Fitting Multidimensional Measurement Data to Tolerance Zones Taking into Account for the Uncertainties of Measurements

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Kostadin Doytchinov
Institute for National Measurement Standards
National Research Council Canada
Kostadin.Doytchinov@nrc-cnrc.gc.ca

Teodor Natchev
Kotem Technologies Inc.
What is Best-fitting?

Best-fitting is the process of finding the best mutual relationship between Measured data and Nominal data when the part is not fully constrained while trying to satisfy a specific goal.
Conditions for Best-fitting

- Profile tolerance when no Datums are given at all
  - Also form error calculation – flatness, cylindricity, etc.
- Datum Reference Frame (DRF) does not fully constrain the part
- If DRF mobility is present due to MMC or LMC modifiers.

In all these cases best-fitting of the measured data to the nominal geometry is mandatory
Conditions for Best-fitting

No Datum Reference Frame at all

\[ \theta = 0.2 \]
Datums do not fully constrain the coordinate system.

Datums K and M together can constrain 5 degrees of freedom. The rotation about the Z axis is unconstrained.
DRF Mobility Requires Best-fitting

DRF - fully constrained, possible **mobility** from MMC, LMC

Without **MMC** modifier on the datum this part would be rejected!
Mathematical Criteria

The best-fitting criterion is the mathematical approximation of the practical goal we are trying to achieve.

- Examples of criteria:
  - Least Squares
  - Sum of the absolute values of deviations
  - Min-Max
    - Straightness, flatness, etc.
  - Uniform deviations
  - Tolerance envelope
  - Tolerance envelope Min-Max
  - Etc.
Least Squares Criterion (LS)

\[ F = \text{Min}(\sum \Delta_i^2) \]

- The most commonly used criterion
- Very stable
- Best averaging effect
- The result is **not influenced** by the prescribed tolerance
  - NOT to be used when tolerances present unless no other possibilities!
The Min-Max Criterion

The goal of this criterion is to reduce the maximum deviation to the minimum possible.

- Directly minimizes the maximum profile deviation.
- Almost equivalent to the Tolerance Envelope with uniform tolerance zone.
- Affected by outliers.
The Tolerance Envelope Criterion

\[ F = \min\left( \sum \Delta_{out}^2 \right) \]

- The goal of this criterion is to bring the measured points in tolerance by reducing the out-of-tolerance portion of the deviations.
- Does not optimize the in-tolerance distribution
  - Only works with the points out of tolerance
  - Separate criterion needed to improve inside the tol.zone
The Tolerance Envelope Min-Max Criterion

\[ F = \text{Max}(\Delta_{\text{Min}}) \]

- This criterion is applied after first successfully running the “tolerance envelope” criterion.
- The goal of this criterion is to maximize the value of the closest (minimum distance) to the tolerance zone deviation.
Decision Rules: Considering Measurement Uncertainty in Determining Conformance to Specifications

The ISO 14253-1:1998

- **Lower specification limit (LSL)**
- **Upper specification limit (USL)**
- **Specification zone**
- **In specification**
- **Out of specification**
- **Non-conformance zone**
- **Acceptance zone**
- **Uncertainty range**
- Increasing uncertainty $U$

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ISO Acceptance Zone

Tol. Zone

Acceptance Zone

Uncertainty Zones, U (k=2)
Example: a 5:1 rule means that $U$ should not be larger than one tenth of the specification zone. If this condition is fulfilled, then the measurement is accepted if the results lies within the specification zone and rejected otherwise. If MPE* specified, then the specification zone is twice the MPE, i.e., $\pm$MPE
Decision Rules:
The ASME B89.7.3.1-2001

Stringent Acceptance and Relaxed Rejection using Z% Guard Band

The acceptance zone is the specification zone reduced by the guard bands. The relaxed rejection allows product rejection even if the result is in the specification zone by the guard band amount. The guard band amount (expressed as a percent of the expanded uncertainty) is determined based on the “acceptable risk” of accepting out-of-specification products.

Note: the guard banding can be one or two sided
Stringent Rejection and Relaxed Acceptance using Z% Guard Band

The rejection zone is the specification zone increased by the guard bands. The relaxed acceptance allows product acceptance even if the result is outside the specification zone by the guard band amount.
Decision Rules:
The ASME B89.7.3.1-2001

Decision Rules With a Transition Zone

The transition zone may be useful if special conditions are agreed when the results are in the transition zone. For example, the product could be accepted at a reduced price.
Starting Conditions

Tol. Zone

Measured points with uncertainties

Desired Result - all points together with uncertainty zones at selected level of confidence within the tolerance zone
When measured points have different uncertainties, each point will have its own separate acceptance zone.

The critical point for the task may not be the one closest to the tol. zone.
Proposed Method

Tol. Zone

Uncertainty zone outside tol. zone

Uncertainty Zone at a specified level of confidence or guard bands

Effective Tol. Zone

Individual measured points may have different uncertainties. Particularly when data collected with different sensors.
When measured points have different uncertainties, each point will have its own separate acceptance zone.
Application of the Method

Data Reduction Application

This is not a CAD model. It is a cloud of 6 million points (1.6 gigabytes)!

Urgent need for metrologically correct data reduction techniques
The goal of the data filtering is to reduce noise, eliminate legitimate outliers and possibly reduce the number of points. Then, see if surface in tolerance.
A part like this could have thousands of nominal surfaces and millions of measured points.

During the best-fitting process each point is being projected to each surface in order to find the nearest surface. This can result in many millions of operations slowing down calculations to unpractical levels.

Major Issue!
Normal Substitution Technique

Measured points being substituted

Area shapes:

• Simple mean
• Weighted mean
• Centroid
• Etc.

Substitution Principle:

• Simple mean
• Weighted mean
• Centroid
• Etc.

All “Normal” substitution techniques result in a loss of information - loss of effective “form” information!
Proposed Substitution Technique

$X_i, Y_i, Z_i$
Measured point being substituted

Substitution point with uncertainty enveloping several points with their uncertainties

No significant information lost!
Adaptation for Data Reduction

- **Tol. Zone**: Uncertainty zone outside tol. zone
- **Effective Tol. Zone**: Affective Uncertainty Zone reflecting the substituted points and their uncertainties at a specified level of confidence

No significant information lost! Reliable decision for acceptance or rejection of the measurands
Most of the material presented is a result of my involvement with Kotem Technologies Inc.

Thank You!