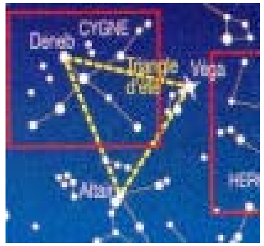
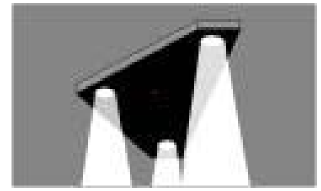
Laboratoire national de métrologie et d’essais

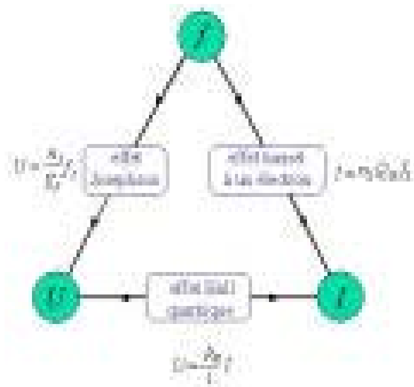
LNE

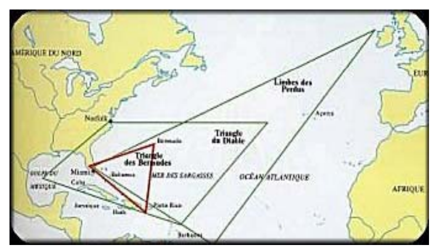
Le progrès, une passion à partager

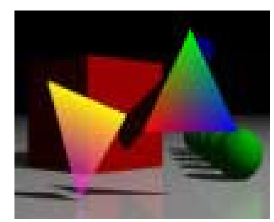




Triangle of enchiladas







Quantum metrology triangle and determination of the charge quantum

François Piquemal

The permanent team for the QMT project at the LNE

SET&CCC: *Laurent Devoille, Nicolas Feltin,*

QHE: *Wilfrid Poirier, Félicien Schopfer*

JE: *Sophie Djordjevic, Olivier Séron*

Former and present PhD students & postdocs :
F. Gay, Y. De Wilde, B. Steck, A. Gonzalez Cano, B. Chenaud, S. Sassine

Collaboration

CEA-Saclay & Grenoble, LPN/CNRS Marcoussis, PTB Braunschweig&Berlin

iMERA+ project REUNIAM: *LNE, METAS, MIKES, NMi/VSL, NPL, PTB*

OUTLINE

I) Introduction

1) Fundamental electrical metrology
2) Aims of the QMT

II) Arguments for closing the triangle

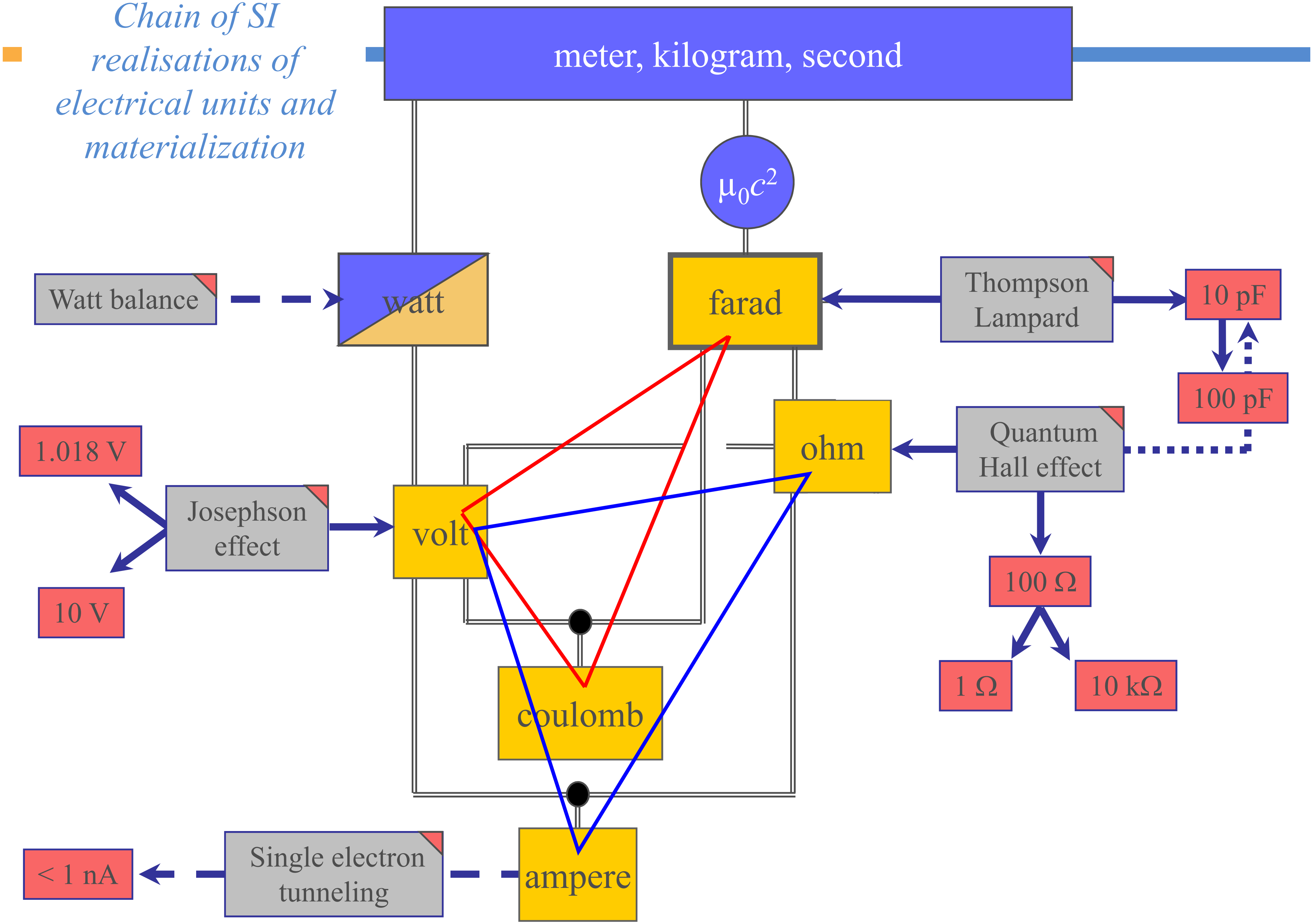
1) Uncertainty thresholds
2) Determination of the charge quantum

III) Experimental set-up based on the CCC: $U = RI$

1) CCC: Cryogenic Current Comparator
2) Overall set-up at LNE
3) First results

IV) Conclusion

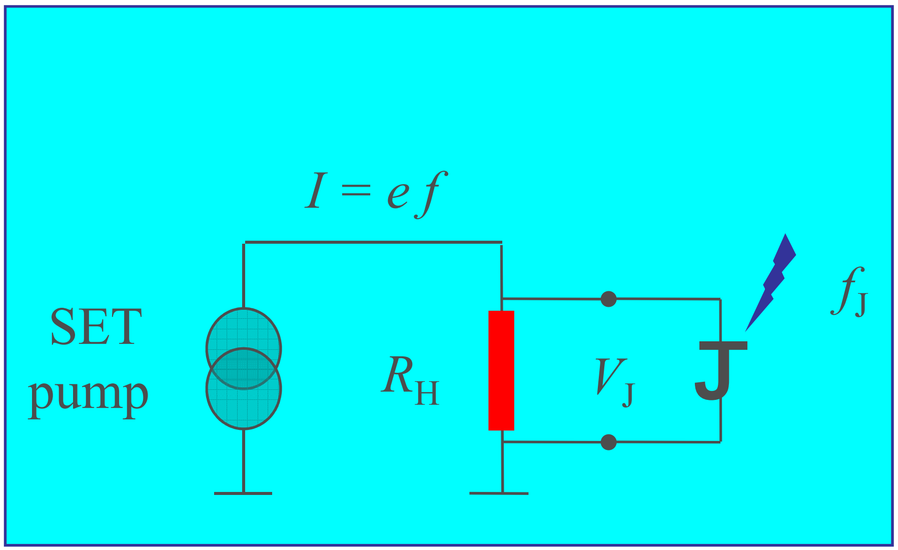
Chain of SI realisations of electrical units and materialization

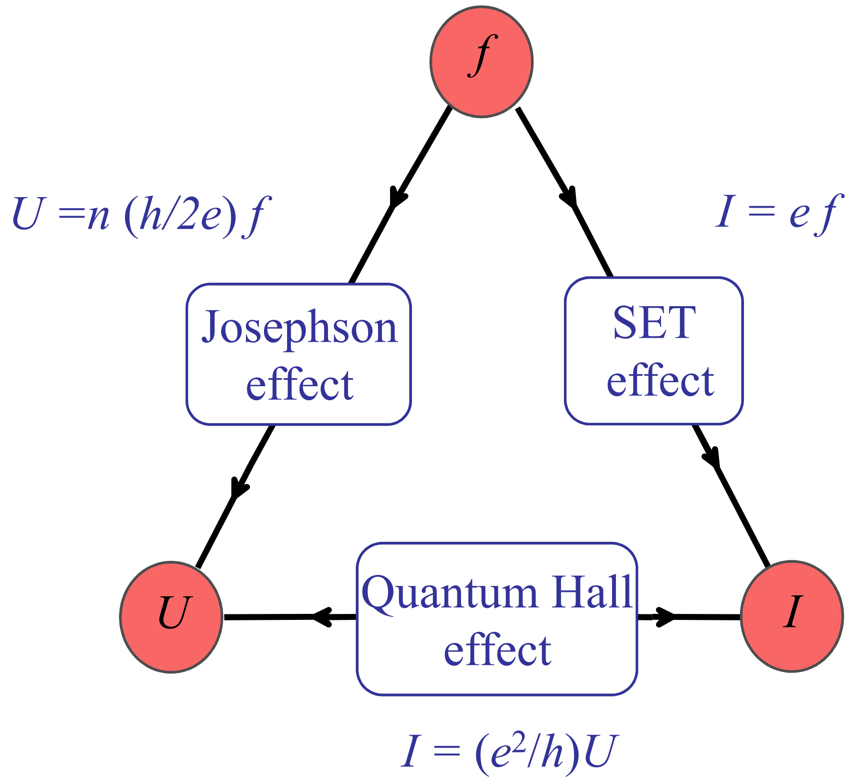


The quantum metrological triangle (QMT) experiment

By means of SET devices such as electron pumps,
a current standard with **quantized** amplitude is available : $I = e\,f$

The experiment originally proposed by *K. Likharev and A. Zorin in 1985* consists of applying Ohm's law, $U = R\,I$ directly to the quantities issued from ac JE, QHE and SET.



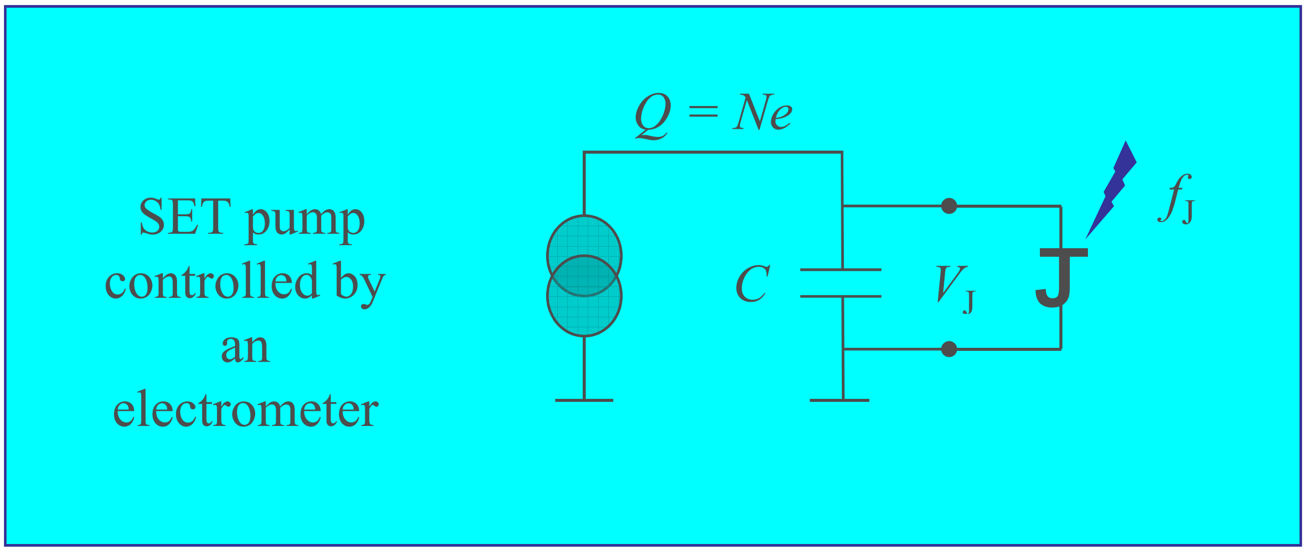


LNE
Le progrès, une passion à partager

The QMT experiment

Another promising approach to close the triangle is to apply $Q = CV$

Charging a capacitor electron per electron with a pump measuring the voltage drop with Josephson voltage standard, calibrating the capacitance by means of QHR standard



$\Rightarrow$ Electron counting capacitance standard (ECCS)

Aims of the QMT

To confirm with a very high accuracy that these three effects of condensed matter physics give the **free space** values of constants $2e/h$, h/e^2 and e.

The ultimate target uncertainty is **one part in 10^8**

- If there is no deviation, our confidence on the three phenomena to provide us with $2e/h$, h/e^2 and e will be strengthened.
- If deviation occurs, some other works both experimental and theoretical will have to be done.

To determine the elementary charge e or, in other words, the charge quantum brought by the SET devices

Last but not least, to establish whether the SET can achieve a high level quantization (ie **one part in 10^8**) in particular when the SET device is connected to an external circuit.

LNE
Le progrès, une passion à partager

OUTLINE

I) Introduction

1) Fundamental electrical metrology
2) Aims of the QMT

II) Arguments for closing the triangle

1) Uncertainty thresholds
2) Determination of the charge quantum

III) Experimental set-up based on the CCC: $U = RI$

1) CCC: Cryogenic Current Comparator
2) Overall set-up at LNE
3) First results

IV) Conclusion

Quantum standards: universality and high reproducibility

- *Test of the universality of relations involved in Josephson and quantum Hall effects:*

U_J / f and $i \times R_H\ (i)$ **independent of materials**

at a level of **2.10^{-16}**

J.S. Tsai *et al* – 1983

JE: In microbridge and planar Nb/Cu/Nb junction

1.10^{-10}

A. Hartland *et al* - 1991
B. Jeckelmann *et al* – 1995

QHE: GaAs/AlGaAs and Si-MOSFET

- *Unique representation of the volt and the ohm:*

The recent international comparisons of complete JE and QHE systems show a high level of consistency: from a few **10^{-11}** to a few **10^{-9}**.

These remarkable results do not prove that the phenomenological constants are exactly $2e/h$ and h/e^2 but they strengthen our confidence in the equalities $K_J = 2e/h$ and $R_K = h/e^2$ in addition to strong theoretical arguments.
If corrections exist, they will be probably of fundamental nature.

Different uncertainty thresholds for closing the QMT (1)

- First critical test of validity for SET: Uncertainty of **1 ppm**

Neither the JE nor the QHE is questionable at that uncertainty level

$\Rightarrow$ *recently completed by NIST with* $\sigma = 9.2$ *parts in* 10^7

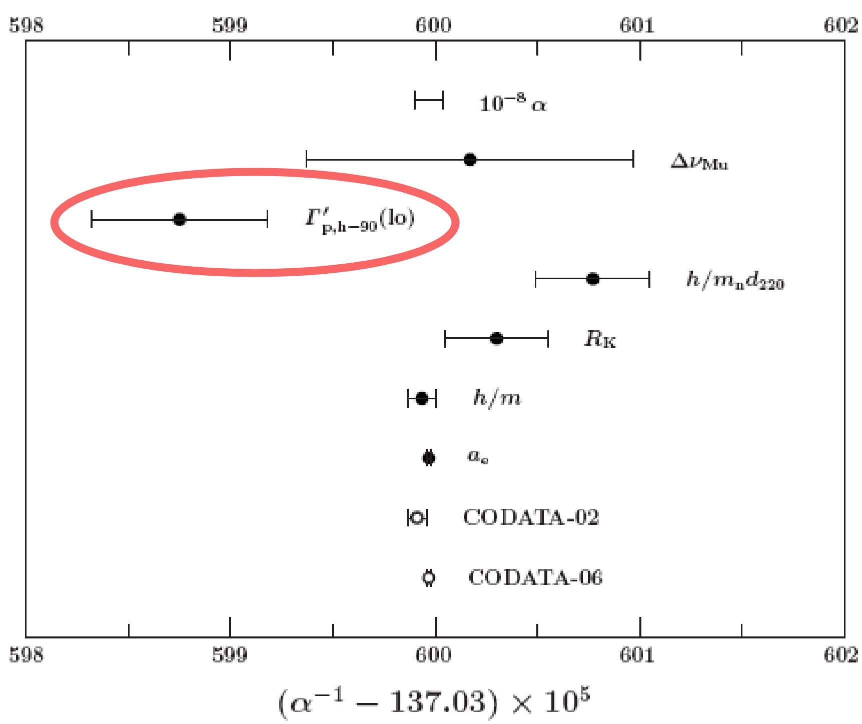
Different uncertainty thresholds for closing the QMT (2)

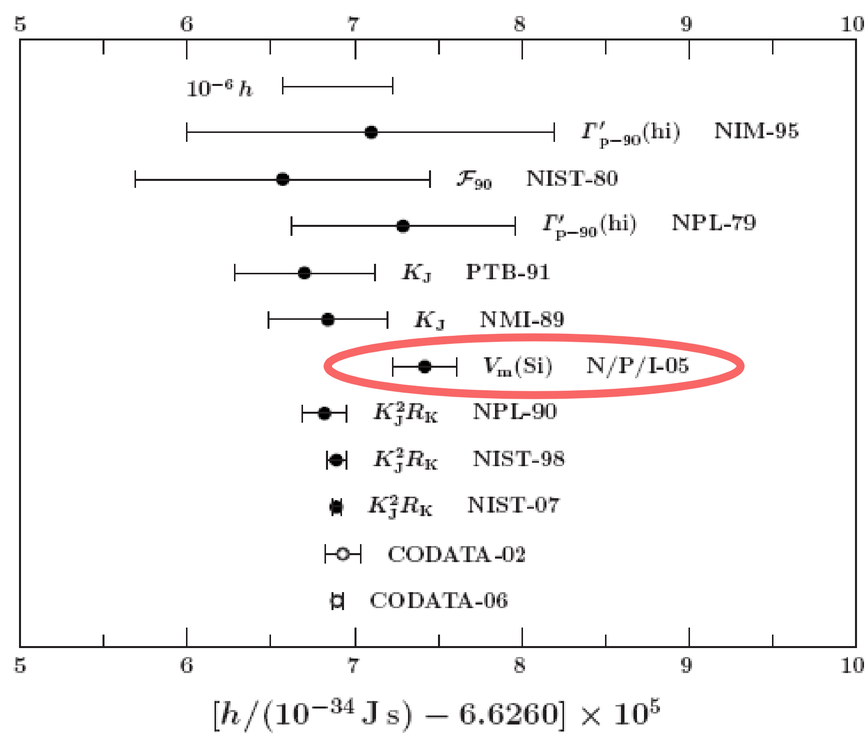
- Second uncertainty level lies between

  7 parts in 10^7 and **2 parts in 10^8**

  $\Rightarrow$ resulting information will be mainly relevant for the JE and the SET

This comes from the present discrepant values of $\Gamma'_{\text{p h-90}}$ (lo) and V_m(Si)





P. Mohr, B.N. Taylor, D.B. Newell, Rev of Mod Phys., vol.80, 2008

Closing the triangle: *first way* $U = R \times I$

As for JE, $\boldsymbol{K_J = (2e/h)|_{JE}}$ and QHE, $\boldsymbol{R_K = (h/e^2)|_{QHE}}$, one can define a phenomenological constant: $\boldsymbol{Q_X = e|_{SET}}$

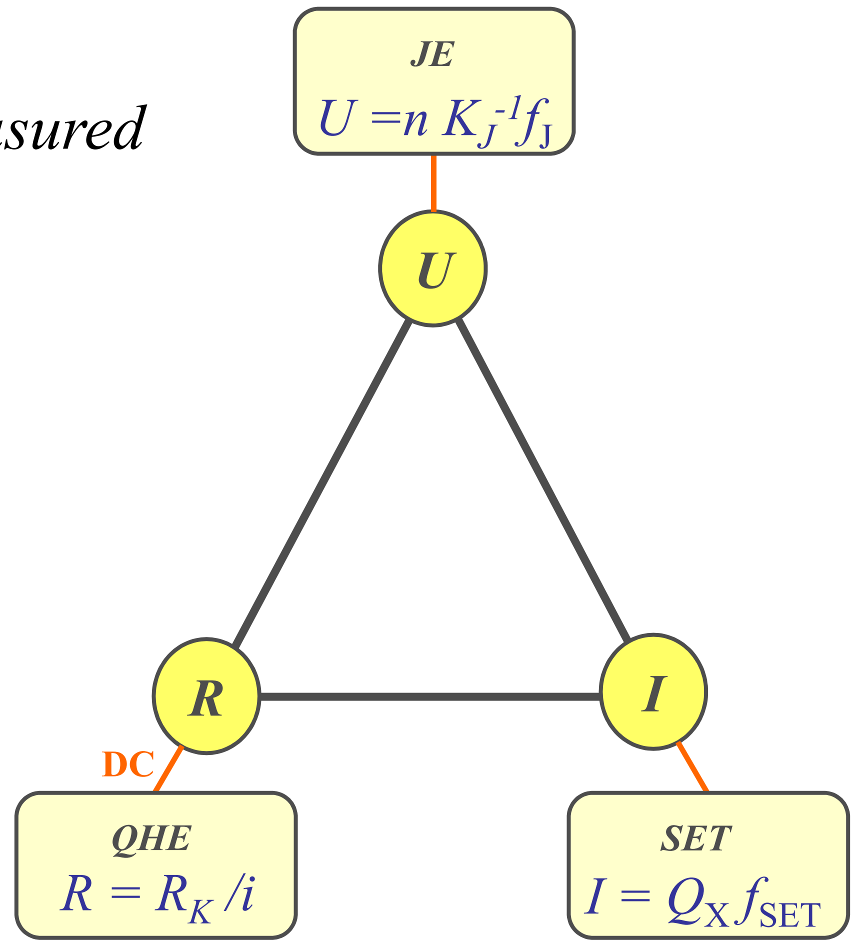
➡ *Dimensionless product to be measured*

$$R_K\, K_J\, Q_X = 2 \quad \text{if :} \quad \begin{cases} R_K = h/e^2 \\ K_J = 2e/h \\ Q_X = e \end{cases}$$

Exactness ?!

➡ $$\boldsymbol{R_K\, K_J\, Q_X = (n\, i/G)\, (f_J\, /\, f_{SET})}$$

$G = N_P/N_S$
gain of the CCC



Closing the triangle: *second way* $Q = C \times U$

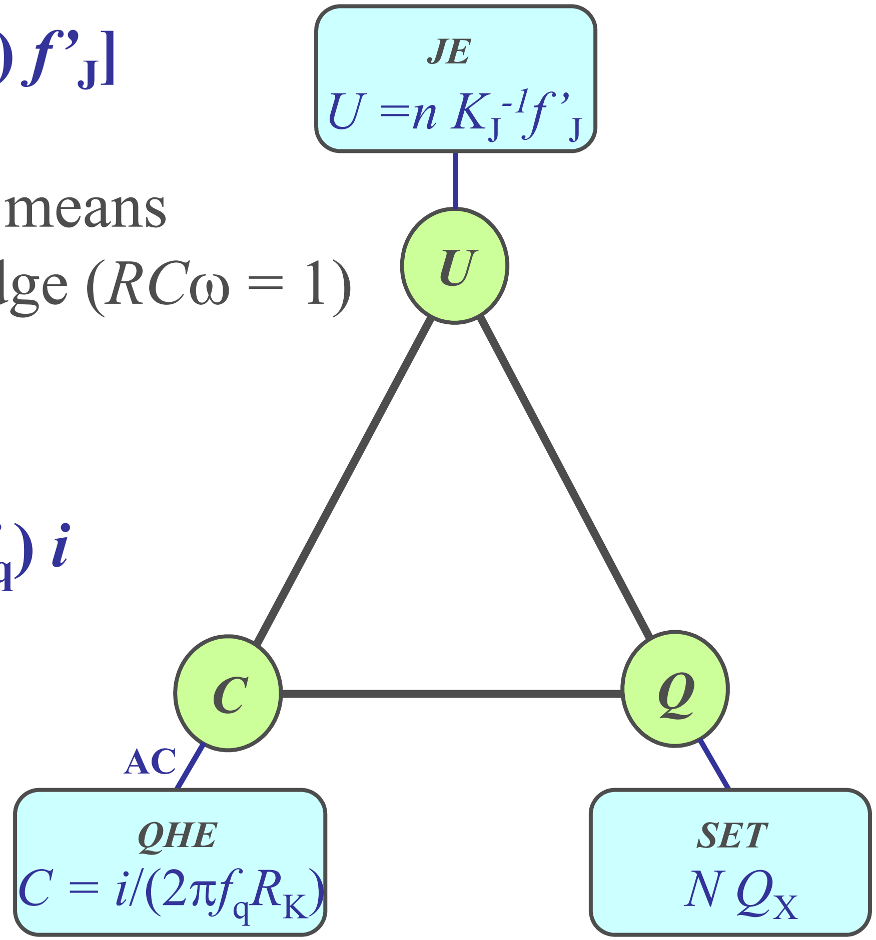
ECCS: Charging a capacitor electron per electron by a SET pump and measuring the voltage drop with Josephson standard

$$C_{ECCS} = K_J\, Q_X /[(n/N)\, f'_J]$$

C_{ECCS} compared to C_X calibrated by means of QHR standard *via* quadrature bridge ($RC\omega = 1$)

$$R_K\, K_J\, Q_X = (n/N)(C_{ECCS}/C_X)\, (f'_J / f_q)\, i$$

Quadrature bridge



LNE
Le progrès, une passion à partager

Determination of the charge quantum

Since a long time (1950's), the evaluation of the elementary charge *e*, is derived from a complex calculation and is no more related to an experiment.

In the framework of the LSA by the CODATA (>1973), **e** is no more an adjustable constant and its value is obtained from the relation giving α:

$$\alpha = \frac{\mu_0 c}{2h/e^2} \Rightarrow e = \sqrt{\frac{2\alpha h}{\mu_0 c}}$$

CODATA 2006: $e = 1.602\ 176\ 487$ C and $\sigma = 2.5\ 10^{-8}$

α from $\boldsymbol{a_e}$, ***h/m*** and $\boldsymbol{R_K}$ (Calc. capacitor + QHE)

h *via* $\boldsymbol{K_J^2 R_K}$ (WB + QHE + JE)

To combine the three experiments QMT, calculable capacitor and watt balance

⇒ a first determination of *e* involved in SET devices **without assuming** that $R_K = h/e^2$ and $K_J = 2e/h$

Determination of the charge quantum

The watt balance provides the SI value of the product $K_J^2 R_K$

$$K_J^2 R_K = A_1\{f^2_J/(Mgv)\}_{SI}$$

A_1 : dimensionless factor, f_J : Josephson freq.
M: suspended mass, g : earth's gravitational accel.
v : constant speed of the moving coil within B.

The determination of R_K from calculable capacitor to the QHR standard

$$R_K = A_2\{(\Delta C f_q)^{-1}\}_{SI}$$

A_2 : dimensionless factor
f_q : frequency of the balanced quadrature bridge
ΔC: capacitance variation of the calc. capacitor

QMT$_{U = RI}$

$$R_K K_J Q_X = n(i/G)(f_J/f_{SET}) \Rightarrow Q_X = A_3\{(\Delta C f_q M g v)^{1/2}/f_{SET}\}_{SI}$$

QMT$_{Q = CV}$

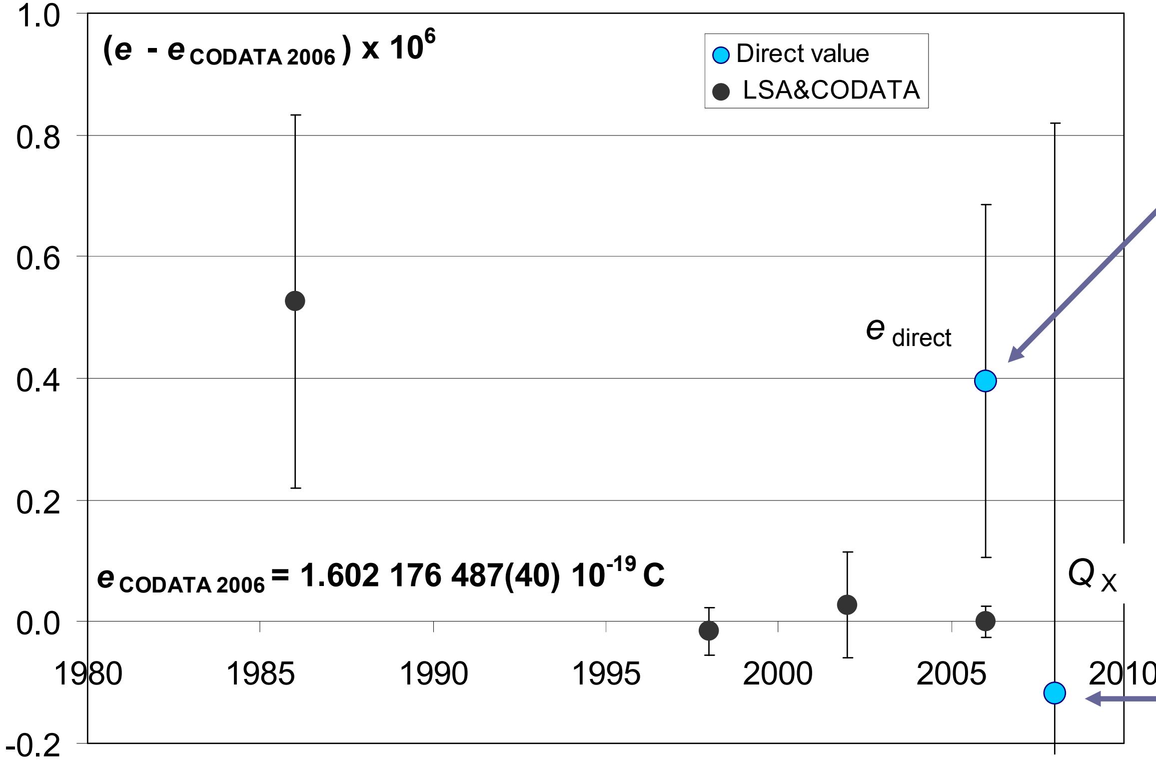
$$R_K K_J Q_X = A_4(n/N)(C_{ECCS}/C_X)(f'_J/f_q) \Rightarrow Q_X = A_5\{(\Delta C M g v/f_q)^{1/2}\}_{SI}$$

A_i dimensionless factors

- Piquemal *et al.*, in *Proc. of the international school of physics "Enrico Fermi"* IOS Press, 2007.
- Keller *et al.*, *Metrologia*, 2008

Determination of the charge quantum

Two direct values **independent** of the QHE and the JE



$e = [\alpha^3 A_r(e) M_u / (\mu_0 R_\infty N_A)]^{1/2}$

- $A_r(e)$: 2006 CODATA
- M_u : $= 10^{-3}$ kg.mol^{-1} exactly.
- R_∞ : 2006 CODATA
- α : $\Leftarrow a_e$ and $h/m_{Cs, Rb}$
- $N_A \Leftarrow N_A = V_m(\mathrm{Si})/(8^{1/2} d_{220}^{\,3})$

Q_X value from QMT at NIST with ECCS

$\sigma = 0.92\ 10^{-6}$

Comparisons: $Q_X \Leftrightarrow e$,

$R_K \Leftrightarrow h/e^2, K_J \Leftrightarrow 2e/h$

LNE
Le progrès, une passion à partager

OUTLINE

I) Introduction

1) Fundamental electrical metrology
2) Aims of the QMT

II) Arguments for closing the triangle

1) Uncertainty thresholds
2) Determination of the charge quantum

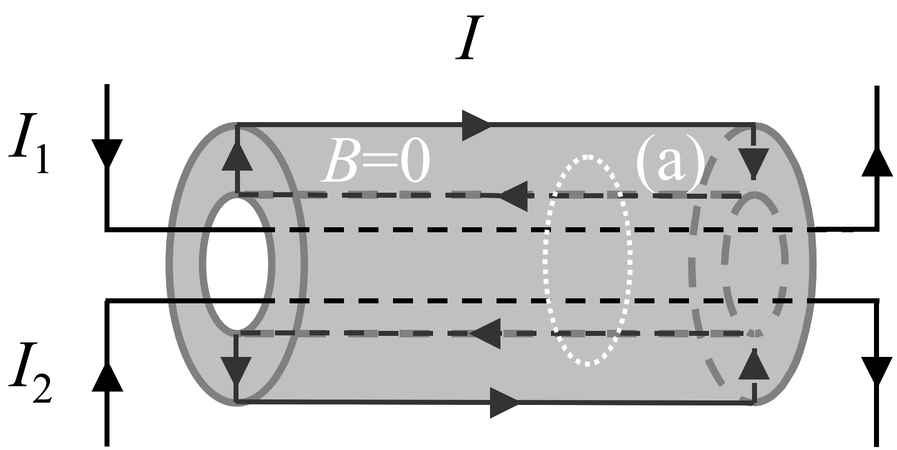
III) Experimental set-up based on the CCC: *U* = *RI*

1) CCC: Cryogenic Current Comparator
2) Overall set-up at LNE
3) First results

IV) Conclusion

Principle of the CCC *Harvey 1972*

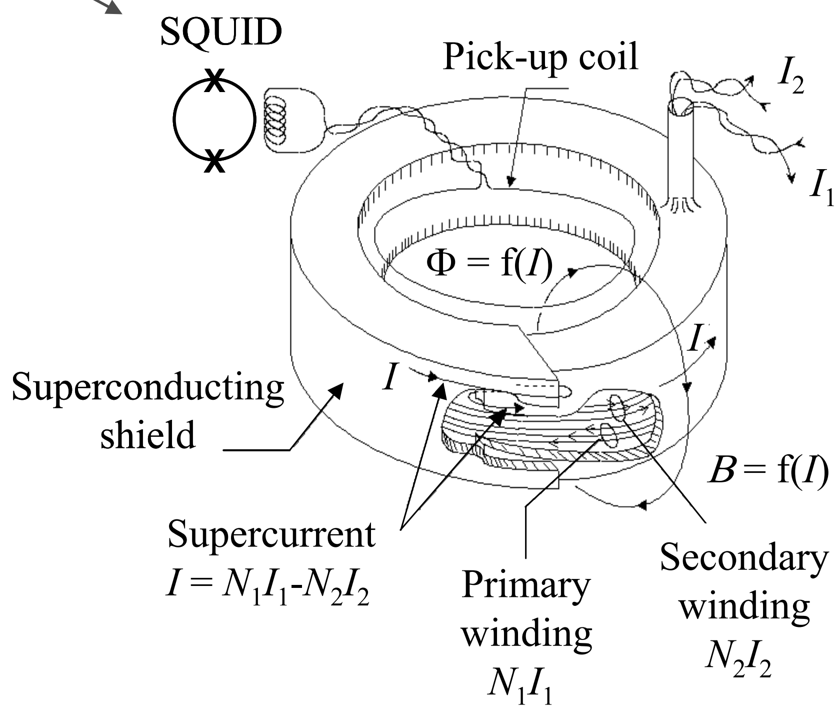
➡ **SQUID:** Superconducting QUantum Interference Device: very sensitive magnetic flux detector **$\delta\Phi \approx$ few $\mu\Phi_0/\mathrm{Hz}^{1/2}$ (typ.)**



Application of the Ampère's law :

$$\int_{(a)} B.\mathrm{d}l = 0 = \mu_0\,(I_1+I_2-I)$$

$$I = I_1+I_2$$



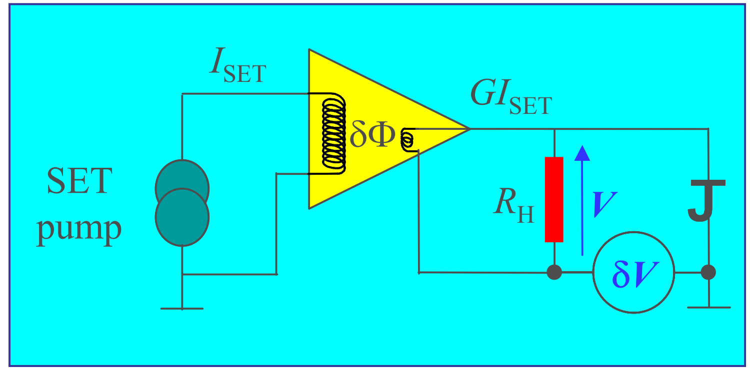
ampere - turn balance : $I = 0$

$$N_2I_2 = N_1I_1$$

$$\mathbf{G = I_2/I_1 = N_1/N_2}$$

Experimental set-ups based on CCC

- **CCC as current amplifier**



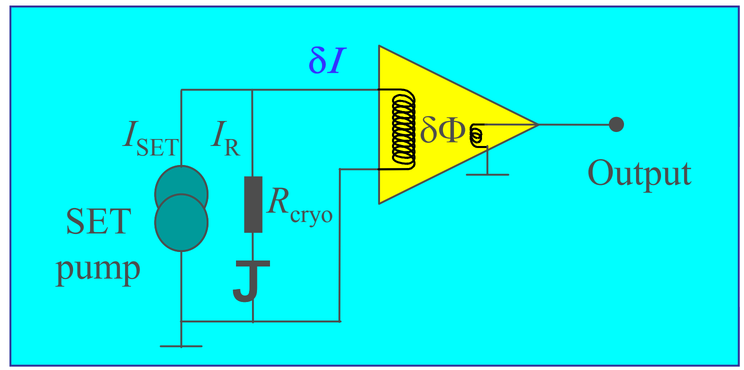
Two detection levels: magnetic flux and voltage

Highly accurate gain: $\sigma_G < 10^{-9}$

$$\delta V/V = ([\delta I^2_{CCC} + (4kT/G^2R_H)]/I^2_{SET} + \delta V^2_{ND}/V^2)^{1/2} \approx \delta I_{CCC}/I_{SET}$$

CCC (G = 40 000): $\delta I < 1$ fA/Hz$^{1/2}$, t_{meas} = 10 hours, $\sigma_{<I>}/I < 10^{-7} \Rightarrow I > 60$ pA

- **CCC as current detector**



Single detection level

$$\delta I/I = [4kT/R_{cryo})]^{1/2}/I_{SET}$$

R_{cryo} = 100 MΩ at 4.2 K: $\sigma_{<I>}/I < 10^{-7} \Rightarrow I > 80$ pA

- **in both cases:** SQUID operates at $\delta\Phi \approx 0$ (Flux Locked Loop)

LNE
Le progrès, une passion à partager

Present status of *U* = *RI* triangle experiments

Up to now, 2 laboratories have carried out measurement of current delivered by a SET device with a CCC

1) NPL: SETSAW device

J.T. Janssen and A. Hartland, 2000

Standard uncertainty: **3 fA** for **1 nA** of current

2) LNE: 3-junctions R pump

From 2000 to 2006, with CCC in **non accurate** mode

B. Steck *et al*, Metrologia 08

Best Type A uncert.: **60 aA** for 1**6 pA** (3.9 ppm)

Since 2007, with CCC in **accurate** mode

B. Steck – N. Feltin *et al*, CPEM' 08

Best Type A uncert.: **24 aA** for **6 pA** (4 ppm)

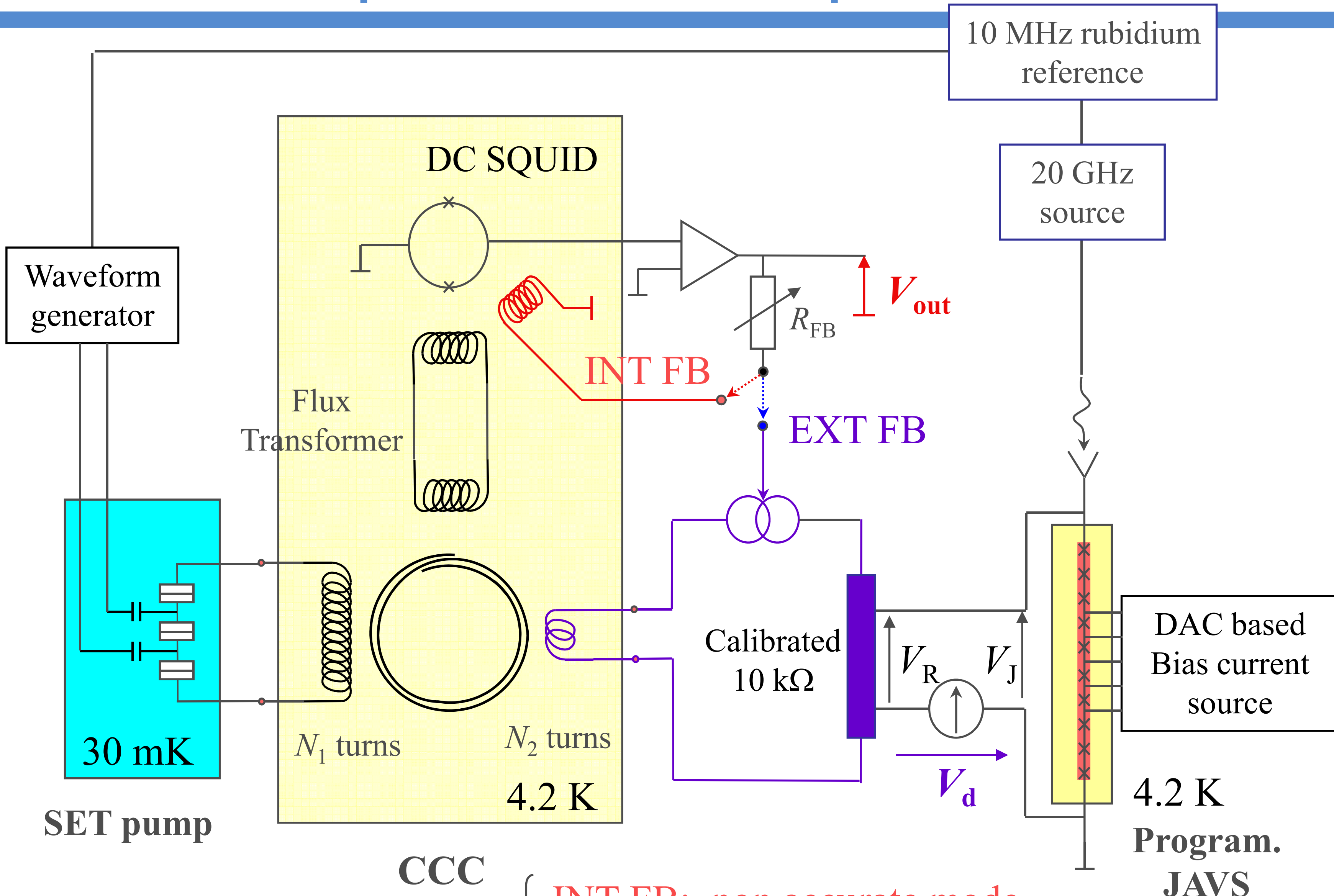
Towards a closure of the triangle via U = RI at 1 ppm next year !

10^{-6} on 1 pA ⟹ *6 electrons per second !*

LNE

Le progrès, une passion à partager

Principle of the QMT set-up at LNE

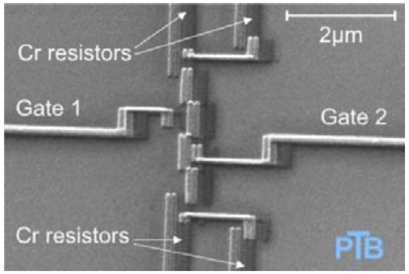


INT FB: non accurate mode

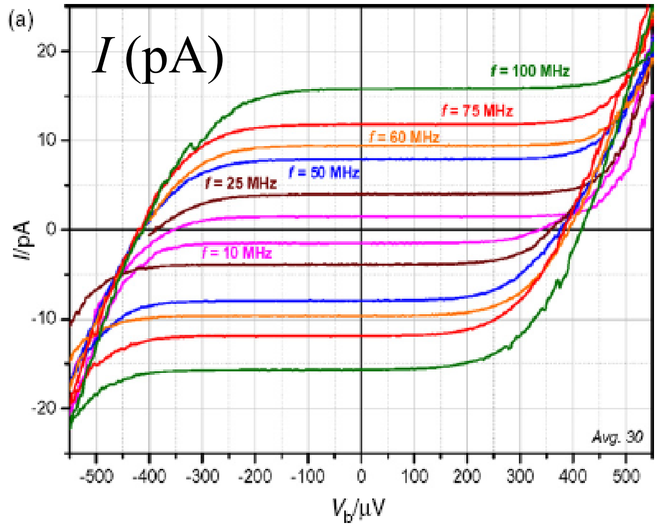
EXT FB: accurate mode

Results on 3-junctions R pump (SQUID in int FB)

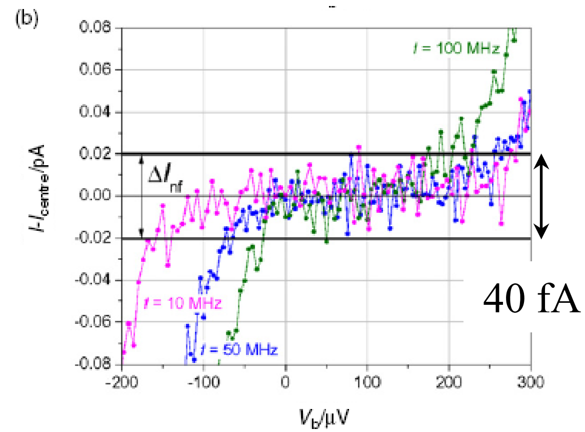
- with PTB pump



Zorin *et al.* , 2000

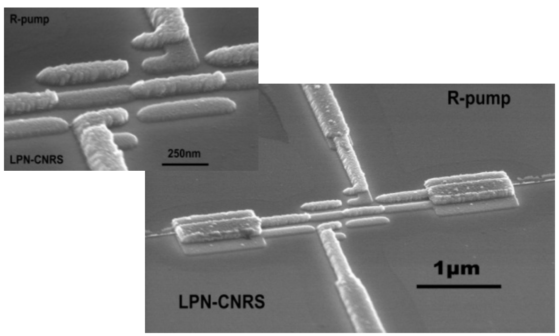


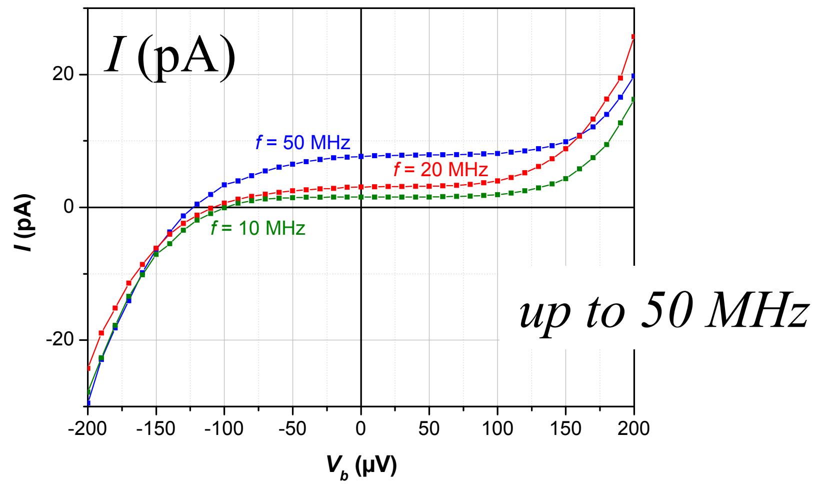
B. Steck *et al*, Metrologia 2008



- with pump from LPN

(Laboratoire de Photonique et de Nanostructure)





LNE
Le progrès, une passion à partager

Results on 3-junctions R pump (SQUID in int FB)

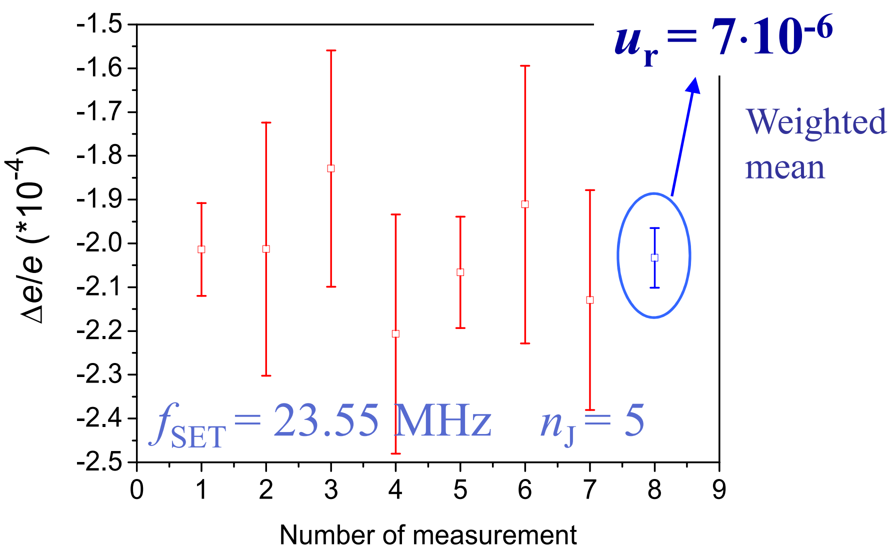
Two key issues:

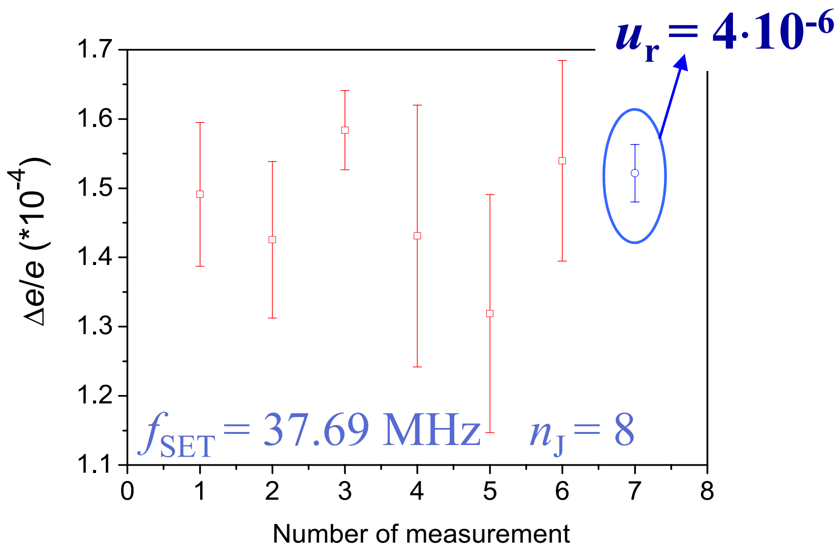
- No real measurement of the flatness of the current step
- No idea of the quantization level !

⇒ go to the accurate mode of the system involving resistance calibrated in terms of R_K *and the JAVS with the target uncertainty of one part in* 10^6

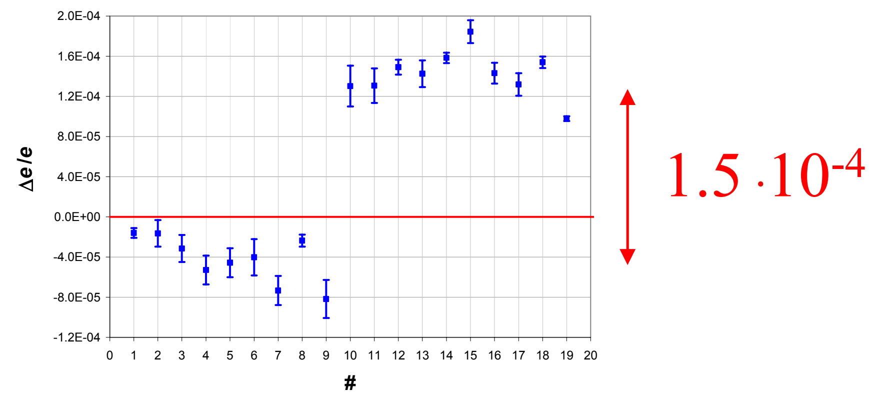
Data recently measured at LNE (SQUID in ext FB)

$$Q_X = (N_2/N_1)(n_J f_J/K_J - V_d)/(f_{SET}R) \quad \Rightarrow \quad \Delta e/e = (Q_X - e_{CODATA})/e_{CODATA}$$





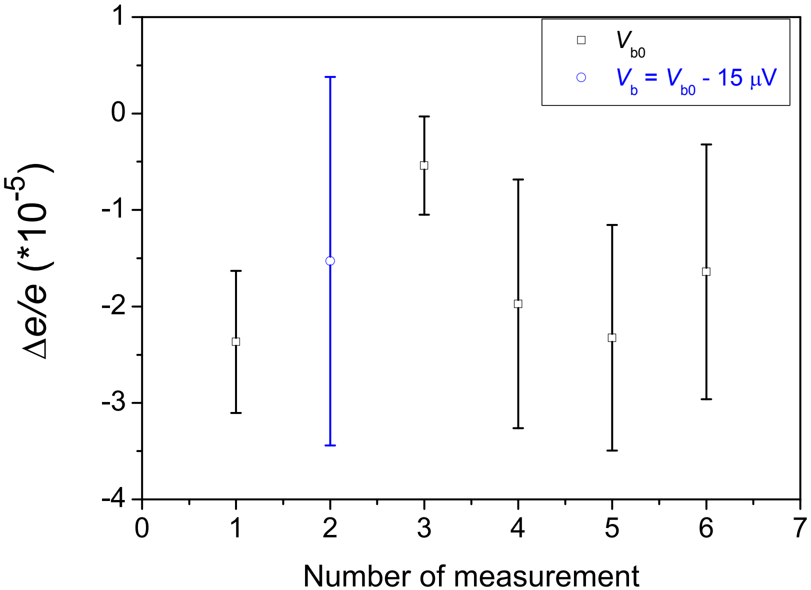
But irreproducibilities !



B. Steck *et al.*, CPEM'08
N. Feltin, satellite meeting- CPEM'08

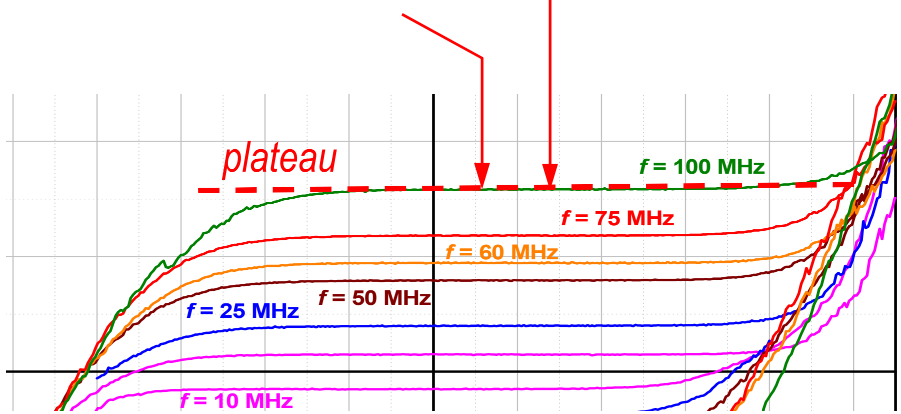
QMT : very *first measurement of flatness*

f_{SET} = 37.69 MHz and n_J=5 f_J=73 GHz



V_o= center of the plateau

V_1= V_o-15 µV



Long time measurements performed with various bias voltages in order to check the flatness of current steps

⇒ The plateau is flat within one part in 10^5

LNE
Le progrès, une passion à partager

IV- Conclusion (1)

- **Technical and technological challenges**
  - Development of CCC as current amplifier
  - Development of single charge transport devices as current source with $I >> 1$ pA
    Growing number of new devices, some are very promising
    e.g. - Hybrid SINIS SET turnstile (Pekola *et al.*)
    - electron pump based on silicon nanowire (Blumenthal *et al.*)

    ⇒ *Current plateaux observed at a level 100 pA*
  - Improvement of metrology relative to ultra low amplitude of current (< 1 nA)

IV- Conclusion (2)

- **Possible contributions**

  - to test and hopefully to enhance our confidence on QHE, JE and SET to provide h/e^2, $2e/h$ and e

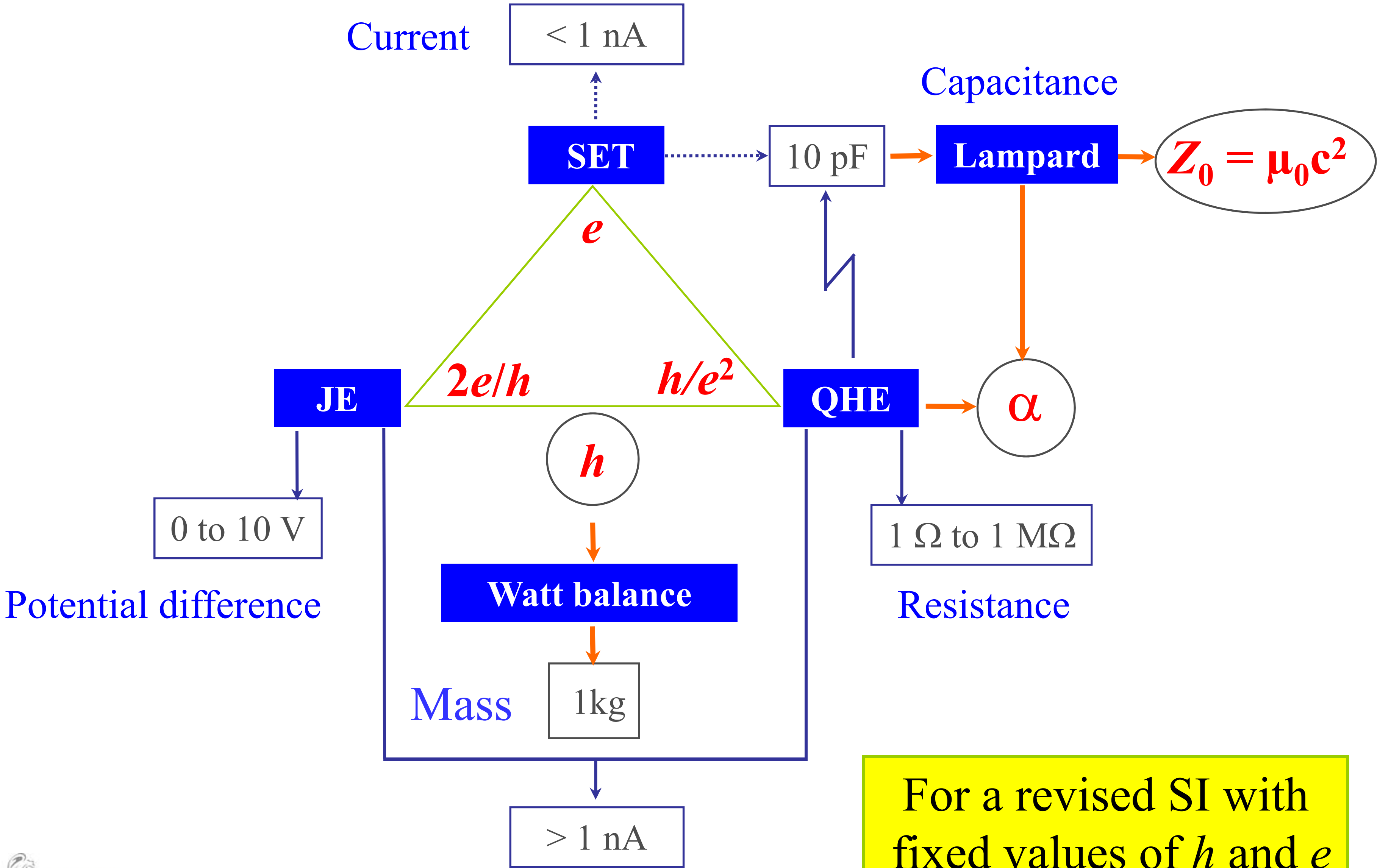
  - to improve knowledge on fundamental constants, in particular the **elementary charge**

  ⇒ This direct determination of ***e*** *via* QMT, calculable capacitor and watt balance links up with the historical experiment of Millikan, early last century

  - to give some elements of thought about a redefinition of electrical units and a revision of the SI

LNE
Le progrès, une passion à partager

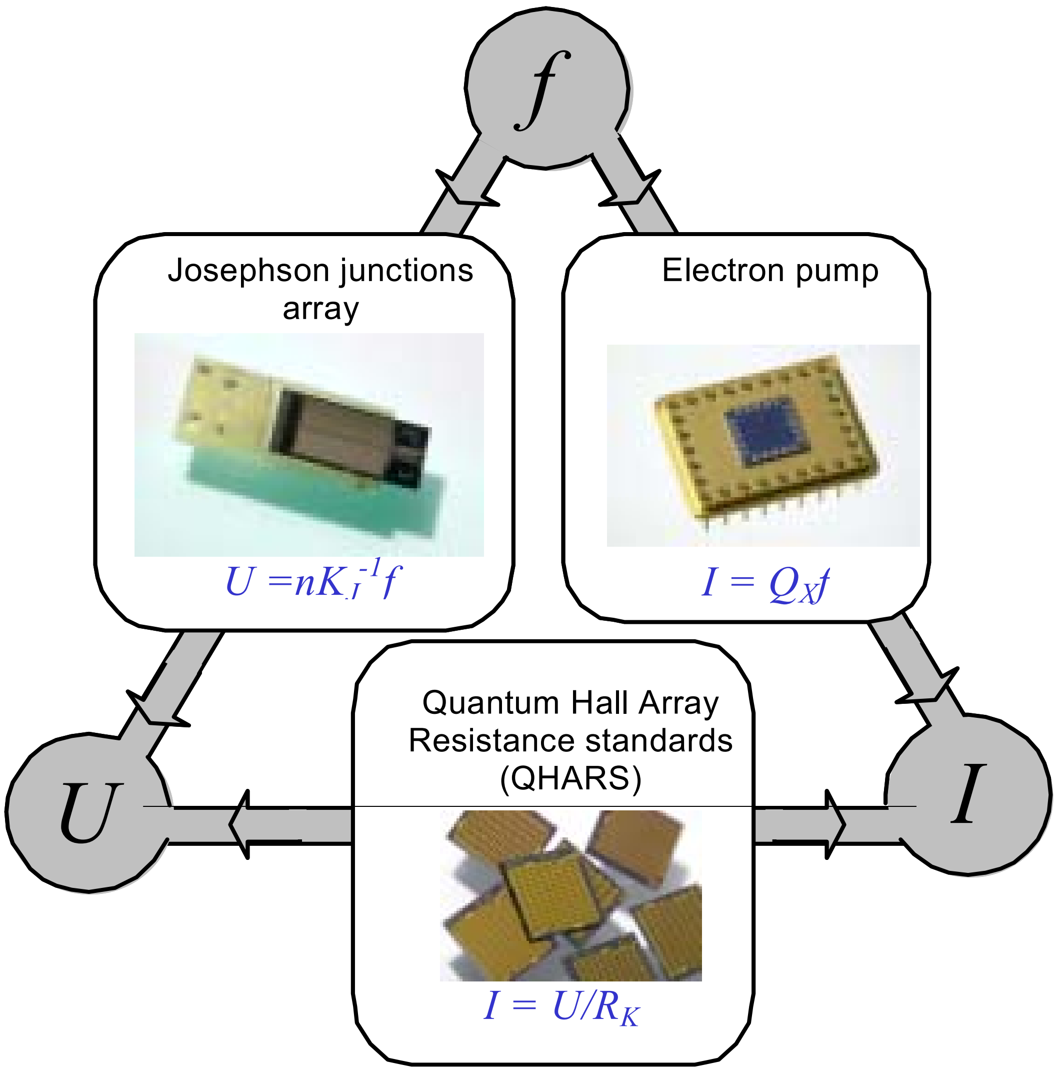
Impact of electrical constants in Metrology



For a revised SI with fixed values of *h* and *e*

LNE
Le progrès, une passion à partager

Muchas gracias !

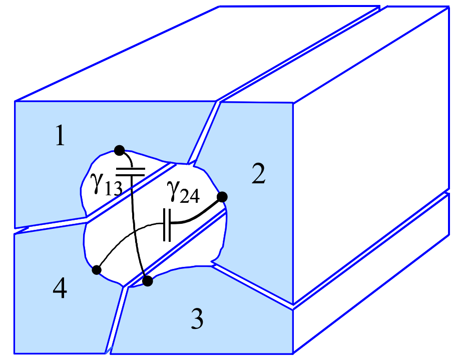


LNE
Le progrès, une passion à partager

Cross calculable capacitance standard

Theorem of A. Thompson and D. Lampard (1956):
For a **cylindrical** system of 4 **isolated** electrodes of **infinite** length and **placed in vacuum**,

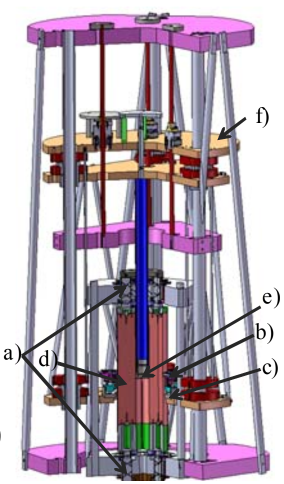
$$\exp(-\pi\,\gamma_{13}/\varepsilon_0) + \exp(-\pi\,\gamma_{24}/\varepsilon_0) = 1$$



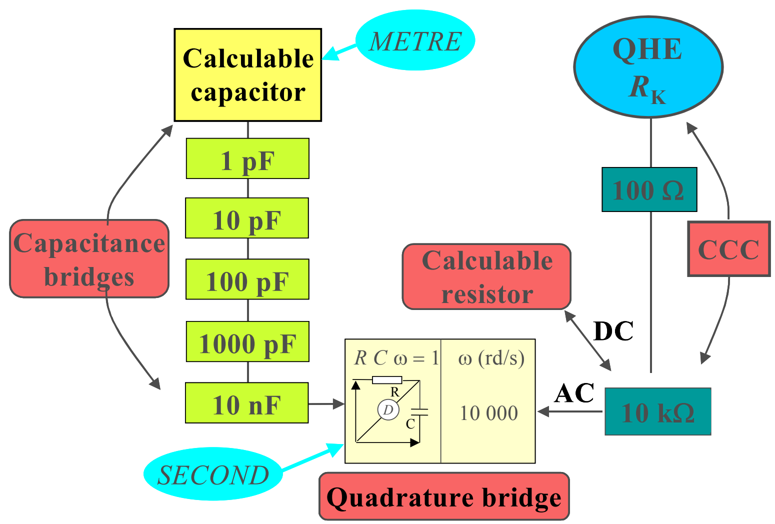
In the case of a **perfect** symmetry with **identical** γ_{ij}

$$\gamma_{13} = \gamma_{24} = \gamma = (\varepsilon_0 \ln 2)/\pi = 1.953\ 549\ 043\ \ldots\ \text{pF/m} \quad \Rightarrow \quad \Delta C = \gamma \Delta L$$

LNE, five-electrodes capacitor



2010

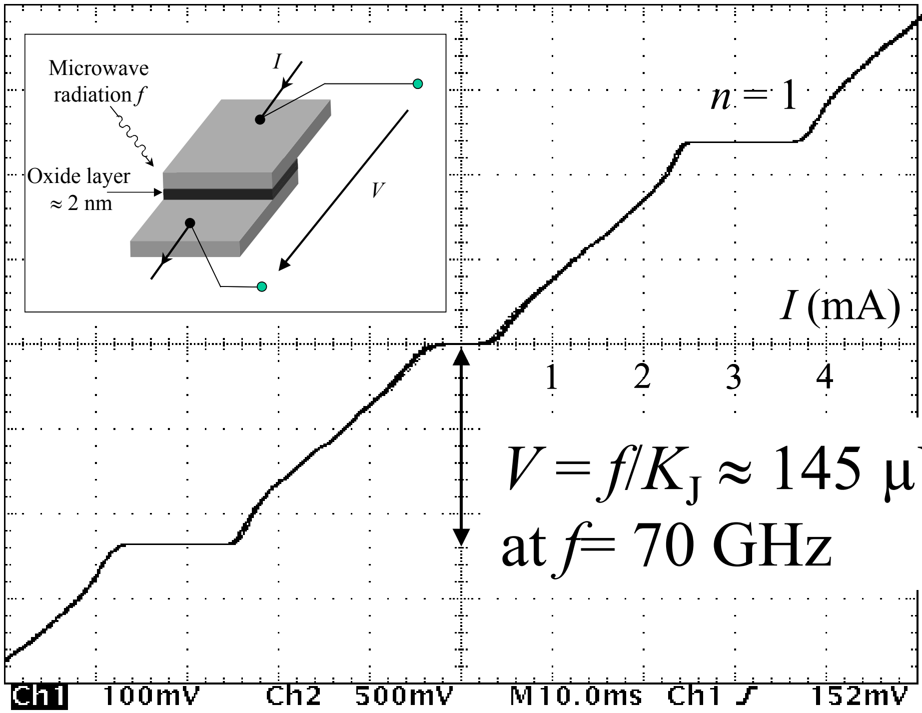


LNE-2000, R_K = 25 812.808 1(14) Ω

Josephson voltage standards

Brian Josephson 1962

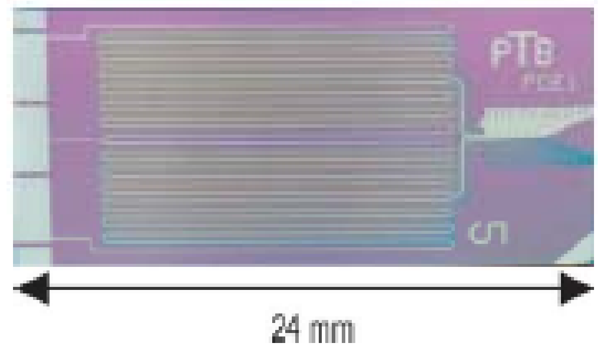
Quantum effects occurring between two superconducting electrodes separated by a small region where the superconductivity is weakened: *thin insulating film*

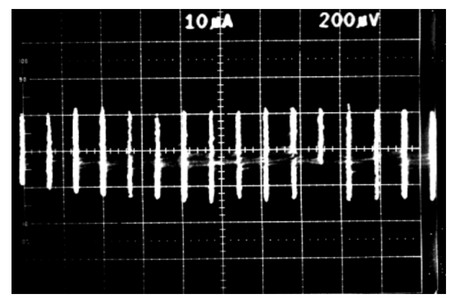


$V = f/K_J \approx 145\ \mu V$
at $f = 70$ GHz

$K_J = 2e/h$, the Josephson constant

10 V Josephson junction arrays





Steps around 10 V

LNE

Le progrès, une passion à partager

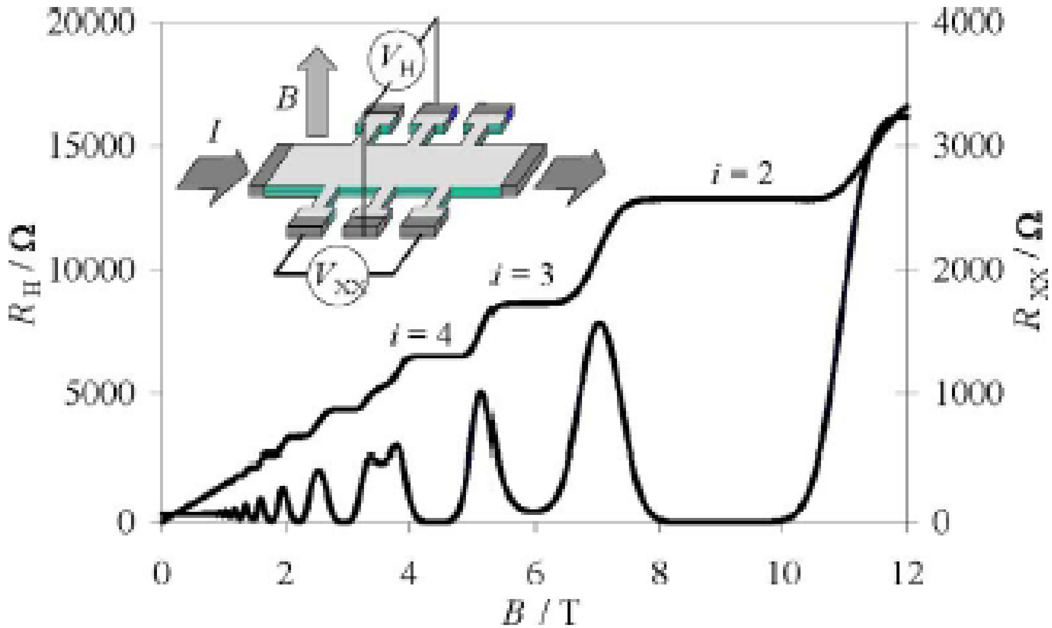
⇒ Programmable arrays for DC and AC applications

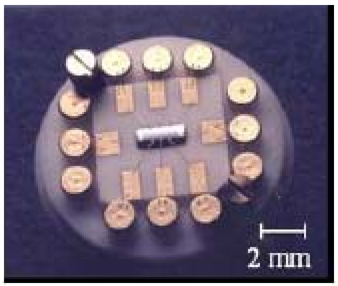
Quantum Hall resistance standards

Klaus von Klitzing, 1980

At low temperature and under high magnetic field,
the Hall resistance of the 2DEG exhibits plateaux centred
on quantized values:

$\boldsymbol{R_{\mathrm{H}}(i) = h/ie^2 = R_{\mathrm{K}}/i}$, where i is an integer,
R_{K} the von Klitzing constant.



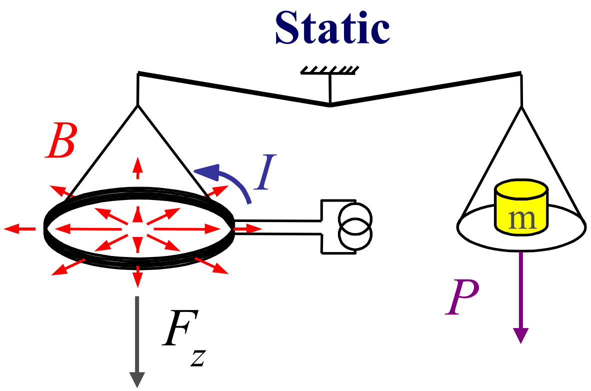


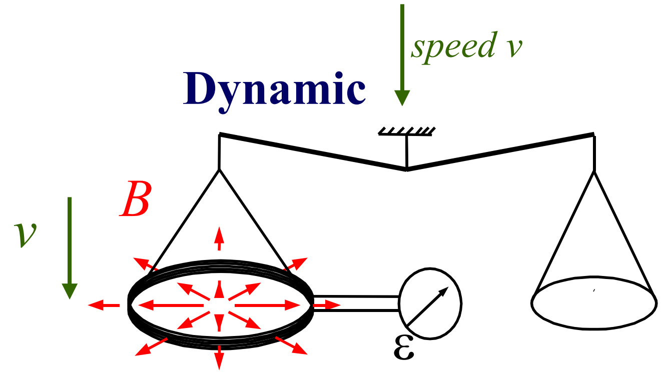
LEP 514 Hall bar sample
GaAs/AlGaAs heterostructure

⇒ Arrays of Hall bars for scaling up to 1.29 MΩ and down to 100 Ω

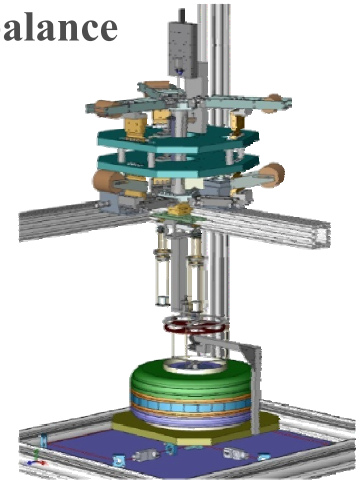
LNE
Le progrès, une passion à partager

Watt balance: monitoring the kilogram  electrically





LNE balance



$F_z = mg = -I\ \partial\Phi/\partial z$ $\qquad$ $\varepsilon = -\ \partial\Phi/\partial t = -\partial\Phi/\partial z\ v$

Equivalence between mechanical and electrical power $\Rightarrow F_z v = \varepsilon I \Rightarrow$ $mgv = \varepsilon V/R$

- ε and V in terms of Josephson effect: $\varepsilon = n_1 f_1/K_J$, $V = n_2 f_2/K_J$
- R in terms of quantum Hall effect: $R = R_K/i$

$\Rightarrow$ $$mgv = \frac{A}{K_J^2 R_K}$$

where $A = n_1 f_1\, n_2 f_2 i$

$\Rightarrow$ $$m = h\,\frac{A}{4gv}$$, assuming $K_J = 2e/h$ and $R_K = h/e^2$

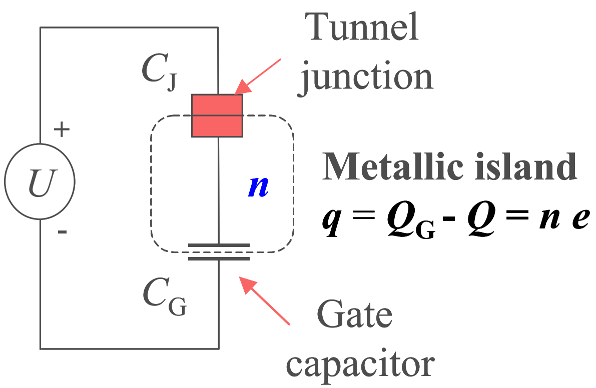
Towards a redefinition of the kilogram in term of the Planck constant h ?!

LNE

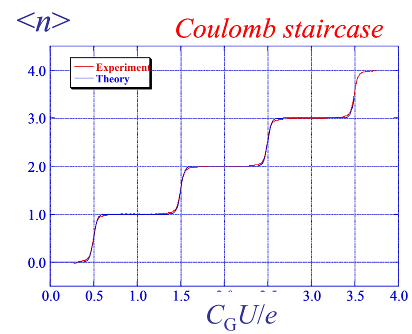
Le progrès, une passion à partager

Single electron tunneling: *towards quantum current standard*

Electron box



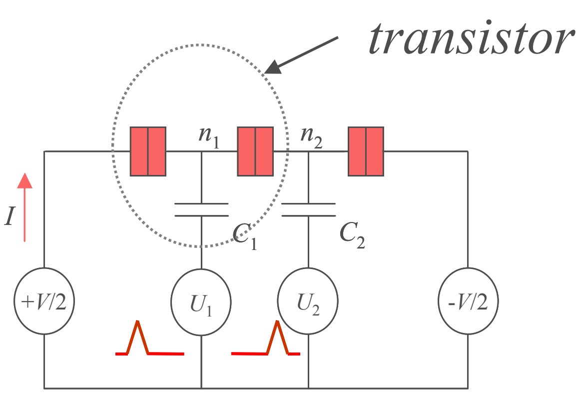
Coulomb Blockade of the tunneling events when $n - 1/2 < C_G U/e < n + 1/2$



- Thermal fluctuations of n are negligible if $k_B T << e^2/C_i$
- Quantum fluctuations of n negligible when $R_j >> R_K$

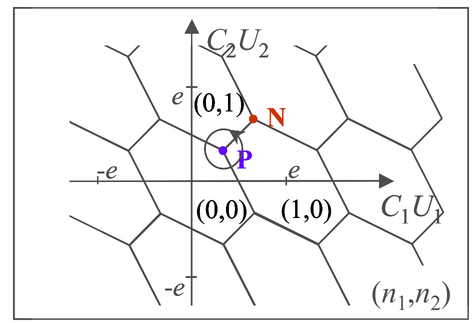
The wave function of electron in excess on the island is well localised

3-junctions electron pump



Two modulation signals at frequency f phase-shifted by $\Phi = \pi/2$

$$\Rightarrow \quad I = e \times f$$



Minimum energy states of the pump As a function of U_1 and U_2

For $f = 100$ MHz $\Rightarrow I = 16$ pA

AnimPompe.exe