



**NRC-CNRC**

*Institute  
for National  
Measurement  
Standards*

# ***Fundamental Constants – the Ultimate Foundation of the SI***

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National Research  
Council Canada

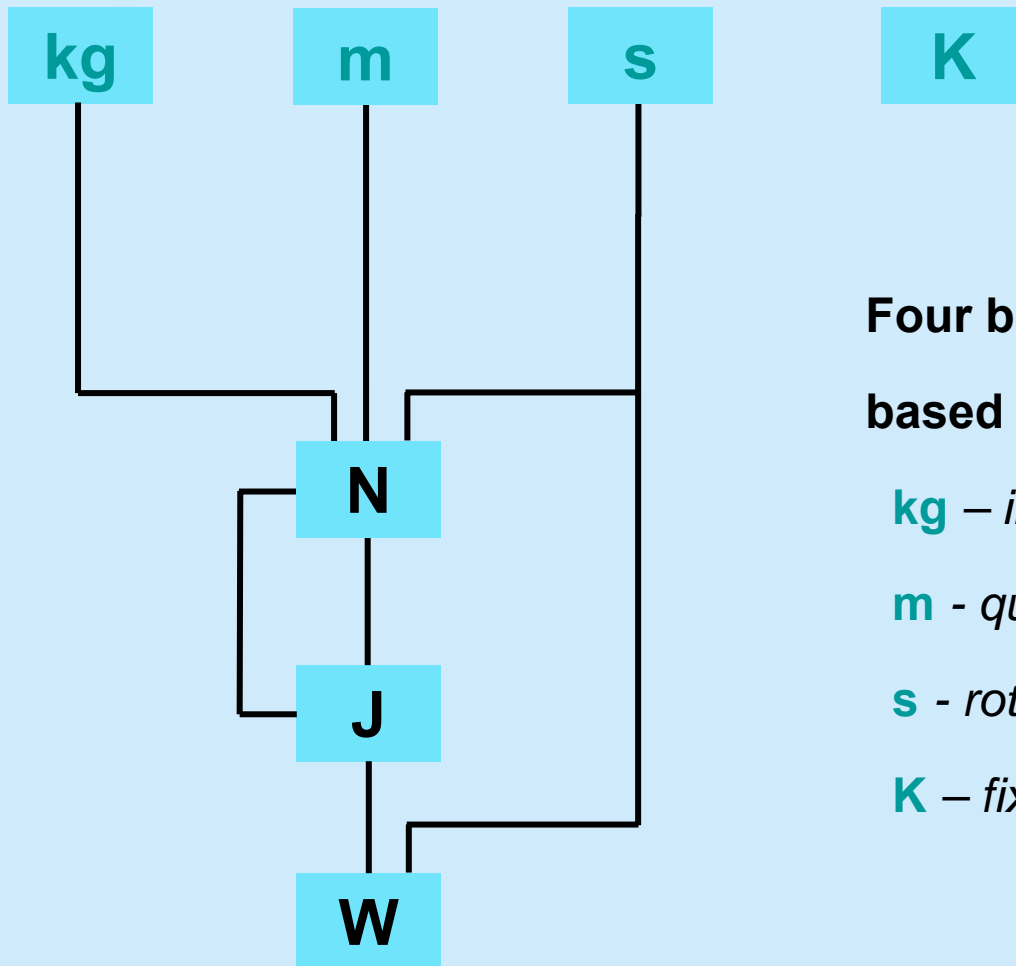
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## *Outline*

- The evolution of the SI to its present state
- Our understanding of the fundamental constants of nature
- Why should we change the SI ?
- The new SI - what, who and when

# The Original SI ~ 1880



**Four base units, all independent and all based on artifacts.**

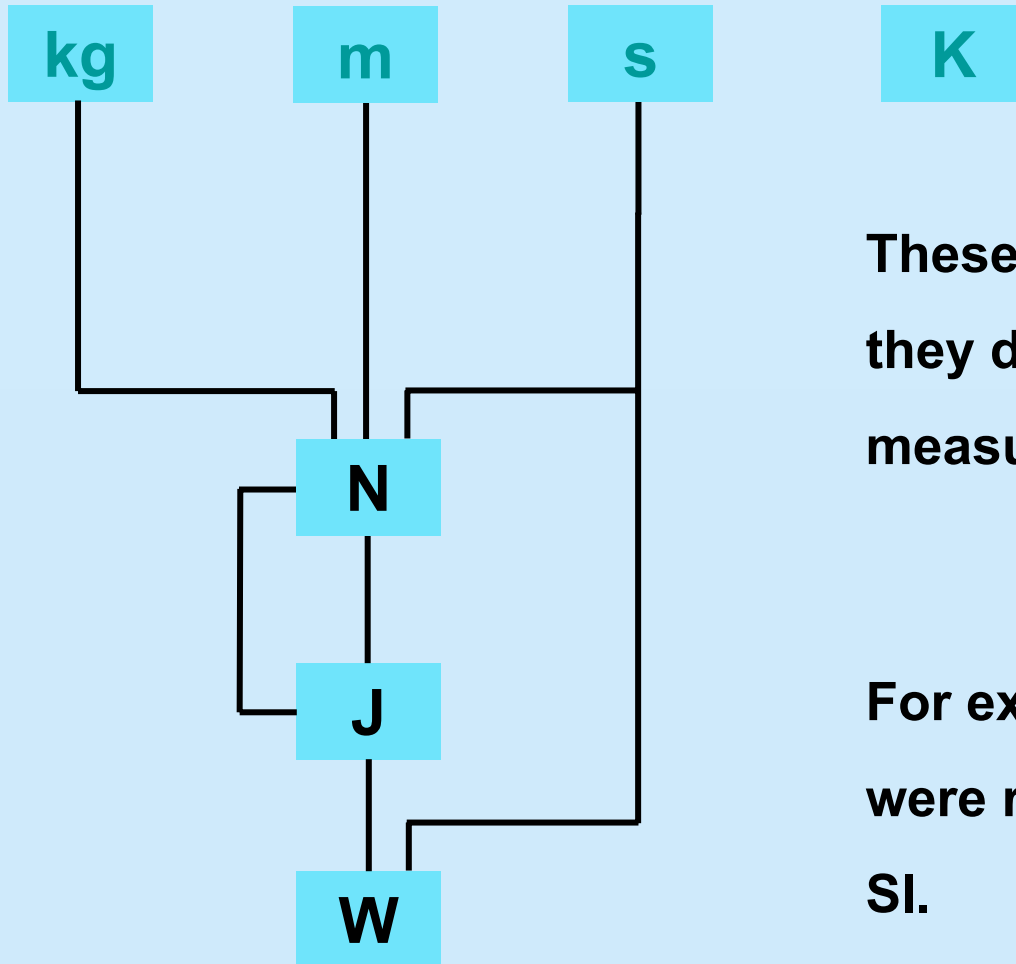
**kg** – *international prototype, a decilitre of water*

**m** - *quadrant of the earth*

**s** - *rotation of the earth*

**K** – *fixed points of water*

# *The Incomplete SI*



These four base units were fine but they did not address all measurements.

For example, electrical measurements were routinely performed outside the SI.

# *Mercury column - 1908 Absolute Ohm*



# *The SI Evolves ...*

*Changes and refinements in definitions of the second, the kelvin, the metre...*

*New base units: mole, candela*

*New SI units*

*radiation and dose*

*photometry, radiometry ....*

*SI recognition of other practical units.*

**Normal Hydrogen Scale:** adopted by the 1<sup>th</sup> CGPM in 1889

Primary thermometer: gas thermometer with H<sub>2</sub>

Fixed points: **ice und boiling point of water**

Interpolation: mercury glass thermometer

**ITS-27:** adopted by the 7<sup>th</sup> CGPM in 1927

Fixed points: **ice point, boiling points of O<sub>2</sub>, H<sub>2</sub>O, S, melting points of Sb, Ag Au**

Interpolation: Pt resistance thermometer, Pt-Rh thermocouple, opt. pyrometer (Wien)

**ITS-48:** adopted by the 9<sup>th</sup> CGPM in 1948

Fixed points: ice point, boiling points of O<sub>2</sub>, H<sub>2</sub>O, S, melting points of Sb, Ag Au

Interpolation: Pt resistance thermometer, Pt-Rh thermocouple, opt. pyrometer (Planck)

**ITPS-48:** adopted by the 11<sup>th</sup> CGPM in 1960

Definition of the degree kelvin: **triple point of water**

Fixed points: boiling points of O<sub>2</sub>, H<sub>2</sub>O, melting points of Zn, Sb, Ag Au

Interpolation: Pt resistance thermometer, Pt-Rh thermocouple, opt. pyrometer (Planck)

**ITPS-68:** adopted by the 13<sup>th</sup> CGPM in 1968

Definition of the kelvin: triple point of water

Fixed points: triple point of H<sub>2</sub>, boiling points of H<sub>2</sub>, Ne, O<sub>2</sub>, H<sub>2</sub>O, freezing points of Sn, Zn, Sb, Ag Au

Interpolation: Pt resistance thermometer, Pt-Rh thermocouple, opt. pyrometer (Planck)

# Definition of the Kelvin 1968

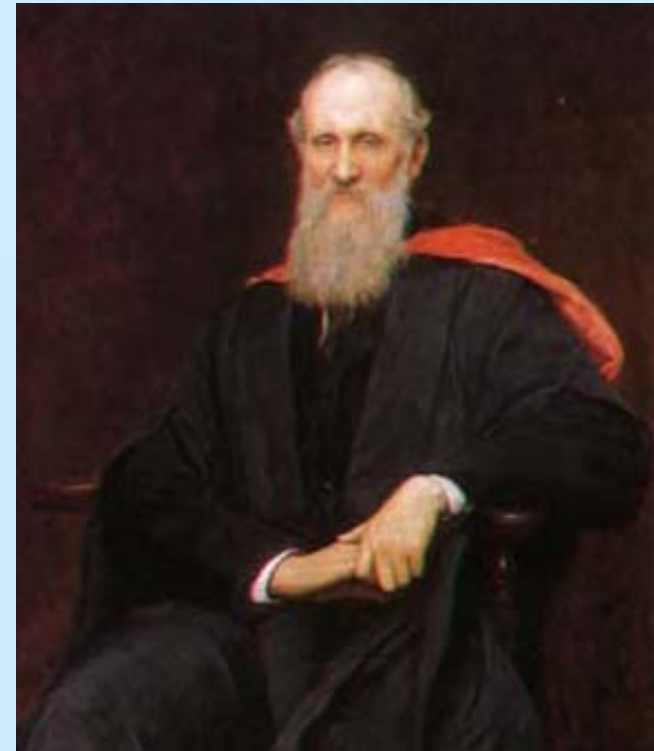
The **kelvin**, unit of the thermodynamic temperature, is the 273.16<sup>th</sup> part of the thermodynamic temperature of the **triple point of water**.

[13<sup>th</sup> CGPM: Metrologia, 4 (1968) 43]

$$T_{\text{TPW}} = 273.16 \text{ K}$$

$$p_{\text{TPW}} = 611.657 \text{ Pa}$$

No uncertainty in the TPW, since it defines the **unit** !



Suggested by Lord Kelvin of Largs  
in 1854 !



# *And in Time (Interval)*

The original definition was to be based on the average solar day divided into 24 hours, with each hour divided into 60 minutes, with each minute divided into 60 seconds.

early years – the second was left in the hands of the astronomers

1800's – general realization that the second must be changing, but slowly

1929 – Danjon suggests a time scale based on planetary motion

1948 – Clemence suggests a practical way of extracting this time scale

1950-52 – astronomers adopt ephemeris second based on the sidereal year 1900

1954 – CIPM (Danjon president) suggests definition in terms of tropical year 1900

1954 – CGPM passed authority to CIPM to redefine the second

1955 – IAU general meeting hears about Cs standard, IAU approves tropical year 1900

1956 – CIPM hears of Cs, refines wording of ephemeris second for Official SI

1958 – IAU approves new wording

1958 – definitive measurement of ephemeris second in terms of Cs standard. This is the only use that was ever made of “the ephemeris second”, and the last input of astronomers to precision timekeeping.

1960 - CGPM ratifies the ephemeris second definition

## And more on Time Interval

- A new definition was based on observations of microwave resonance as the Caesium Atom Time Standard (CATS).

1958 – definitive measurement of ephemeris second in terms of Cs standard. This is the only use that was ever made of “the ephemeris second”, and the last input of astronomers to precision timekeeping: Cs hyperfine transition  $9\,192\,631\,770 \pm 20 \text{ Hz}_{\text{ephemeris}}$

1964 – CGPM empowers the CIPM to name atomic frequency standards

1964 – CIPM chooses Cs hyperfine transition to be  $9\,192\,631\,770 \text{ Hz}$

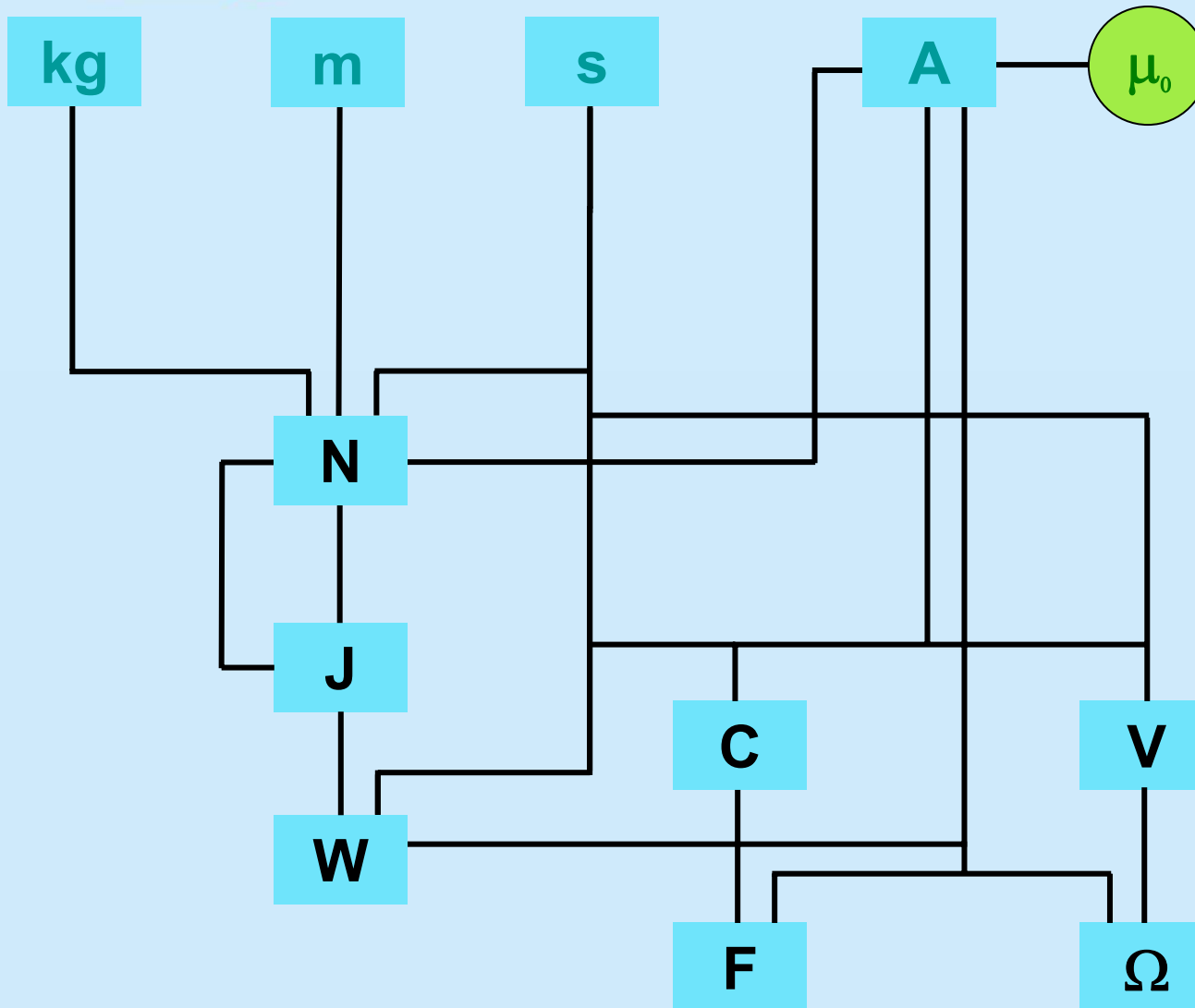
1967 – CGPM redefines the second in terms of the Cs hyperfine transition

1983 – the speed of light in vacuum is defined in terms of the (Cs) second

*“Over the course of the past four or five decades, the time interval unit has seen its uncertainty improved by some ten million times!”*

*Smaller uncertainty is ONLY achieved through change.*

# The SI & Electrical - 1948

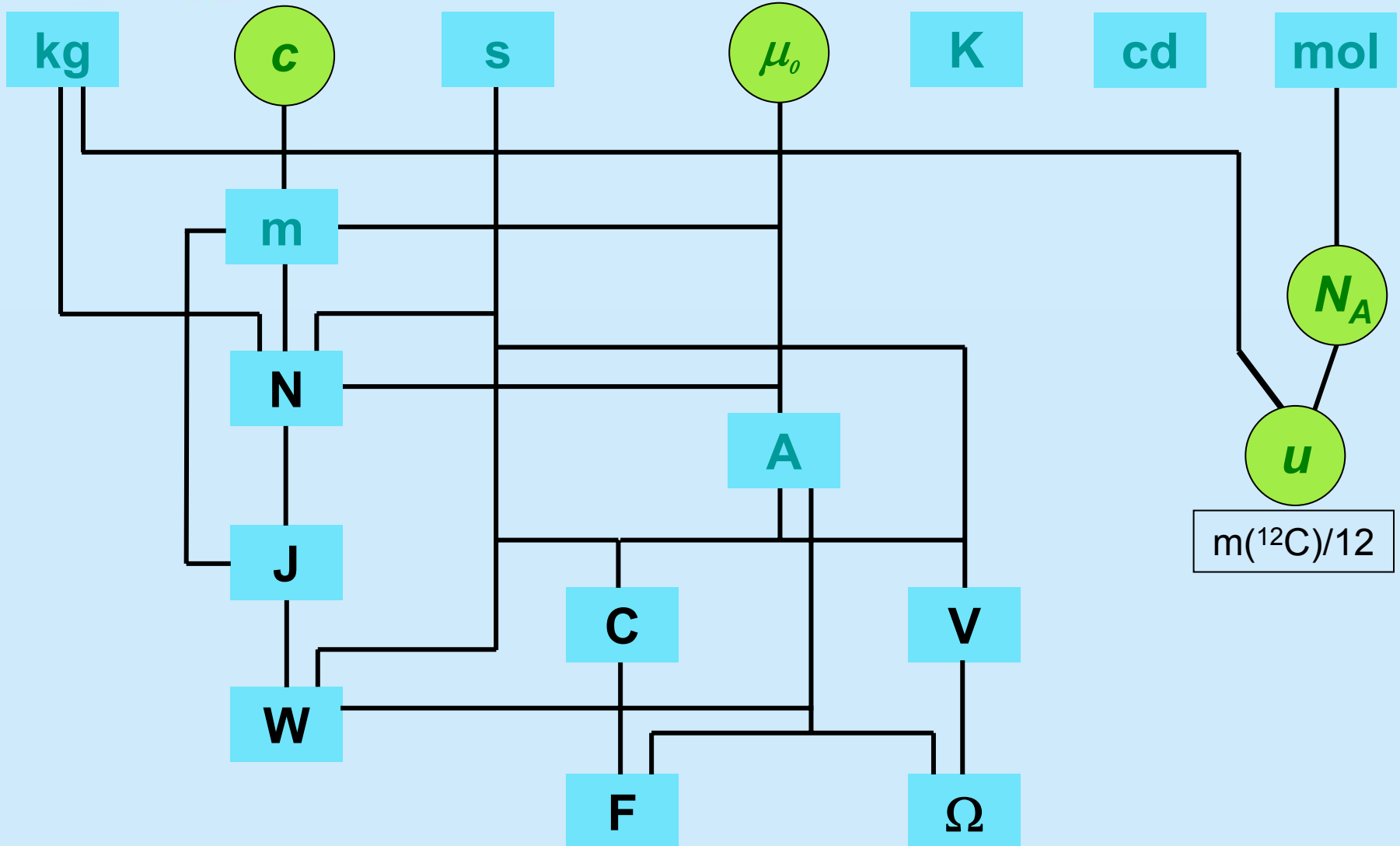


$\mu_0$  permeability  
of vacuum

Five base units, no longer independent and one fundamental constant.

Ampere is defined as the equivalence of mechanical and magnetic force.

# The SI in 1989



# Fundamental Constants

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## Our 2<sup>nd</sup> Theme

- What is a fundamental constant ?
- CODATA Task Group on Fundamental Constants
- Theories
- Experiments for important constants
- The present status

## What is a fundamental constant?

- Fundamental physical properties - but what is fundamental?
- There are many 'constants', but most are inter-related.

**They are the basis of how we describe and model all of our observations of physical processes. They frame**

The **BIG** Picture!

# The BIG Picture

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Dynamic ranges in metrology, measurements and science.

## Resistance

- Resistance Calibrations  $10 \mu\Omega$  to  $10 \text{ P}\Omega$  →  $10^{22}$
- Resistance Measurements  $10^{-27}\Omega$  to  $10^{18}\Omega$  →  $10^{45}$

## Distance

- E. Cornell, asymmetry of electron  $10^{-14} \text{ fm} = 10^{-29}\text{m}$   
& Diameter of earth orbit  $300 \times 10^6 \text{ km} = 3 \times 10^{11}\text{m}$  →  $10^{40}$
- Planck length ( $10^{-35} \text{ m}$ ) to solar system  $\sim 10^{12} \text{ m}$  →  $10^{47}$

Meaningful measurements over more than 40 orders of magnitude are a major challenge. Fundamental constants and quantum standards are essential for scaling over these ranges.

# Some obvious fundamental constants

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## Properties of things

- mass of the electron
- mass of the proton
- charge of the electron...

## Properties of quantized things

- flux quanta
- universal conductance

## Properties of space

- speed of light  $c$
- magnetic constant  $\mu_0$
- electric constant  $\varepsilon_0$

## Relationships between things

- gravitation constant -  $G$
- between energies -  $h, k, R_\infty$
- between impedances -  $\alpha$

But some are already fixed and most are inter-related.  
What can we do?



# Method of Least Squares

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## The First Least-Squares Adjustment of the Fundamental Constants

- Raymond T. Birge, 1929
- Cohen and Du Mond, "1965
- The 1973, 1986, 1998, 2002, 2006 CODATA Adjustment of the Fundamental Constants.

The 2006 LSA had 150 input data values, 135 distinct types or observational equations, 79 adjusted constants or unknowns.

It is a variance weighted, generalized, multivariate least squares adjustment with accounting of covariances.

**But with what equations and from what theories??**

# Theories of theories

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- Most theories are incremental – building on a previous theoretical frameworks.
- Thus newer, more detailed theories actually rely on, or modify, previous assumptions and approximations.
- In this manner modern theories are still critically dependent on older and much simpler classical models.
  - Many, many equations !!

# Classes of Theories

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Classical Newtonian mechanics  
Thermodynamics  
Electrodynamics  
Quantum mechanics  
Special relativity  
Quantum field theory  
Quantum electrodynamics  
Quantum chromodynamics  
The standard model.  
...

Newton, Huygens, Lagrange  
Carnot, Kelvin, Boyle, Joule  
Faraday, Maxwell  
Planck, Heisenberg, Dirac  
Einstein  
Born, Wigner  
Feynman, Schwinger, ...  
Gell-Mann, Yang, Mills, Gross  
Glashow, Weinberg  
*The pyramid of giants*

Each theory builds upon and incorporates what came before and all are tied to fundamental constants.

*We stand on the top of a pyramid of scientific giants.*

# Theories

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We assume that all these theories are right! Provided that they are :  
generally accepted  
within their assumptions  
within their approximations

Note - uncertainty budgets can be created for theories just like experiments.

We assume theories are right even though we also know that our theories are incomplete. No Grand unification theory. No link to gravity etc.

This is a necessary approximation to ask questions about fundamental constants.

# A Quick Reminder About Some Important Constants

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- The speed of light
- The Rydberg
- Relative atomic masses – (not a constant but important)
- The fine structure constant
- The Planck Constant
- The Avogadro Constant
  
- But there are others
  - The Boltzmann Constant, Gravitational Constant, the gas constant,  
...

# The speed of light

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- The speed of light has not always been a fundamental constant. Even after 1905 many believed that it simply could not be constant.
- Now (since 1983) we take it for granted and use a fixed value with zero uncertainty.  
$$c = 299\,792\,458 \text{ m / s}$$
- Strangely, this was not the first exactly fixed fundamental constant. Yes, fundamental constants do evolve, as our understanding evolves.

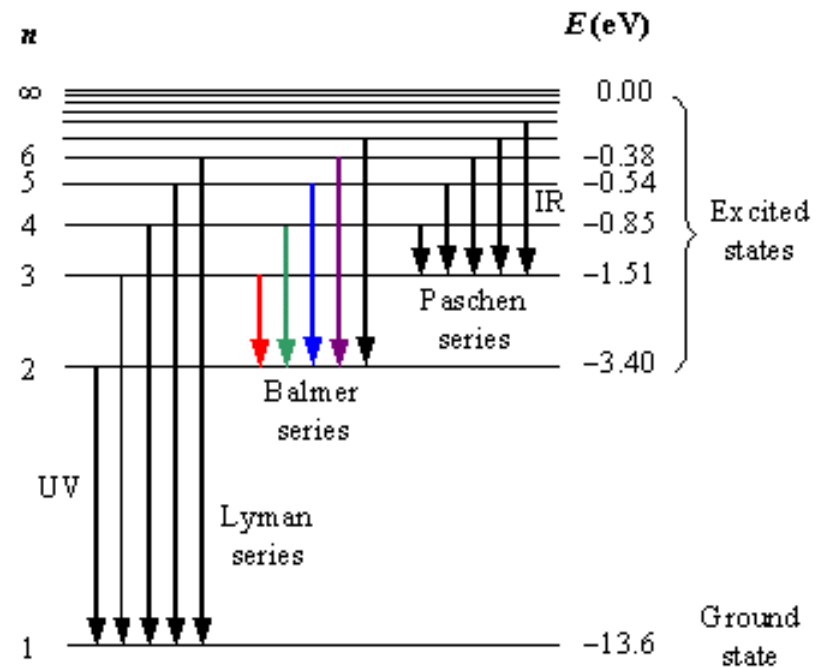
# What is the Rydberg?

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- Rydberg – Johannes Rydberg – in the 1880s described the frequency of light radiated when an electron changes bound states in hydrogen.
- originally applied only to hydrogen but now extends to other ‘simple molecules’.
- involves classical electrodynamic forces, nuclear forces, effects of virtual particles (or states), ...

# The Coarse Hydrogen Spectrum

- $1/\lambda = R_{\infty}(1/n_1^2 - 1/n_2^2)$
- relates the frequency of the transition to initial and final states  $n_1$  and  $n_2$
- Note depends on  $E = h\nu$



Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.



# The Rydberg

For different elements there is a different 'Rydberg'.

$R_M = R_\infty (1 + m_e/M)$ ,  $M$  is the mass of the protons

$$\begin{aligned} R_\infty &= m_e e^4 / ((4\pi\epsilon_0)^2 h^3 4\pi c) = m_e e^4 / (8\epsilon_0^2 h^3 c) \\ &= 10973731.568525 (73) \text{ m}^{-1} \end{aligned}$$

And

$$R_\infty = \alpha^2 m_e c / (4\pi h) = \alpha^2 e \lambda_e$$



# Experimental Determinations of the Rydberg

Summary of measured transition frequencies  $\nu$  considered in the present work for the determination of the Rydberg constant  $R_\infty$  (H is hydrogen and D is deuterium).

Authors, Laboratory,	Frequency interval(s)	Reported value ( $\nu$ /kHz) , Rel. stand uncert. $u_r$
(Fischer et al., 2004), MPQ	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.074(34) <b><math>1.4 \times 10^{-14}</math></b>
(Weitz et al., 1995), MPQ	$\nu_H(2S_{1/2} - 4S_{1/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	4 797 338(10) $2.1 \times 10^{-6}$
	$\nu_H(2S_{1/2} - 4S_{5/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	6 490 144(24) $3.7 \times 10^{-6}$
	$\nu_D(2S_{1/2} - 4S_{1/2}) - 1/4 \nu_D(1S_{1/2} - 2S_{1/2})$	4 801 693(20) $4.2 \times 10^{-6}$
	$\nu_D(2S_{1/2} - 4D_{5/2}) - 1/4 \nu_D(1S_{1/2} - 2S_{1/2})$	6 494 841(41) $6.3 \times 10^{-6}$
(Huber et al., 1998), MPQ	$\nu_D(1S_{1/2} - 2S_{1/2}) - \nu_H(1S_{1/2} - 2S_{1/2})$	670 994 334.64(15) $2.2 \times 10^{-10}$
(de Beauvoir et al., 1997), LKB/SYRTE	$\nu_H(2S_{1/2} - 8S_{1/2})$	770 649 350 012.0(8.6) $1.1 \times 10^{-11}$
	$\nu_H(2S_{1/2} - 8D_{3/2})$	770 649 504 450.0(8.3) $1.1 \times 10^{-11}$
	$\nu_H(2S_{1/2} - 8D_{5/2})$	770 649 561 584.2(6.4) $8.3 \times 10^{-12}$
	$\nu_D(2S_{1/2} - 8S_{1/2})$	770 859 041 245.7(6.9) $8.9 \times 10^{-12}$
	$\nu_D(2S_{1/2} - 8D_{3/2})$	770 859 195 701.8(6.3) $8.2 \times 10^{-12}$
	$\nu_D(2S_{1/2} - 8D_{5/2})$	770 859 252 849.5(5.9) $7.7 \times 10^{-12}$
(Schwob et al., 1999, 2001), LKB/SYRTE	$\nu_H(2S_{1/2} - 12D_{3/2})$	799 191 710 472.7(9.4) $1.2 \times 10^{-11}$
	$\nu_H(2S_{1/2} - 12D_{5/2})$	799 191 727 403.7(7.0) $8.7 \times 10^{-12}$
	$\nu_D(2S_{1/2} - 12D_{3/2})$	799 409 168 038.0(8.6) $1.1 \times 10^{-11}$
	$\nu_D(2S_{1/2} - 12D_{5/2})$	799 409 184 966.8(6.8) $8.5 \times 10^{-12}$
(Bourzeix et al., 1996), LKB	$\nu_H(2S_{1/2} - 6S_{1/2}) - 1/4 \nu_H(1S_{1/2} - 3S_{1/2})$	4 197 604(21) $4.9 \times 10^{-6}$
	$\nu_H(2S_{1/2} - 6D_{5/2}) - 1/4 \nu_H(1S_{1/2} - 3S_{1/2})$	4 699 099(10) $2.2 \times 10^{-6}$
(Berkeland et al., 1995), Yale	$\nu_H(2S_{1/2} - 4P_{1/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	4 664 269(15) $3.2 \times 10^{-6}$
	$\nu_H(2S_{1/2} - 4P_{3/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	6 035 373(10) $1.7 \times 10^{-6}$
(Hagley and Pipkin, 1994), Harvard	$\nu_H(2S_{1/2} - 2P_{3/2})$	9 911 200(12) $1.2 \times 10^{-6}$
(Lundeen and Pipkin, 1986), Harvard	$\nu_H(2P_{1/2} - 2S_{1/2})$	1 057 845.0(9.0) $8.5 \times 10^{-6}$
(Newton et al., 1979), U. Sussex	$\nu_H(2P_{1/2} - 2S_{1/2})$	1 057 862(20) $1.9 \times 10^{-5}$

# The Rydberg Theory (or theories!)

- Bohr model ca. 1915
- Sommerfeld fine structure correction 1922
- Evidence of QED effect since 1929
- Invention of QED and resolution of infinities 1948
- Lamb shift 1951
- and more....

It uses ALL of the classes of our theories!

Some theories can be VERY detailed and still agree very well with experiment.

Fig. 1. It is a relation between the 1s-2s transition frequency  $\nu_{H}(1s-2s)$  and the Rydberg constant  $R_{\infty}$ . A correction for the difference between the center of gravity of the 1s and 2s hyperfine multiplets and their triplet component is not included. This figure is an example of a complicated relationship, and is not intended to be read.

$$\begin{aligned} \nu_{H}(1s-2s) = \frac{1}{4} R_{\infty} \left\{ 1 + \left[ \frac{11}{48} (Z\alpha)^2 + \frac{43}{288} (Z\alpha)^4 + \frac{851}{12288} (Z\alpha)^6 + \dots \right] \right. \\ + \frac{m_e}{m_p} \left[ -1 - \frac{13}{24} (Z\alpha)^2 - \frac{37}{64} (Z\alpha)^4 + \dots \right] \\ + \left( \frac{m_e}{m_p} \right)^2 \left[ 1 + \frac{43}{24} (Z\alpha)^2 + \dots \right] \\ + \left( \frac{m_e}{m_p} \right)^3 \left[ -1 + \dots \right] \\ + \frac{(Z\alpha)^2 m_e}{\alpha} \left[ -\frac{7}{2} \ln \frac{1}{(Z\alpha)^2} - \frac{8}{3} \ln k_0(2s) + \frac{64}{9} \ln k_0(1s) - \frac{112}{3} \ln 2 - \frac{803}{54} \right] \\ + \frac{(Z\alpha)^2}{\alpha} \left( \frac{m_e}{m_p} \right)^2 \left[ \frac{7}{2} \ln \frac{1}{(Z\alpha)^2} + \frac{8}{3} \ln k_0(2s) - \frac{64}{9} \ln k_0(1s) + 112 \ln 2 + \frac{883}{18} \right] + \dots \\ + \frac{\alpha}{(Z\alpha)^2} \left[ -\frac{29}{3} \ln \frac{1}{(Z\alpha)^2} - \frac{4}{3} \ln k_0(2s) + \frac{32}{3} \ln k_0(1s) - \frac{266}{15} \right] \\ + \frac{\alpha}{(Z\alpha)^2} \frac{m_e}{m_p} \left[ \frac{29}{3} \ln \frac{1}{(Z\alpha)^2} + \frac{4}{3} \ln k_0(2s) - \frac{32}{3} \ln k_0(1s) + \frac{14}{5} \right] \\ + \frac{\alpha}{(Z\alpha)^2} \left( \frac{m_e}{m_p} \right)^2 \left[ -\frac{56}{3} \ln \frac{1}{(Z\alpha)^2} - \frac{8}{3} \ln k_0(2s) + \frac{64}{9} \ln k_0(1s) - \frac{14}{15} \right] \\ + \alpha (Z\alpha)^2 \left[ \frac{16}{3} \ln 2 - \frac{2093}{216} \right] \\ + \alpha (Z\alpha)^2 \frac{m_e}{m_p} \left[ -14 \ln 2 + \frac{2093}{90} \right] \\ + \frac{\alpha}{(Z\alpha)^2} \left[ \frac{7}{2} \ln^2 \frac{1}{(Z\alpha)^2} + \left( -\frac{208}{9} \ln 2 + \frac{247}{90} \right) \ln \frac{1}{(Z\alpha)^2} + 71.6262974 \right] + \dots \\ + \left( \frac{\alpha}{Z\alpha} \right)^2 (Z\alpha)^2 \left[ \frac{7}{2} \ln^2 \ln 2 + \frac{70\alpha^2}{81} + \frac{10211}{1848} + \frac{31}{4} \zeta(3) \right] \\ + \left( \frac{\alpha}{Z\alpha} \right)^2 (Z\alpha)^2 \frac{m_e}{m_p} \left[ \frac{21}{2} \ln^2 \ln 2 - \frac{70\alpha^2}{27} - \frac{10211}{648} - \frac{63}{4} \zeta(3) \right] \\ + 10.2976 \left( \frac{\alpha}{Z\alpha} \right)^2 (Z\alpha)^2 \\ + \left( \frac{\alpha}{Z\alpha} \right)^2 (Z\alpha)^4 \left[ \frac{56}{81} \ln^2 \frac{1}{(Z\alpha)^2} + \frac{1}{27} \ln^2 \frac{1}{(Z\alpha)^2} \right. \\ \left. + \left( \frac{246337}{25920} - \frac{395\alpha^2}{81} + \frac{112}{135} \ln 2 - \frac{7\alpha^2}{4} \ln 2 - \frac{242}{27} \ln^2 2 - 34.943333 \right) \ln \frac{1}{(Z\alpha)^2} \right. \\ \left. + 147(2\alpha) \right] + \dots \\ + \left( \frac{\alpha}{Z\alpha} \right)^2 (Z\alpha)^4 \left[ \frac{246673631}{27936} + \frac{1767717\alpha^2}{20160} - \frac{11127\alpha^4}{27} + \frac{7913}{27} \ln 2 - \frac{33208\alpha^2}{324} \ln 2 \right. \\ \left. + \frac{26718\alpha^2}{405} \ln^2 2 + \frac{497}{81} \ln^2 \alpha^2 + \frac{586497}{8312} \zeta(3) + \frac{847\alpha^2}{216} \zeta(3) - \frac{190}{72} \zeta(3) \right] + \dots \\ + \frac{\alpha (Z\alpha)^2 m_e}{\alpha^2} \left[ \frac{112}{81} - \frac{265\alpha^2}{108} + \frac{16\alpha^2}{3} \ln 2 - 14 \zeta(3) - \frac{14}{3} \alpha (Z\alpha) \ln^2 \frac{1}{(Z\alpha)^2} \right] \\ - \frac{14}{3} (Z\alpha)^2 \left( \frac{m_e}{m_p} \frac{2\alpha}{3} \right)^2 \\ + \dots \left. \right\} \end{aligned}$$

# The Rydberg

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- The Rydberg is about the binding of an electron to a proton. It quantifies the elemental binding energy of all atoms.
- It will likely become the ultimate definition for time interval sometime in the future.

## (Relative) Atomic Masses

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Our understanding of matter is that atoms are made up of protons, neutrons and electrons and energy.

This is true up to a point but we are not yet able to accurately predict the mass of particular atoms. However, it is possible to accurately measure the mass of a single atom in comparison with another single atom.

Values of the relative atomic masses are often related to the arbitrarily defined atomic mass unit,  $1 \text{ amu} = {}^{12}\text{C} / 12$ . This is more correctly referred to as the atomic Relative standard mass  $A_r(X)$ .

# Relative Atomic Masses

Values of the relative atomic masses of the neutron and various atoms as given in the 2003 atomic mass evaluation together with the defined value for  $^{12}\text{C}$ .

Atom	Relative atomic mass $A_r(X)$	uncertainty $u_r$
– n	1.008 664 915 74(56)	$5.6 \times 10^{-10}$
– $^1\text{H}$	1.007 825 032 07(10)	$1.0 \times 10^{-10}$
– $^2\text{H}$	2.014 101 777 85(36)	$1.8 \times 10^{-10}$
– $^3\text{H}$	3.016 049 2777(25)	$8.2 \times 10^{-10}$
– $^3\text{He}$	3.016 029 3191(26)	$8.6 \times 10^{-10}$
– $^4\text{He}$	4.002 603 254 153(63)	$1.6 \times 10^{-11}$
– $^{12}\text{C}$	12 (exact)	
– $^{16}\text{O}$	15.994 914 619 56(16)	$1.0 \times 10^{-11}$
– $^{28}\text{Si}$	27.976 926 5325(19)	$6.9 \times 10^{-11}$
– $^{29}\text{Si}$	28.976 494 700(22)	$7.6 \times 10^{-10}$
– $^{30}\text{Si}$	29.973 770 171(32)	$1.1 \times 10^{-9}$
– $^{36}\text{Ar}$	35.967 545 105(28)	$7.8 \times 10^{-10}$
– $^{38}\text{Ar}$	37.962 732 39(36)	$9.5 \times 10^{-9}$
– $^{40}\text{Ar}$	39.962 383 1225(29)	$7.2 \times 10^{-11}$
– $^{87}\text{Rb}$	86.909 180 526(12)	$1.4 \times 10^{-10}$
– $^{107}\text{Ag}$	106.905 0968(46)	$4.3 \times 10^{-8}$
– $^{109}\text{Ag}$	108.904 7523(31)	$2.9 \times 10^{-8}$
– $^{133}\text{Cs}$	132.905 451 932(24)	$1.8 \times 10^{-10}$

# Measuring Relative Atomic Masses

A charged single atom in a Penning trap is trapped by an ac voltage between the vertical electrodes.

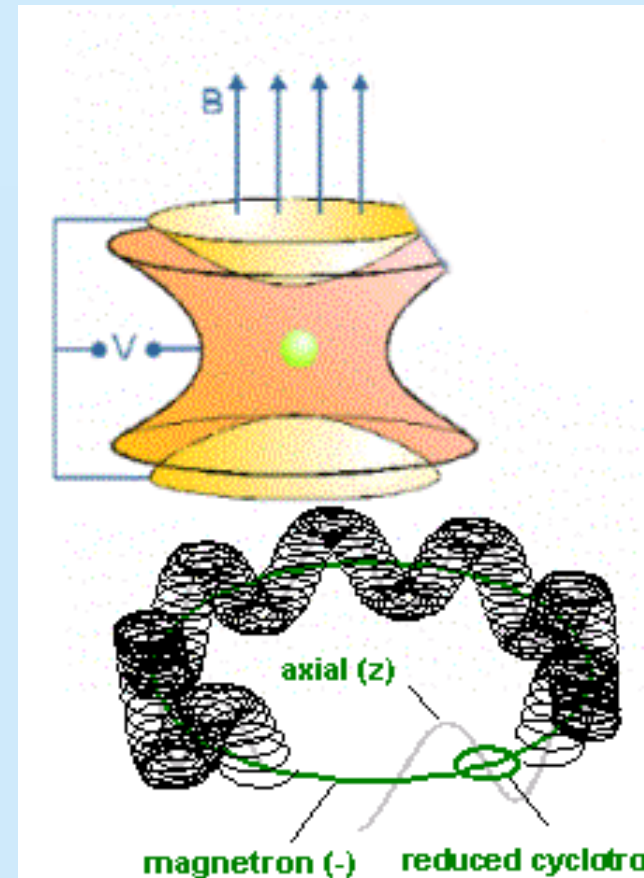
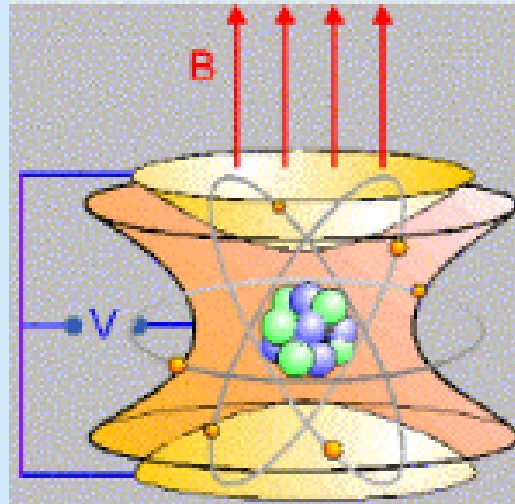
A magnetic field then causes three types of motion, axial, cyclotron and magnetron described by:

Relative masses can then be determined by frequency ratios.

$$\omega_z = \sqrt{\frac{qV_0}{md^2}}$$

$$\omega_c = \frac{qB}{m}$$

$$\omega_m = \frac{V_0}{2d^2B} = \frac{\omega_z^2}{2\omega_c}$$





# Fine Structure Constant

The simple Bohr model of hydrogen explained the coarse spectra of hydrogen. Sommerfeld explained the further splitting of this 'coarse structure' by considering elliptical electron orbits in a relativistic model.

The fine structure constant,  $\alpha$ , is a measure of the strength of the interaction between the electron and photons.

$\alpha$  is also the ratio of the impedance of vacuum and the universal conductance and is related to several other constants.

$$\alpha = \frac{e^2}{\hbar c 4\pi\epsilon_0} = 7.297\,352\,5376(50) \times 10^{-3} = \frac{1}{137.035\,999\,679(94)}$$

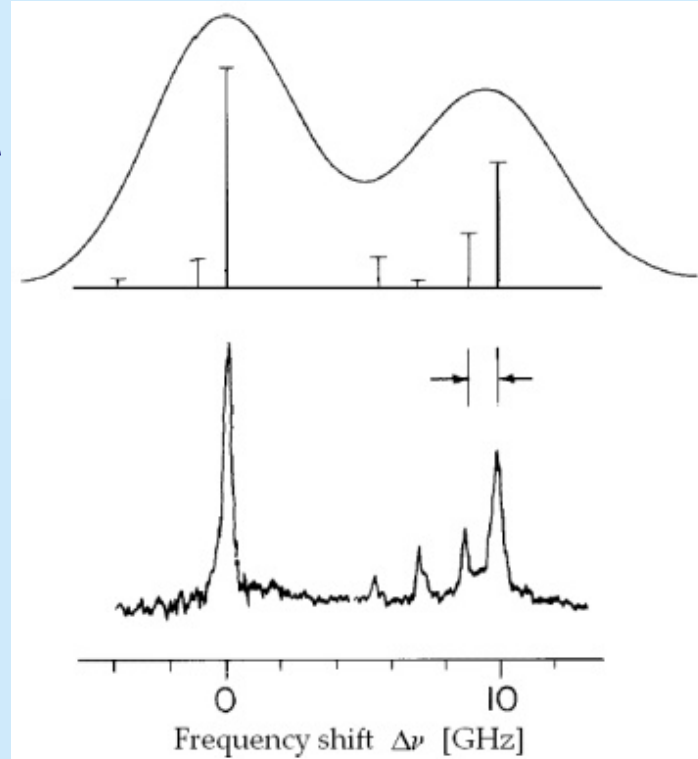




# Fine Structure Constant

Thus,  $\alpha$  appears in all models that incorporate quantum and relativistic properties of charged particles.

If electron and light did not interact the fine structure constant would be zero.



"one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man." Richard Feynman

# Measuring $\alpha$

---

Anomalous magnetic moment of the electron or muonium.

Atomic recoil of various atoms.

Calculable capacitor, capacitance to resistance scaling and QHR.

$d_{220}$  and  $h$  / neutron mass

Gyromagnetic moment of the proton

...

Very different measurements and diverse theories !

# Measuring $\alpha$

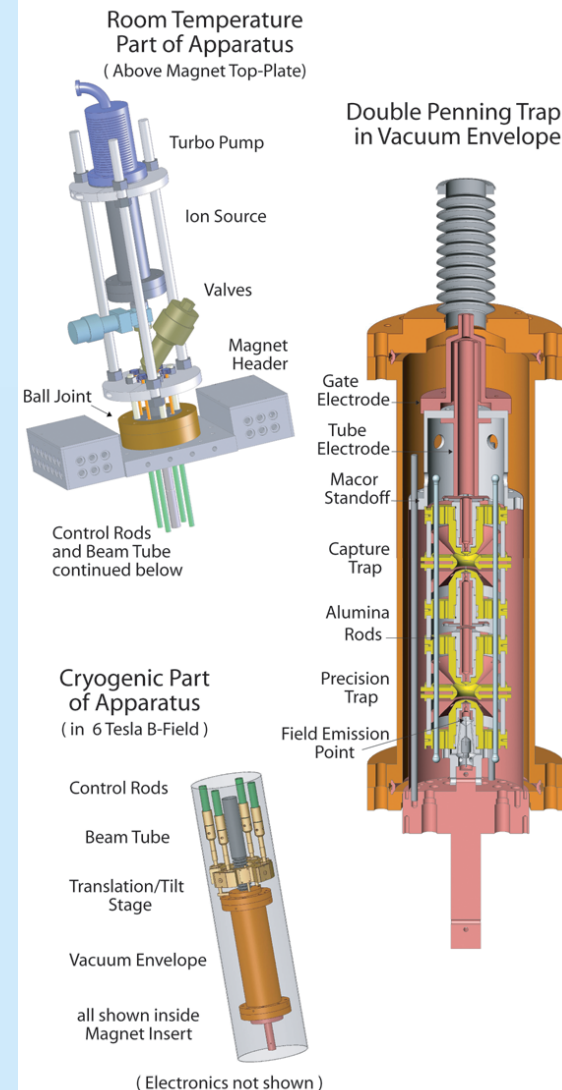
Anomalous magnetic moment of the electron,  $a_e$ .

a penning trap experiment  
measuring mass ratios combined  
with QED theory

$$a_e(\text{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau). \quad (\text{B3})$$

$$A_i = A_i^{(2)} \left( \frac{\alpha}{\pi} \right) + A_i^{(4)} \left( \frac{\alpha}{\pi} \right)^2 + A_i^{(6)} \left( \frac{\alpha}{\pi} \right)^3 + A_i^{(8)} \left( \frac{\alpha}{\pi} \right)^4 + \dots \quad (\text{B4})$$

the U of W trap



# Planck Constant

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$E = h\nu$  the energy of a photon

a link between heavy and  
massless particles

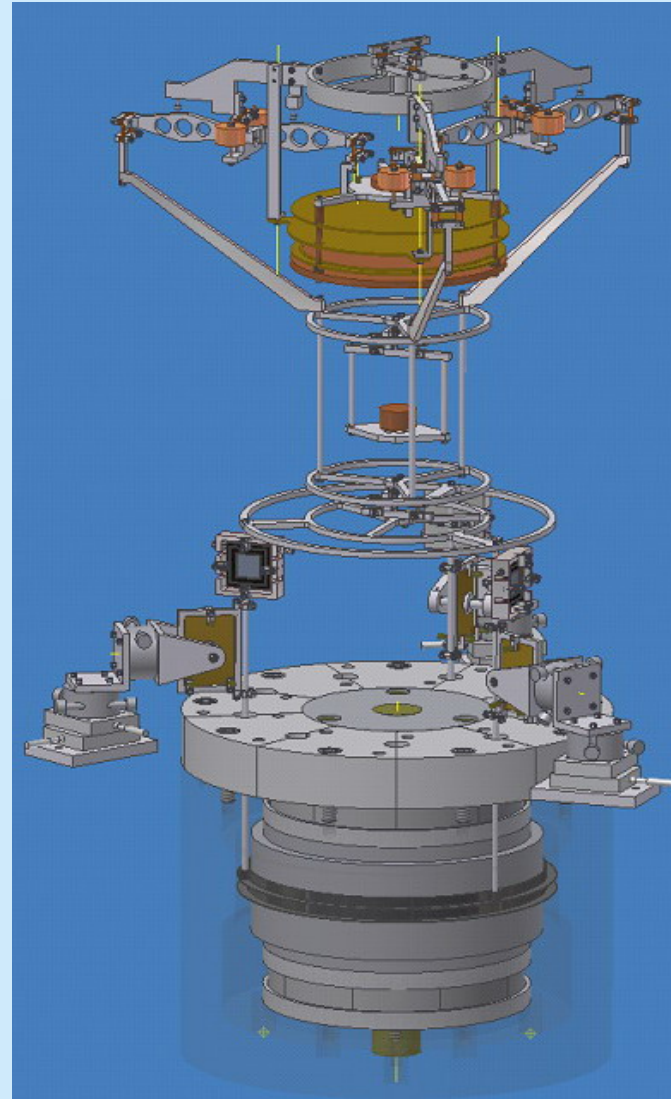
Because of the accuracy and ease with which electromagnetic and optical frequencies can be measured, the Planck constant plays a critical role in physics and in the LSA of the Fundamental Constants.



# Measuring $h$

- Watt balances – with JVS and QHR
- Volt balances
- Molar mass of silicon (Avogadro)
- Gyromagnetic ratio of the proton
- Faraday constant
- ...

The BIPM watt balance





# Avogadro Constant

The Avogadro constant,  $N_A$ , is simply the number  $^{12}\text{C}$  atoms that is equivalent to a mass of 12 g.

Or the number of atoms in a mole...



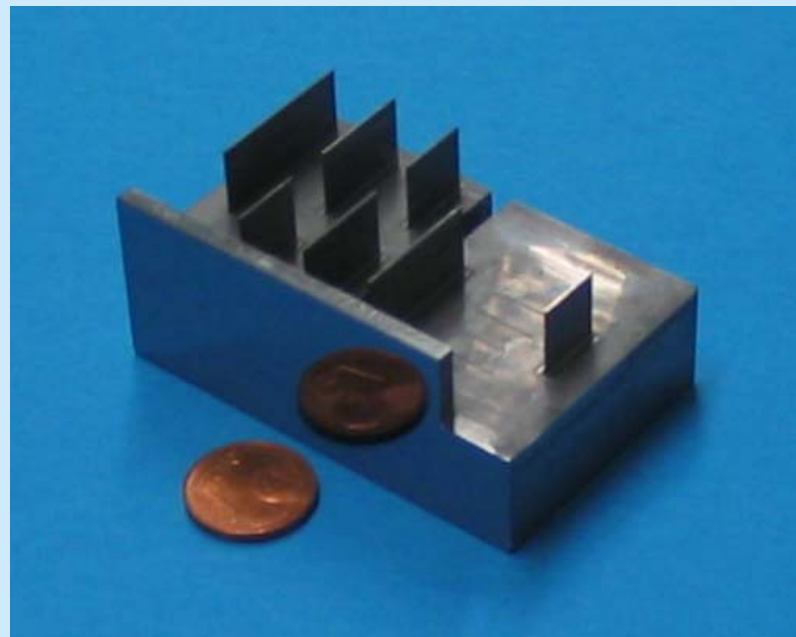
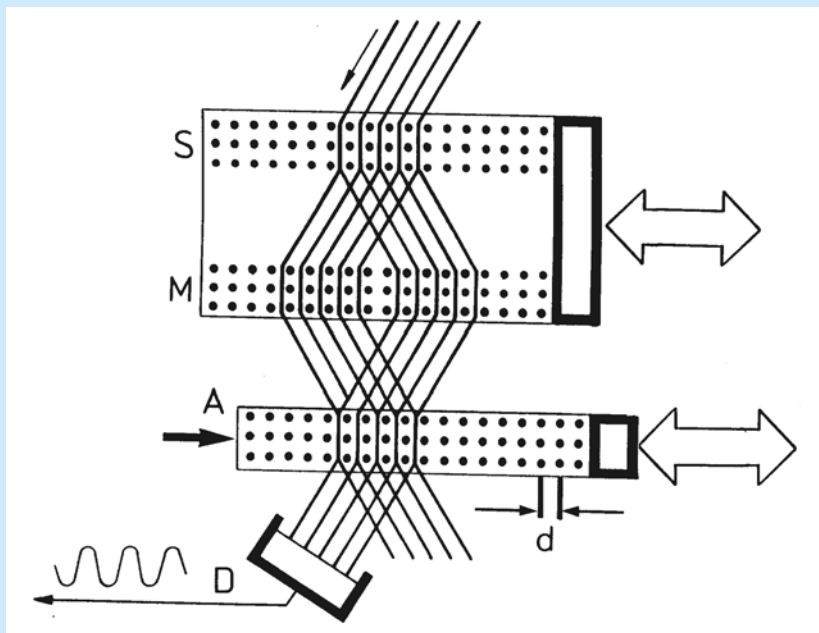
It is the scaling parameter between atomic masses and the macroscopic mass of the SI kg.



# IAC - the premiere Avogadro project

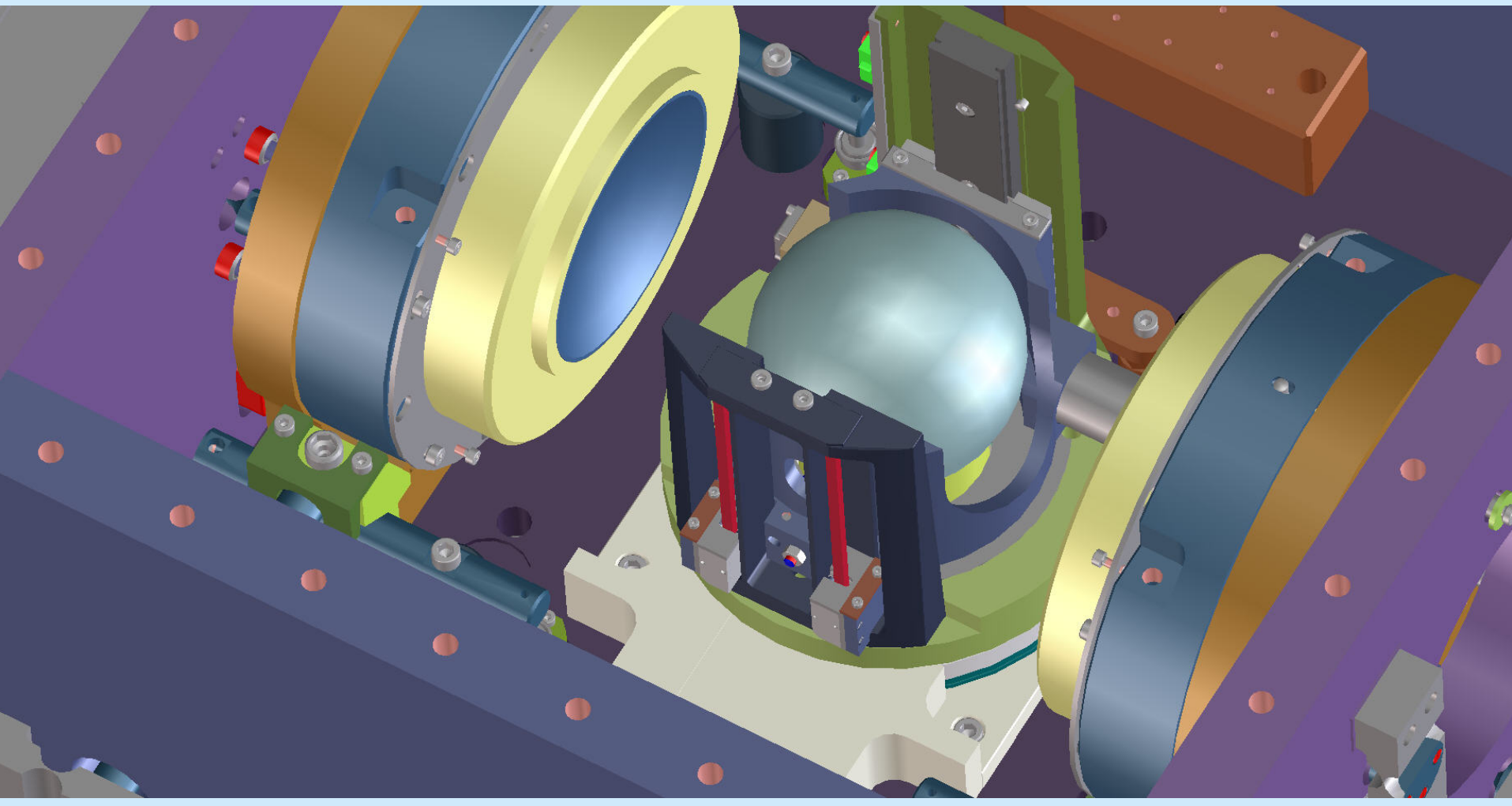
X-ray Interferometry measuring the Si lattice spacing

From a single crystal grind out two thin walls and move them with respect to a third in an X-ray beam.



# Measuring a Si Sphere

A Fizeau Optical Interferometer to measure diameter (PTB)





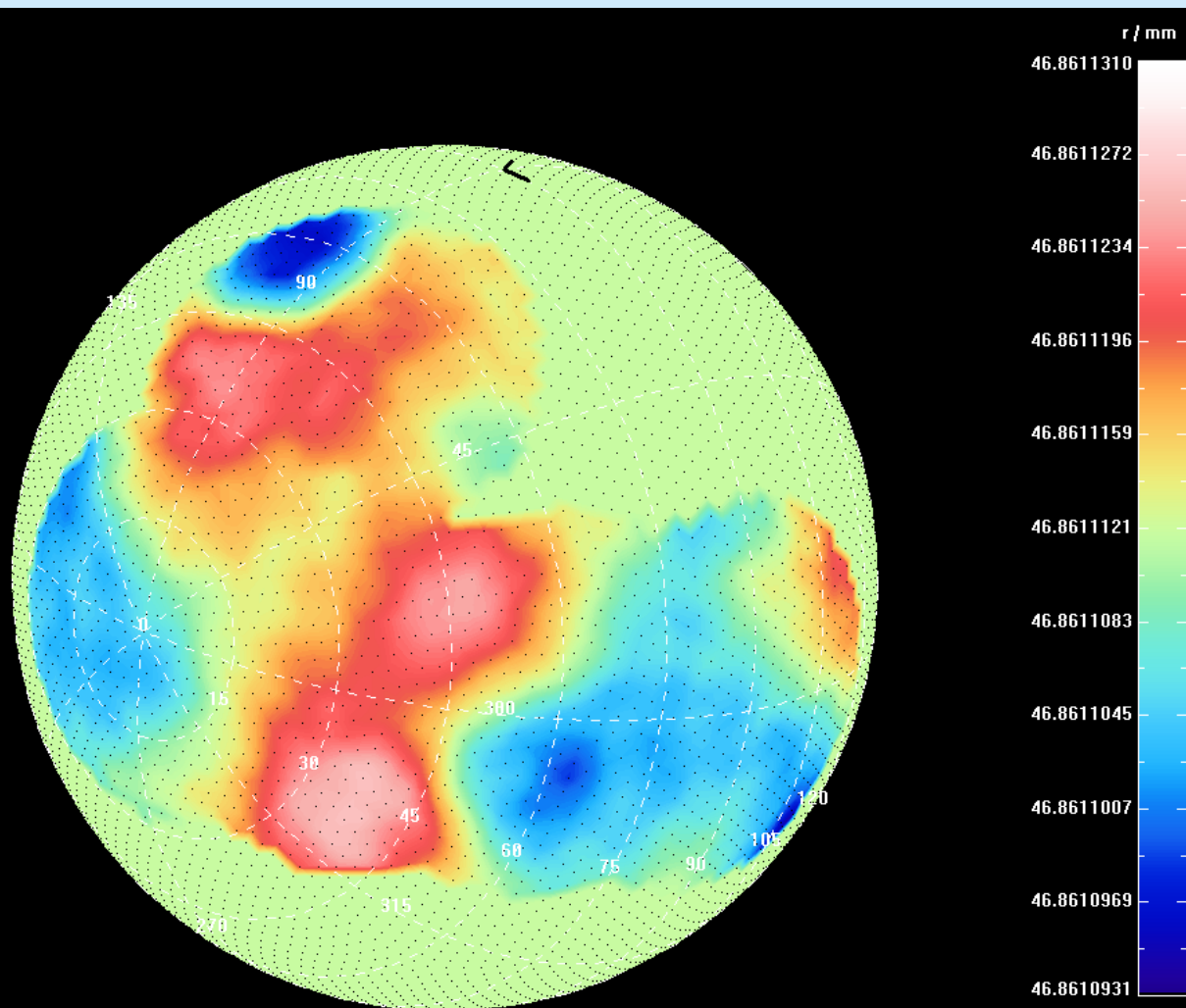
# The dimensions of a Si sphere

Volume

$d_{220}$

Mass

$N_A$



Max: 46.8611265 mm  
Min: 46.8610947 mm

Mean: 46.8611120 mm  
StdDev: 0.0000071 mm

# CODATA

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- Committee on Data for Science and Technology (CODATA) – was established in 1966 - is an interdisciplinary **Scientific Committee of the International Council for Science (ICSU)**, which works to improve the quality, reliability, management, and accessibility of data of importance to all fields of science and technology.
- **CODATA Task Group on Fundamental Constants** - established in 1969 - “to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all of the relevant data available at a given point in time.”
- The Task Group sanctions the data selection and methodology of the adjustment of the recommended values of the constants.

# The NIST Reference on Constants, Units, and Uncertainty

Information at the foundation of modern science and technology from the [Physics Laboratory of NIST](#)

URL: [physics.nist.gov/constants](http://physics.nist.gov/constants)

## CODATA Internationally recommended values of the Fundamental Physical Constants

### Latest (2006) values of the constants

[Version history](#) and [disclaimer](#)

- Constants Topics:
- [Values](#)
- [Energy Equivalents](#)
- [Searchable Bibliography](#)
- [Background](#)

[Constants Bibliography](#)

[Constants, Units & Uncertainty home page](#)

(e.g., [electron mass](#), most misspellings okay)

Search for value by name

Display  alphabetical list,  table (image), or  table (pdf)

by clicking a category below

- Universal
- Adopted values
- Frequently used constants
- Electromagnetic
- Non-SI units
- Extensive listings
- Atomic and nuclear
- Conversion factors for energy equivalents
- All values (ascii)
- Physico-chemical
- X-ray values

Find the [correlation coefficient](#) between any pair of constants  
Data from the [least-squares adjustment](#) of the values of the constants

### See also

- [Searchable bibliography](#) on the constants
- [Background information](#) related to the constants
- [Links](#) to selected scientific data

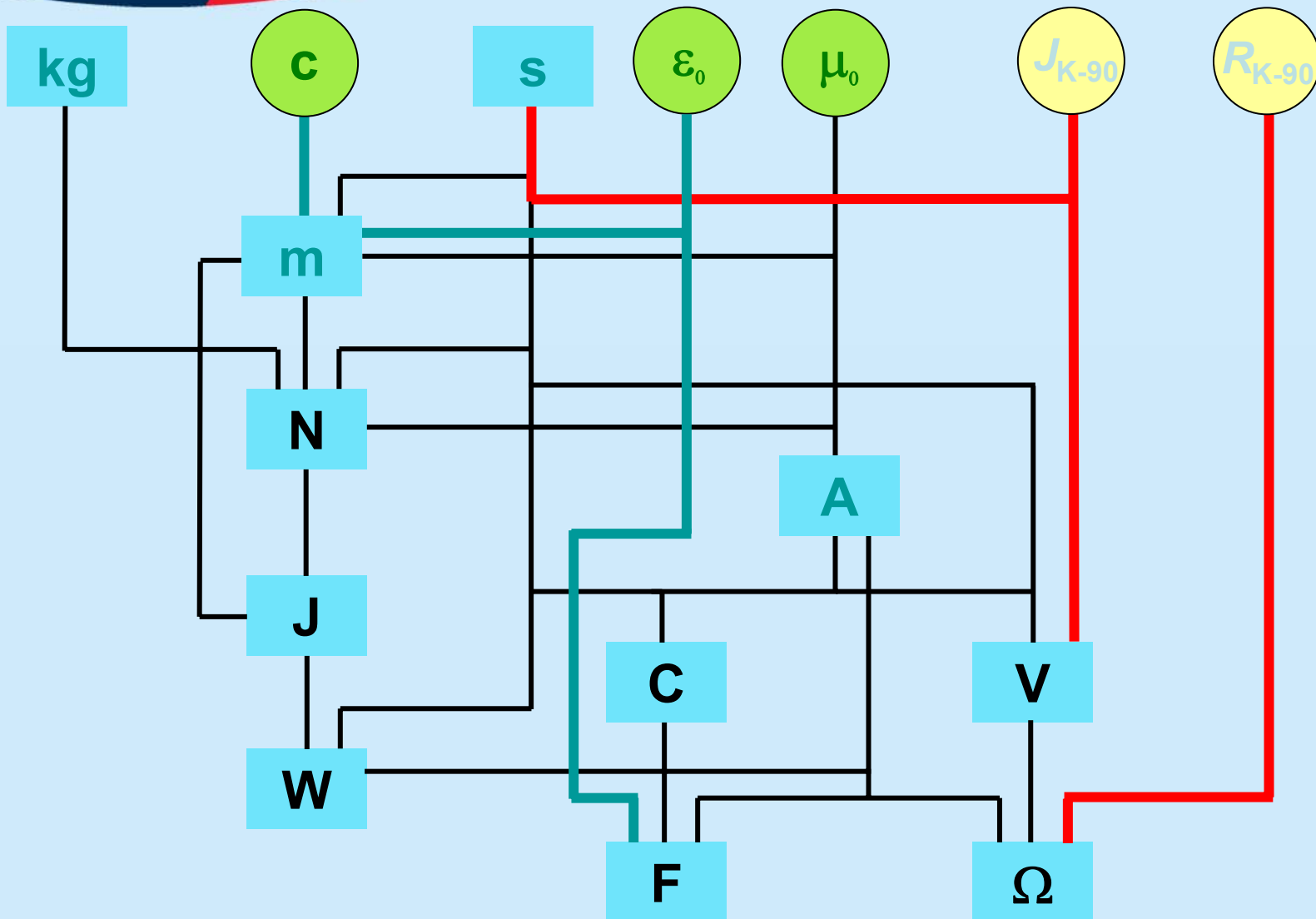
[Detailed contents](#). [About this reference](#). [Feedback](#).

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NIST Standard Reference Database 121

Online: October 1994 - Last update: March 2007

# Electrical since 1990



# *The 1990 Electrical Representations*

- Improved reproducibility to  $<10^{-10}$ .
- Allowed all industrialized countries to realize electrical units at the lowest possible uncertainty.
- Made the electrical units invariant in time and location.
- Provided redundancy- samples, steps, frequency,...
- **But they are not really SI units.**

# Evolution of the SI towards Fundamental Constants

Fundamental constants are the 'best standards' that we have

- $\mu_0$  permeability of vacuum (1947)
- $c$  speed of light (1983) and  $\epsilon_0$  permittivity of free space
- $N_A$ , Avogadro constant was set through the mole in 1971
- $h/2e$  and  $h/e^2$  unofficially set with the 1990 representations

## *The Proposed Changes to the SI*

Modify the SI by exactly fixing the values of a set of fundamental constants, such as:

*$c$ , speed of light*

*$h$ , Planck's constant*

*$e$ , elementary charge*

*$k$ , Boltzmann's constant*

*$N_A$ , Avogadro's number*

and to make the SI units consistent with these values.



# *CIPM Recommendation 1 (CI-2005)*

Preparative steps towards new definitions of the kilogram, the ampere, the kelvin and the mole in terms of fundamental constants

(adopted by CIPM at 94th meeting, 4-7 October 2005).

## *One of several Responses*

‘Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005)’, I. M. Mills, T. J. Quinn, P. J. Mohr, B. N. Taylor and E. R. Williams, *Metrologia* **43** (2006) 227-246



## *Why change it if it's not broken?*

- **It is broken** – 1990 representations of volt and ohm
- It can be fixed
- It would improve science
- It would improve the SI
- Its honest

## *Major Advantages of this Change in the SI*

- Reduced uncertainties of many fundamental constants.
- Improved usage of the SI in some scientific measurements.
- Reduced uncertainties of electrical quantities and the practical realizations become part of the SI.
- Invariance in time and space.
- The SI becomes more accessible at the highest level of accuracy.

**But with increased uncertainty in mass related quantities.**

**And the meaning of base units may have to change.**

# *The Recent Proposed Changes to the SI*

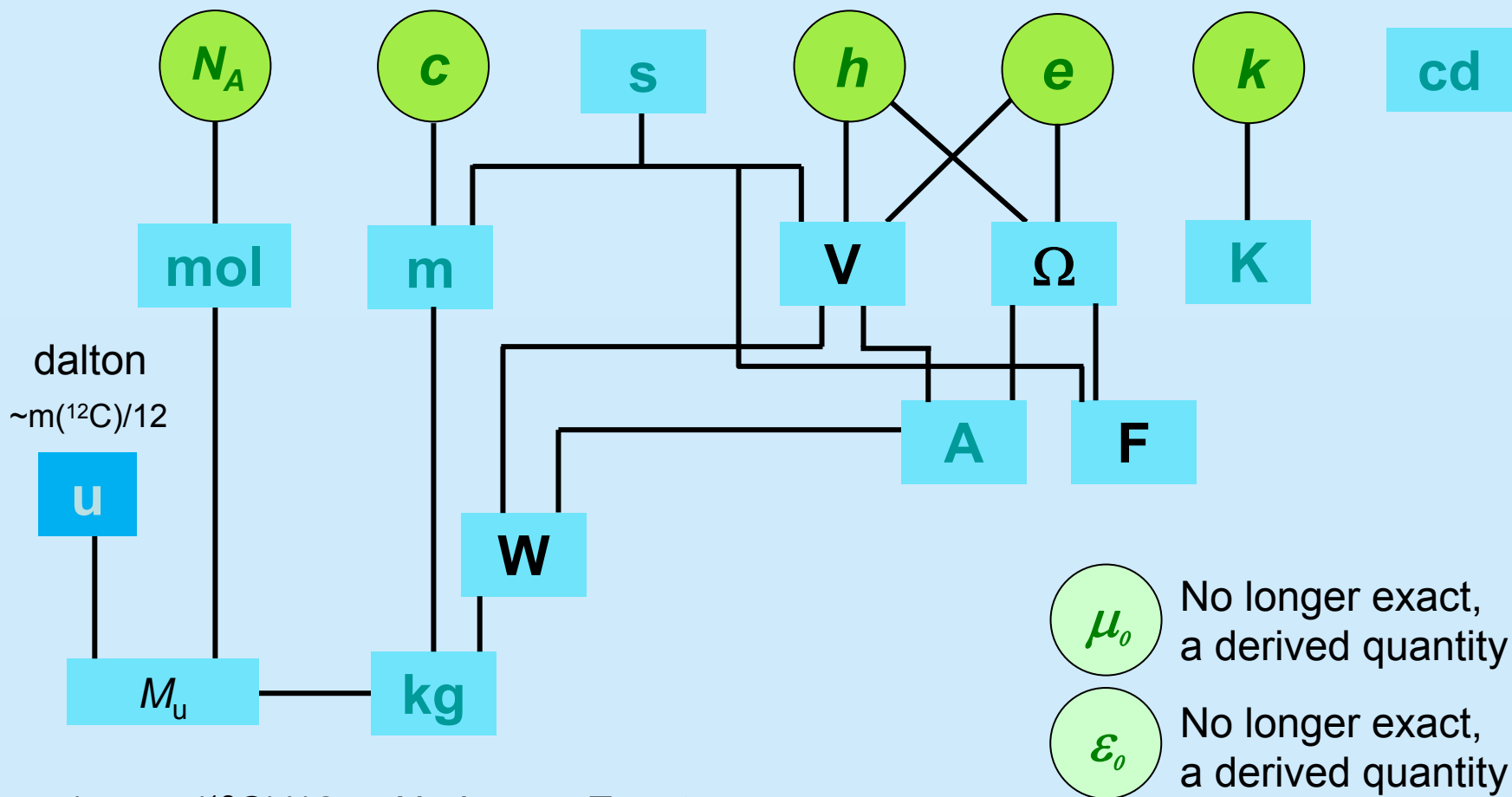
*The proposed changes primarily impact:*

- *Electrical*
- *Mass*
- *Thermometry*

*Impacts on CCQM, CCPR, CCL, CCAUV,  
CCIR are minimal.*

# The Proposed Change

The most likely option (in my opinion).



$1u \sim m(^{12}\text{C})/12$  No longer Exact  
an experimentally derived quantity

# Changes in relative uncertainties $\times 10^{-9}$

Constant or Unit		Present SI (CODATA 2002)	Define $h, e, k, N_A$
Mass	$m(\kappa)$	exact	170 (>20)
Planck constant	$h$	170	exact
Avogadro	$N_A$	170	exact
Elementary charge	$e$	85	exact
Mass of electron	$m_e$	170	1.4
Flux quantum	$2e/h$	85	exact
Mass of proton	$m_p$	170	1.4
dalton (amu)	$u$	170	1.4
Klitzing constant	$h/e^2$	0.68	exact
Triple point of water	$T_{\text{TPW}}$	exact	920 (0.25 mK)

## *Typical Implementation of the new SI*

- second – same as before, using caesium clocks
- metre – same as before, using  $c$  and the second
- volt – Josephson, using second and  $h$  and  $e$ ;  $(h/2e)$
- ohm – Quantum Hall, using  $h$  and  $e$ ;  $(h/e^2)$
- kilogram - watt balance, using all of the above
- ampere – ratio of volt and ohm
- kelvin – primary thermometry using  $k$

### **Other possibilities within the same system**

ampere – single electron tunneling, using the second and  $e$

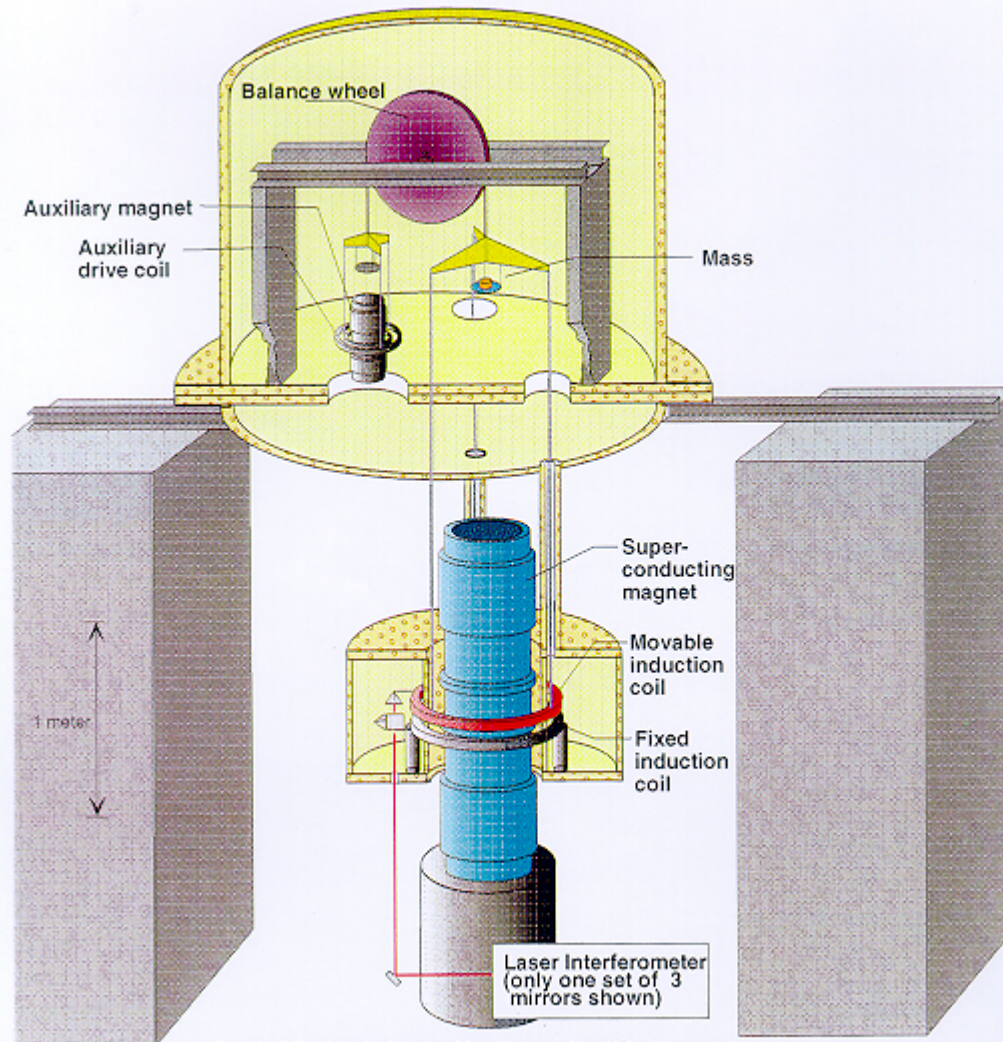
kilogram – from the  $N_A$ ,  $d_{220}$  and dalton



# NIST Watt Balance

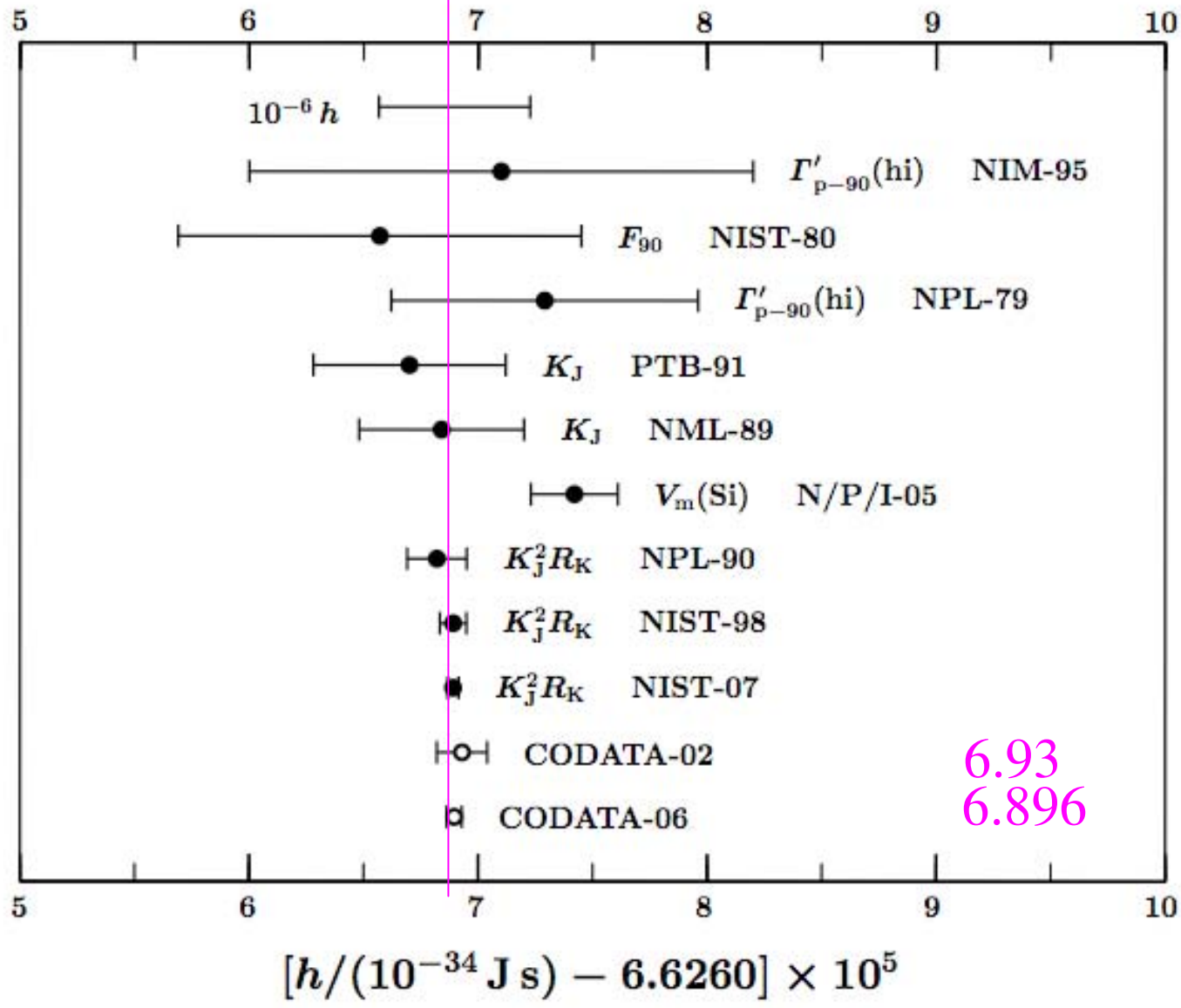
Electrical energy  
(voltage \* current)  
balanced against  
mechanical energy

With the Josephson  
and quantum Hall  
indirectly measures  
 $h$ , Planck's constant



The NIST Watt Balance: Monitoring the Kilogram  
A schematic showing the basic elements of the experiment  
and the future vacuum enclosure.

$$h_{90} = 6.885$$





# CCU Report to 2007 CIPM

“... In our report from the CCU we made the explicit recommendation that the kilogram, ampere, kelvin and mole should be redefined to fix the Planck constant  $h$ , the elementary charge  $e$ , the Boltzmann constant  $k$ , and the Avogadro constant  $N_A$  respectively. There remain the alternative possibilities.....”

# Recommendations of CCM WGS1

*‘The CCM recognizes that the new definitions of the kilogram so far suggested are equivalent with respect to their impact on mass metrology.’*

*‘...after redefinition of the kilogram, means of making an experimental link to **the Planck constant** or the **mass of a** suitable atomic or **elementary particle** must be maintained and simplified;...’*

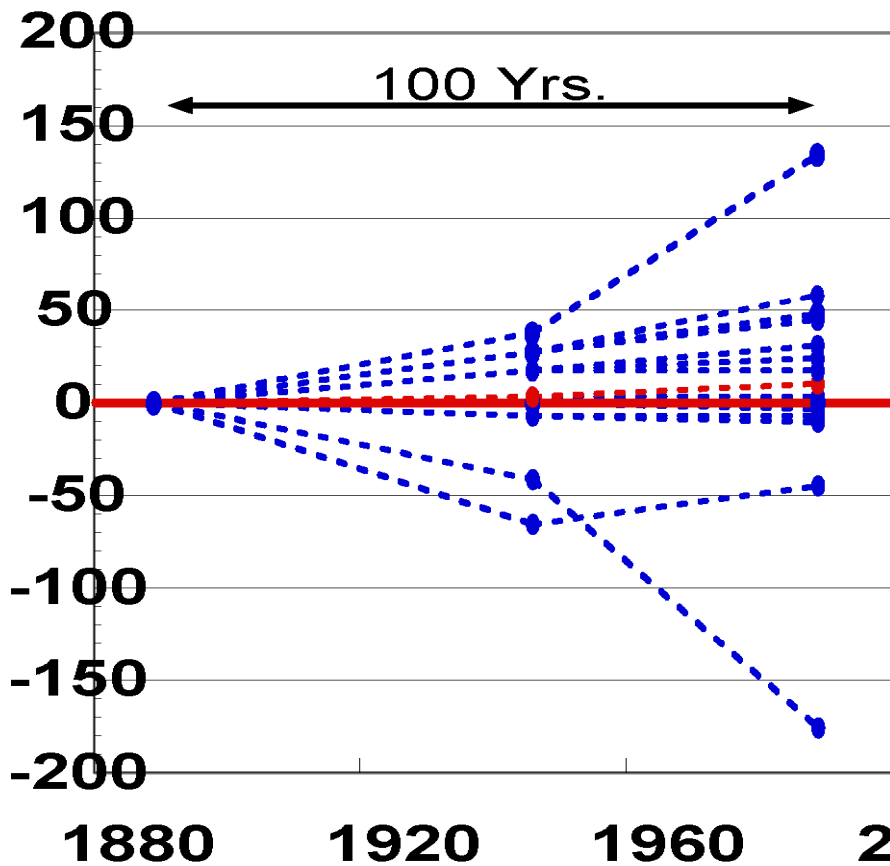
## *The Status to Date*

- CIPM 2005 recommended to prepare for the change
- CODATA TG on fundamental constants endorses the change
- CCM – recommends a change when the data is in agreement
- CCEM – recommends fixing both  $h$  and  $e$
- CCT – recommends fixing  $k$ , Boltzmann's constant
- CCU – in June 2007 recommends fixing  $h$ ,  $e$ ,  $k$ , and  $N_A$  and proceeding with the change by 2010 if experimental disagreements can be resolved.

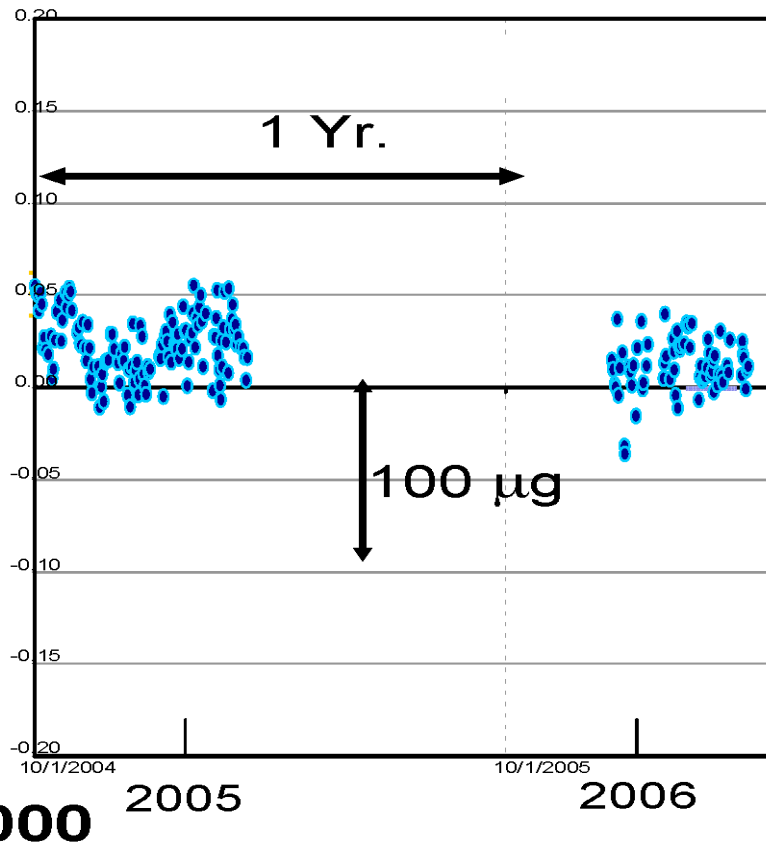
The soonest practical date for such a change is the end of 2010.

# Is Now the Time?

$\times 10^{-9}$  Mass Results



Watt Balance Results



# Is Now the Time?

Assuming that the density of mercury used in two experiments separated by thirty years (1950s to 1980s) is invariant, one can draw the following conclusion as to the stability of the unit of mass:

“....that the NPL realization of the kilogram has been changing on average by less than 2 parts in  $10^8$  per year over the past 30 years. Based on the close ties between realizations of the unit of mass at NPL and BIPM, this upper bound appears to apply to BIPM as well.”

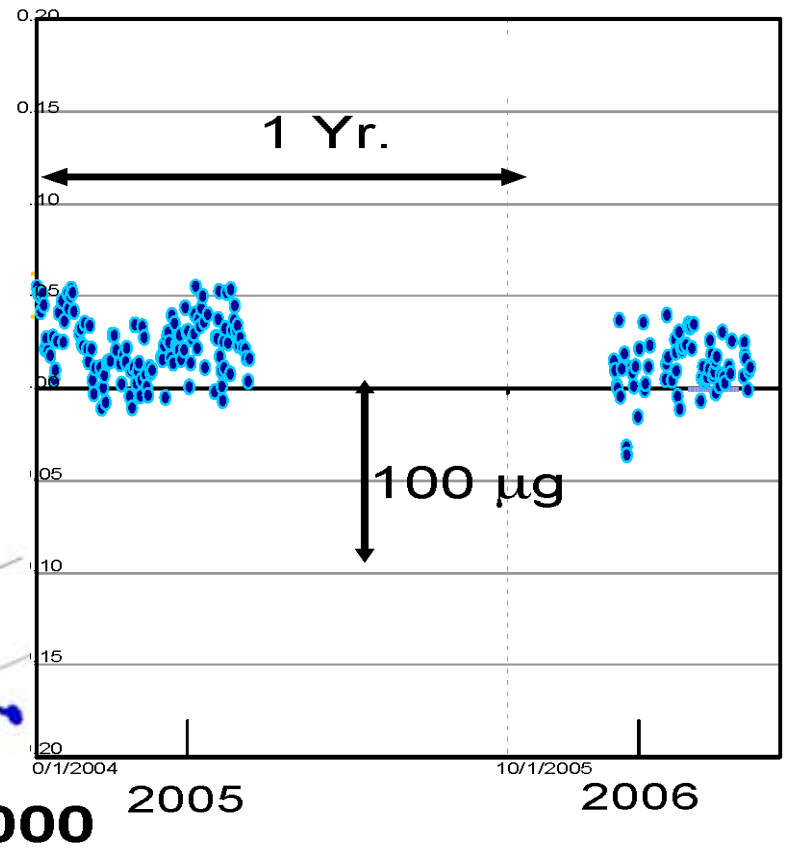
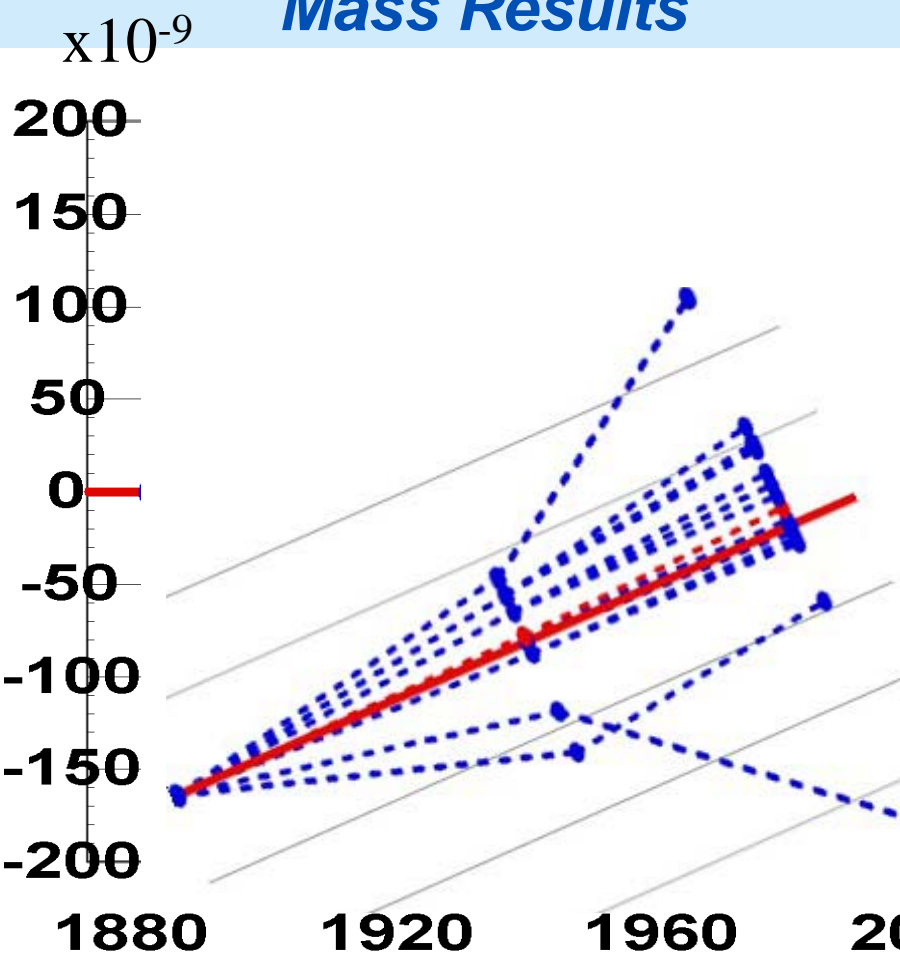
From R.S.Davis *Metrologia* 26, 75-76 (1989)



# Is Now the Time for the Earth to Move?

## Mass Results

## & Watt Balance Results



# Conclusion

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## What are Fundamental Constants?

Fundamental Constants are the ultimate consistency check of physical measurement and scientific theory.

But these check standards are better than our primary standards.

We can base the SI on some of these Fundamental Constants with improvements to both the SI and Fundamental Constants.