, necessary to obtain the molar mass. A new project has been started recently, eliminating this problem by using isotopically enriched ^{28}Si . Results are expected by 2008. The target uncertainty is 2 parts in 10^8 .

Initiated by a recent publication which proposed to redefine the kilogram immediately [12], the Consultative Committees for Electricity and Magnetism (CCEM) and for Mass and Related Quantities (CCM) discussed this situation in 2005 and defined conditions which would allow a redefinition of the kilogram. The major obstacle is currently the discrepancy between the watt balance and the XRCD results. It is, however, expected that

this discrepancy will be resolved during the next few years. The uncertainty of the best realization of the new definition should not exceed 2 parts in 10^8 , to ensure that the new definition is consistent with the present one within this uncertainty. It is expected that a *mise en pratique* would explain how the new definition can be realized in practice. Besides the operation of a watt balance, this could include the comparison of an unknown mass standard with a known mass standard, whose mass itself was determined with a watt balance.

In addition to this, redefinitions of the ampere, the kelvin and the mole are proposed [13]. The International Committee for Weights and Measures (CIPM) made a recommendation in 2005 which describes the preparative steps towards these new definitions. In particular it recommends that National Metrology Institutes should pursue vigorously their current projects to obtain values for the fundamental constants with sufficiently low uncertainty and that they should prepare for long-term maintenance of the experiments needed to realize the new definitions.

6. CONCLUSIONS

Preparations have started recently for new definitions of the kilogram, the ampere, the kelvin and the mole. Further steps will depend on the progress with the related experiments, especially with watt balances, to determine the value of the Planck constant.

As shown in this article, the progress to date is encouraging, with one result having an estimated uncertainty of 5 parts in 10^8 . For the future, this uncertainty still needs to be reduced by a factor of 2 or 3, and the discrepancy between watt balance and XRCD results should be resolved. Other watt balance results with comparable uncertainties would also be highly desirable.

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