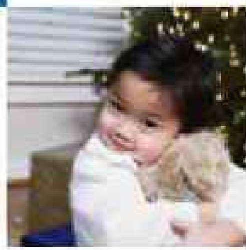
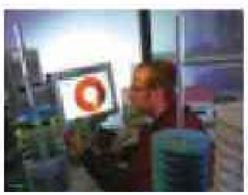








Laboratoire national de métrologie et d'essais


LNE

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Toward a determination of R_K in term of the new LNE calculable cross capacitor

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J. David, S. Leleu (ENSAM, France)

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Summary

Introduction

- The SI electrical base unit - Ampere definition
- Filiation of the electrical units
- Determination of R_K

Part 1 - Thompson Lampard calculable capacitor

- Presentation
- Mise en pratique at LNE
- Last results in 2000

Part 2 – Present developments

- Dedicated measurement apparatus
- Fabrication of a new set of electrodes
- Design of the new LNE Thompson Lampard calculable capacitor

Conclusion

The SI electrical base unit : the ampere

- The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed **1 m** apart in vacuum, would produce between these conductors a force equal to **2.10^{-7}** newton per metre of length.

This force has for expression : $F = (\mu_0/2\pi)\cdot(I^2/d)$

⇨ $\mu_0 = 4\pi\ 10^{-7}\ \mathrm{N/A^2}$ (permeability of free space)

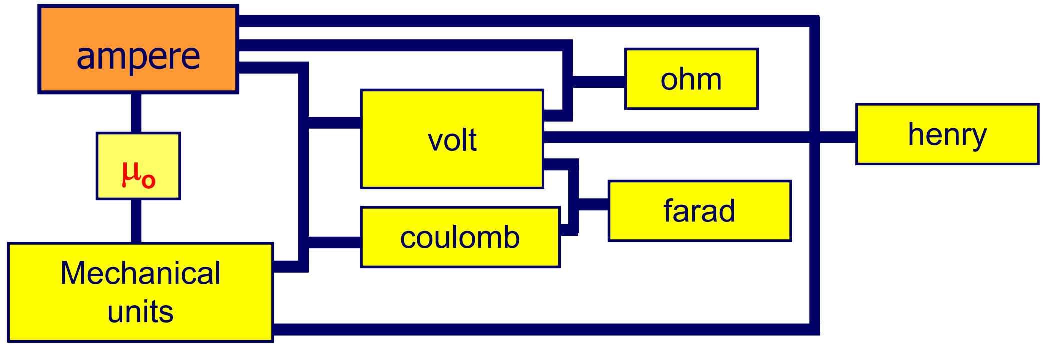
The SI definition of the ampere gives simultaneously those of the vacuum impedance :

⇨ $Z_0 = \mu_0 \cdot c$ ($Z_0 \approx 377\ \Omega$)

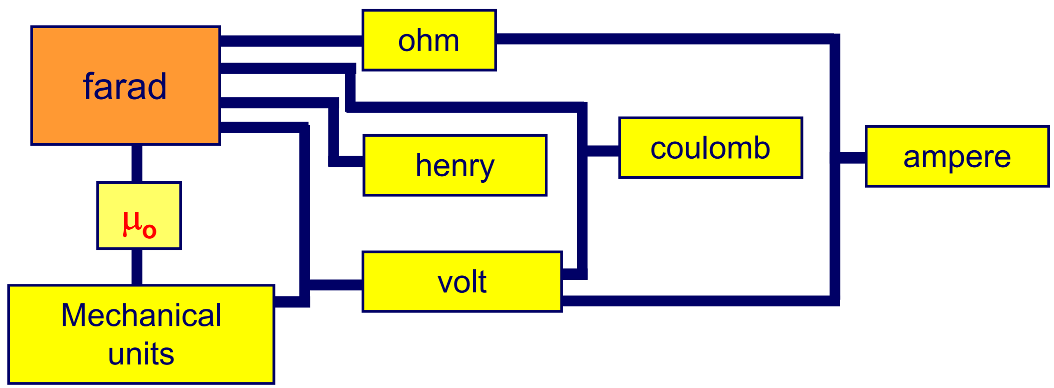
and the vacuum permittivity :

⇨ $\varepsilon_0 = 1/(\mu_0 \cdot c^2)$ ($\varepsilon_0 \approx 8.85\ \mathrm{pF/m}$)

Realization chain of the electrical units

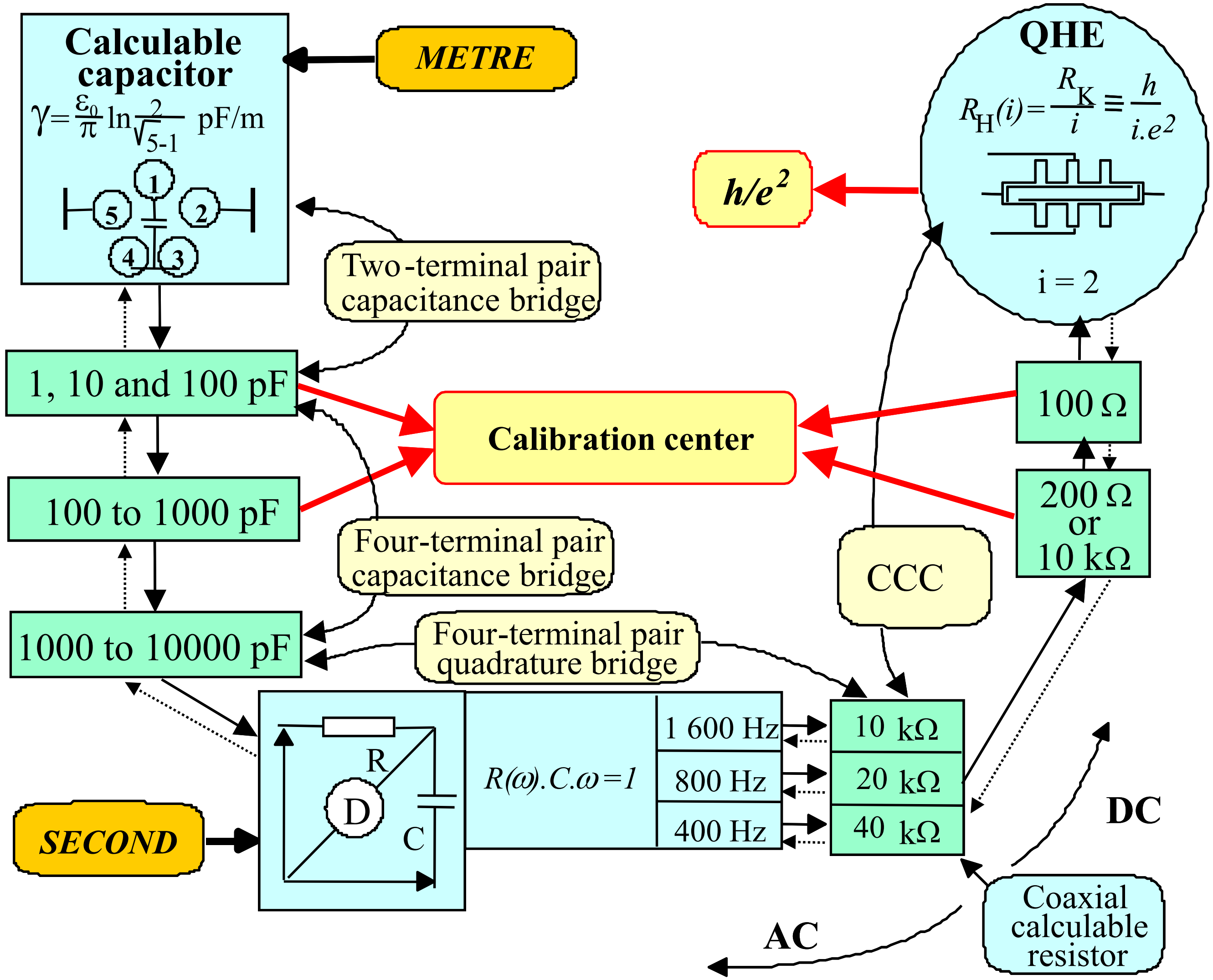


Filiation of the electrical units definition



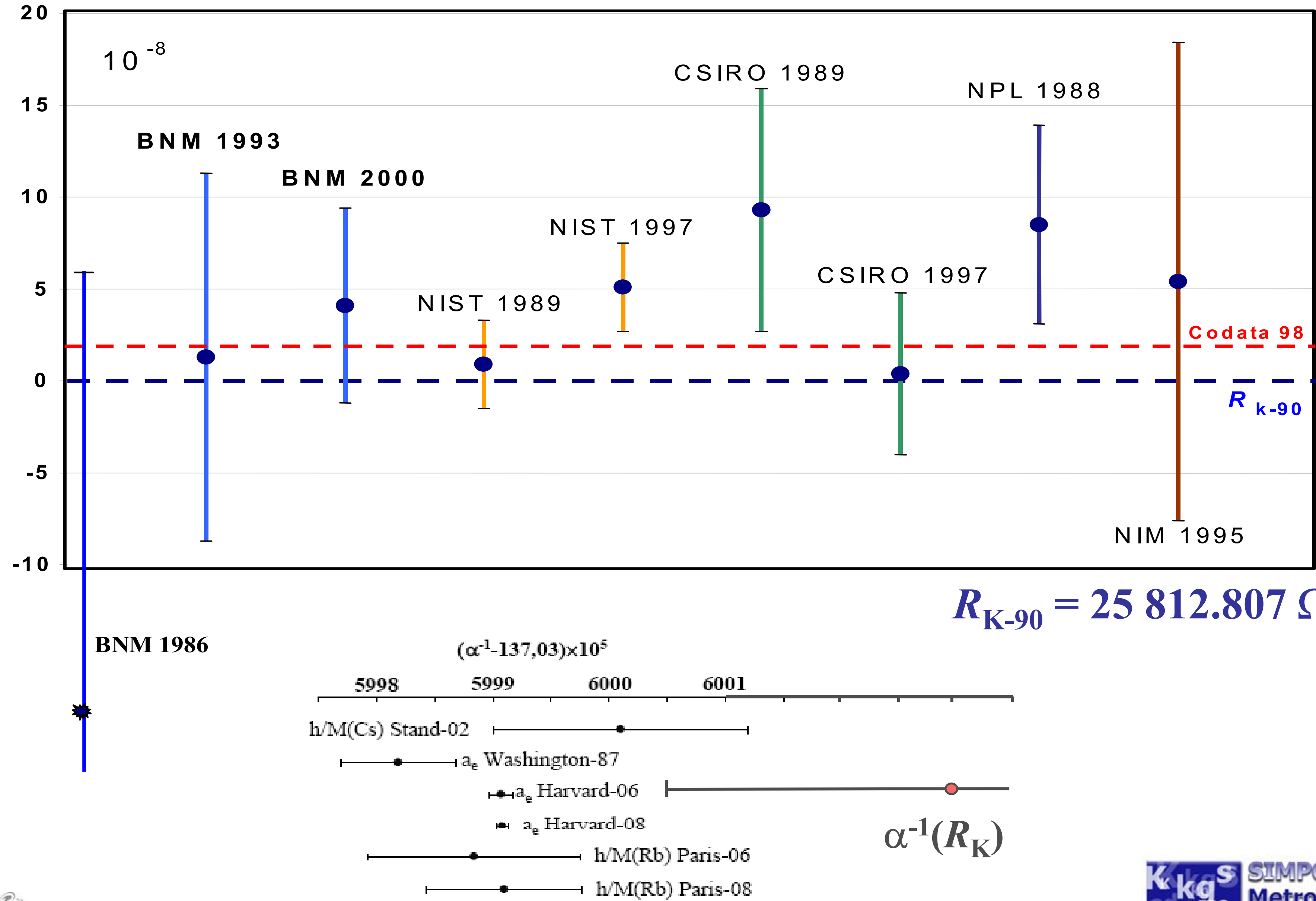
Filiation of the realisations

Determination of R_K at LNE : *linking the ohm to the farad*



… and checking $R_K = h/e^2$

Determination of R_K in National Metrology Institutes



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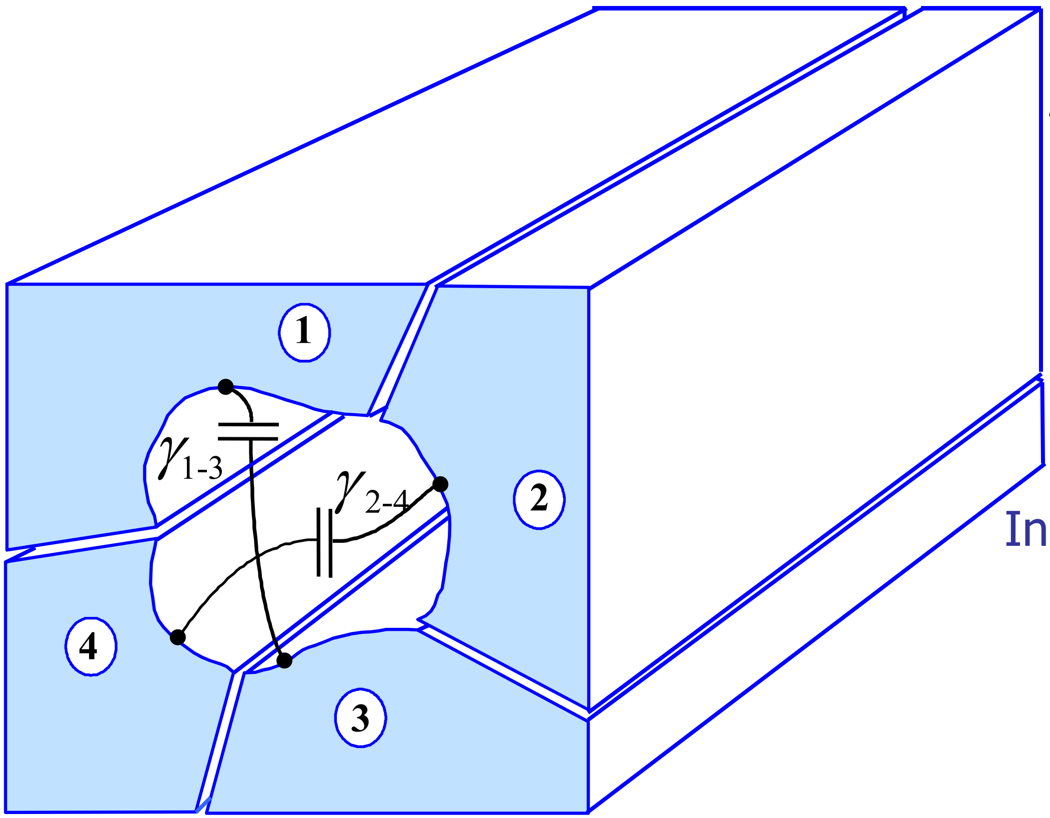
The Thompson Lampard calculable capacitor

The Lampard theorem

Assume four electrodes of **infinite length**, separated by a **null thickness insulator**, delimiting a volume perfectly **cylindrical :**



$\gamma_{1\text{-}3}$ and $\gamma_{2\text{-}4}$: lineic capacitances (pF/m)

$$\boxed{e^{(-\pi/\varepsilon)\gamma_{1\text{-}3}} + e^{(-\pi/\varepsilon)\gamma_{2\text{-}4}} = 1}$$

In a case of a symmetrical arrangement :

$$\gamma = \frac{\varepsilon_0 \ln 2}{\pi} \cong 1.953\ldots \text{ pF/m}$$

*Lampard D.G, IEE, **104C** (1957) 271-280*

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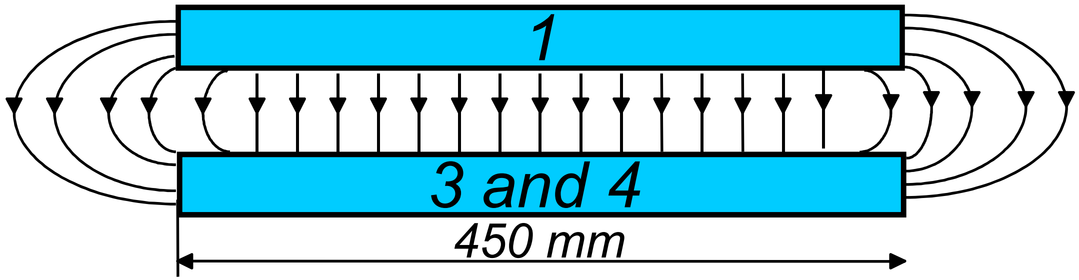
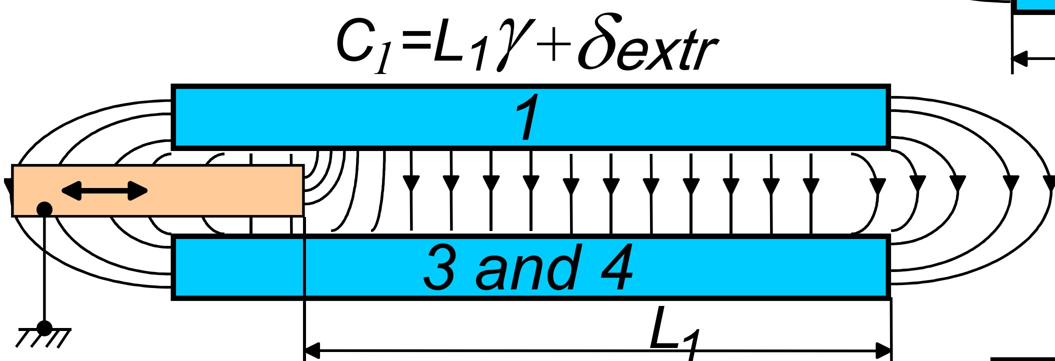
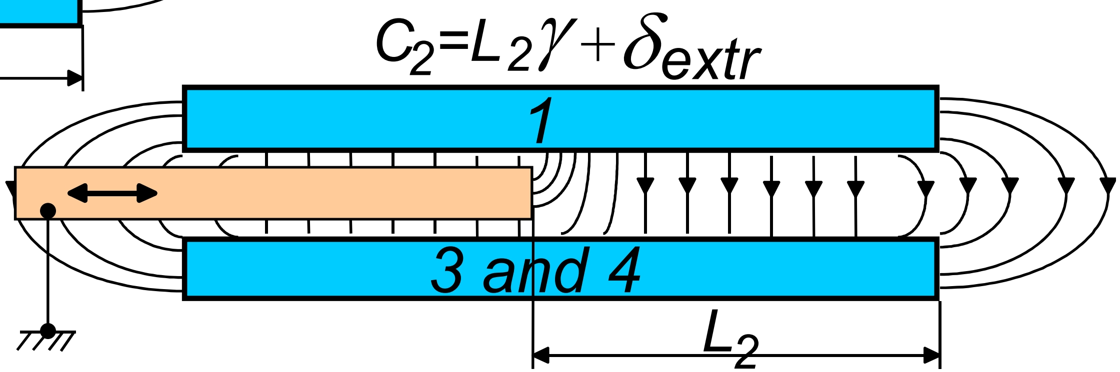
The Thompson Lampard calculable capacitor

➢ Infinite lenght of the electrodes

The Lampard theorem is valid only for a uniform field repartition (out of extremity effect)





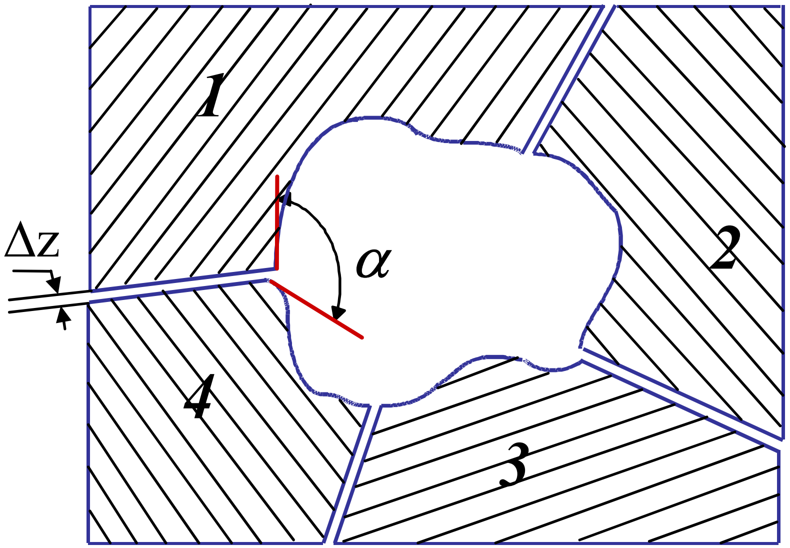


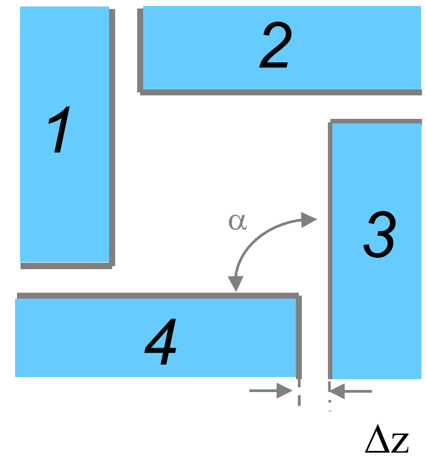
$$\boxed{\Delta C = C_1 - C_2 = \gamma(L_1 - L_2)}$$

[3/8 pF for 138.25 mm]

The Thompson Lampard calculable capacitor

➢ Null thickness inter-electrodes insulator



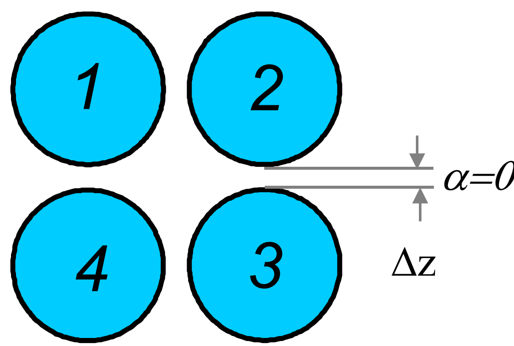


$$\alpha = \frac{\pi}{2}$$

$$k = \frac{1}{2}$$

$$\delta = \frac{\Delta C}{C} \approx \left(\frac{\Delta z}{z}\right)^2$$

It can be demonstrated that the error due to the non null thickness of the insulator, δ, varies as a function of angle, α, between the electrodes' tangents, at their contact to the insulator (of thickness Δz)



$$\alpha = \frac{\pi}{n}, \alpha \to 0$$

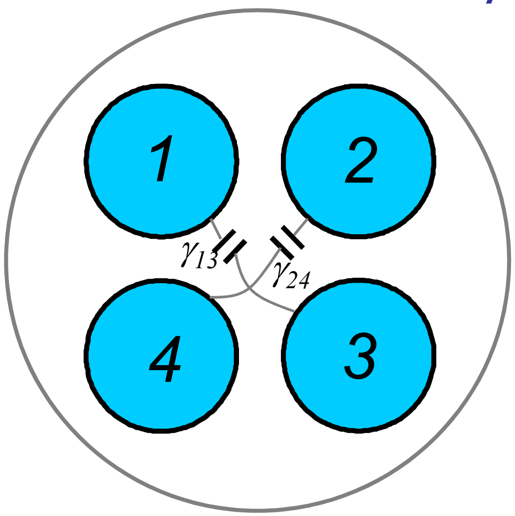
$$k = \frac{1}{n}, n \to \infty$$

$$\delta = \frac{\Delta C}{C} \approx \left(\frac{\Delta z}{z}\right)^n \approx 0$$

The Thompson Lampard calculable capacitor

➢ Mise en pratique

- For a 4 electrodes system :



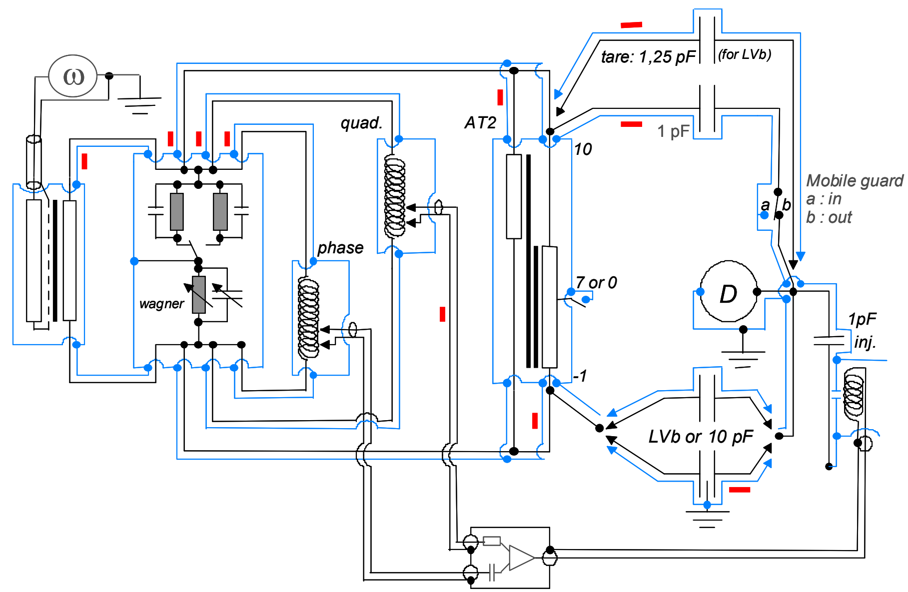
theorem : $$\exp\left(-\frac{\pi\gamma_{13}}{\varepsilon_0}\right)+\exp\left(-\frac{\pi\gamma_{24}}{\varepsilon_0}\right)=1$$

measurement : $K = \gamma_{1\text{-}3} / \gamma_{2\text{-}4}$

The capacitance variation allows to calibrate a capacitance C_x :

$$C_x = k_i . \Delta L \gamma_{ij}$$

Comparison system ratio



This kind of standard has been developped in different NMIs (NMIA, PTB, BNM, NPL, NIST…)

The Thompson Lampard calculable capacitor

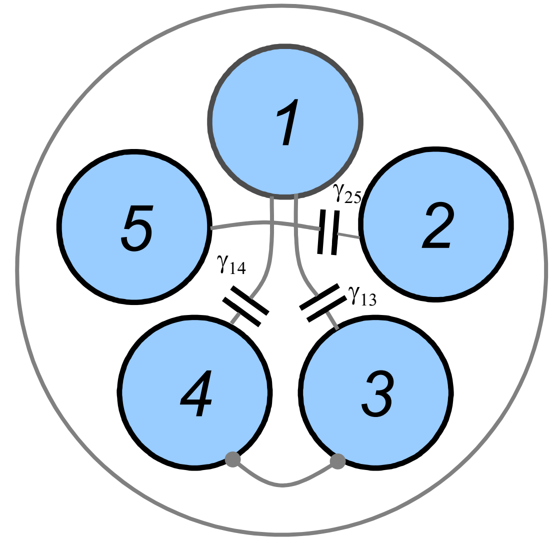
➢ Lampard theorem extension

N. Elnekavé (BNM, 1973): Capacitor with **5** electrodes

$\exp(-\pi(\gamma_{13}+\gamma_{14})/\varepsilon_0) + \exp(-\pi\gamma_{25}/\varepsilon_0) = 1$

In case of a perfect symmetry

$\gamma = (\varepsilon_0/\pi)\cdot\ln[2/(\sqrt{5}-1)]$ F/m = 1. 356 235 626 … pF/m



Circular permutation ➔ 5 Lampard configurations

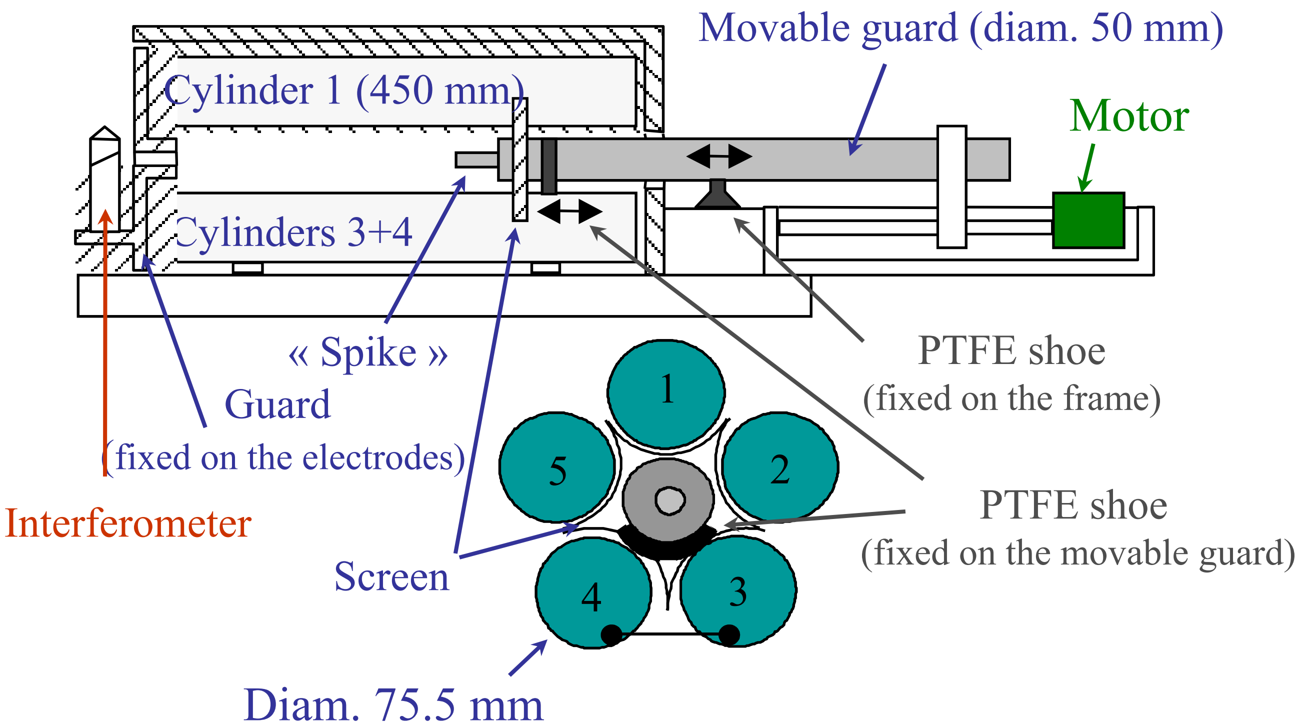
$$\left.\begin{aligned} C_x &= k_1 \Delta L(\gamma_{1,3}+\gamma_{1,4}) \\ &\dots \\ C_x &= k_5 \Delta L(\gamma_{5,2}+\gamma_{5,3}) \end{aligned}\right\}$$

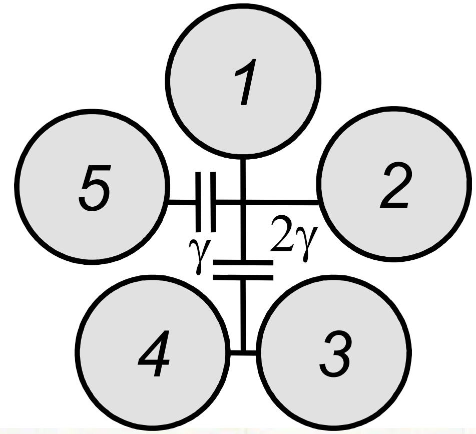
These 5 equations are successively associate with the 5 possible Lampard relations :

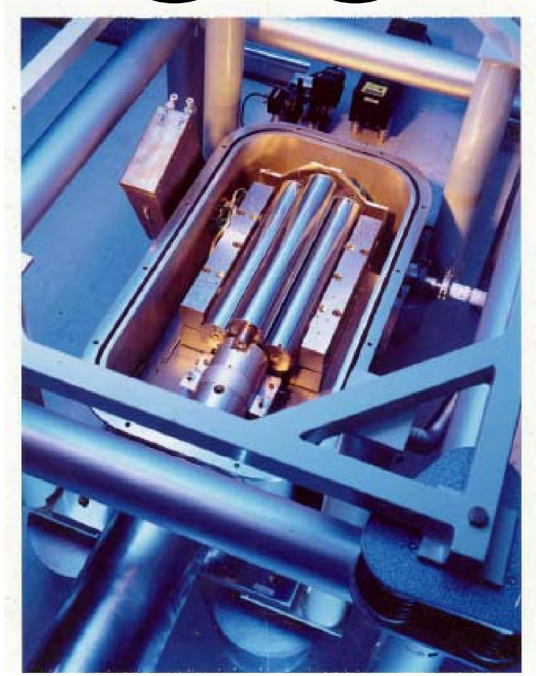
$$\exp\left[(-\pi/\varepsilon_0)(\gamma_{i,i\oplus 2}+\gamma_{i,i\oplus 3})\right]+\exp\left[(-\pi/\varepsilon_0)(\gamma_{i\oplus 1,i\oplus 4})\right]=1$$

➔The resolution of this 6 equations system gives the $\gamma_{i,j}$ values and the C_1 to C_5 values of C_x

The French Thompson Lampard calculable capacitor







➔Usable length ≈ 138.25 mm
ΔC ≈ 0.375 pF

Standard uncertainty : 3.8 10^{-8}

G. Trapon *et al*, Metrologia, **40** (2003) 159-171

Determination of R_K in 2000

Uncertainty components	1σ relative uncertainty x 10^8
Laser wavelenght	0.003
Air refractive index	0.013
Laser alignment	0.09
Defect of the movable corner cube reflector	0.02
Deformation of the pentagon	0.02
Cylindrical defect	2.4
Efficiency of the movable guard	0.2
Lateral shifting of the spike	3
Bridge ratio (used 5 times)	1.5
Injection signal	0.3
Bridge coaxiality defect	0.5
Loading	0.3
Voltage effect (10 pF, 100 pF, 1000 pF)	1.0
10 nF connection effect	0.3
Frequency (quad bridge)	0.02
Resistor frequency effect	1.5
DC QHE	0.3
Total type B uncertainties	**4.6**

$$R_K = R_{K-90}(1+4.1\ 10^{-8})\ (1\sigma=5.3\ 10^{-8})$$

Realisation of a new set of electrodes

Fabrication of a new standard in vertical position

New ratio-transformers

New optimized coaxial calculable resistors and resistance transfer standards for quad. bridge (already available)

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Present improvements

Objective: total uncertainty $\leq 1\ 10^{-8}$

- **Realisation of a new set of electrodes**

  Δ(rect. + diam.)< 0.1µm ➔ 5 10^{-9} (instead of 2.4 10^{-8})

  ➢ **Dedicated measurement apparatus**

  Measurement of the cylindricality defect of the new set of electrodes

- **New Thompson-Lampard calculable capacitor :**

  5 electrodes in vertical position

  Lateral shifting of the movable guard < 50 nm ➔ 5 10^{-9} (instead of 3 10^{-8})

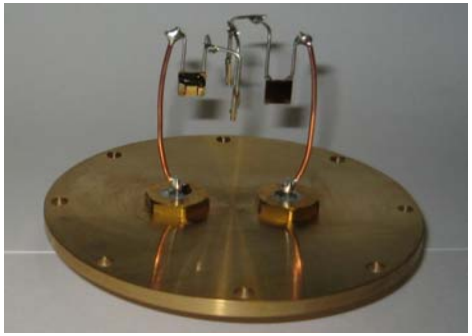
  Positionning of the electrodes better than 100 nm

- **Improvements of the measurement chain**

  ➢ Construction of new ratio transformers

  ➔ target uncertainty on ratio calibration ≈ 1 10^{-9}

  ➢ Fabrication of new calculable resistors (1 kΩ Haddad type) and transfer standards (10, 12.9, 20 and 40 kΩ) for quadrature bridge



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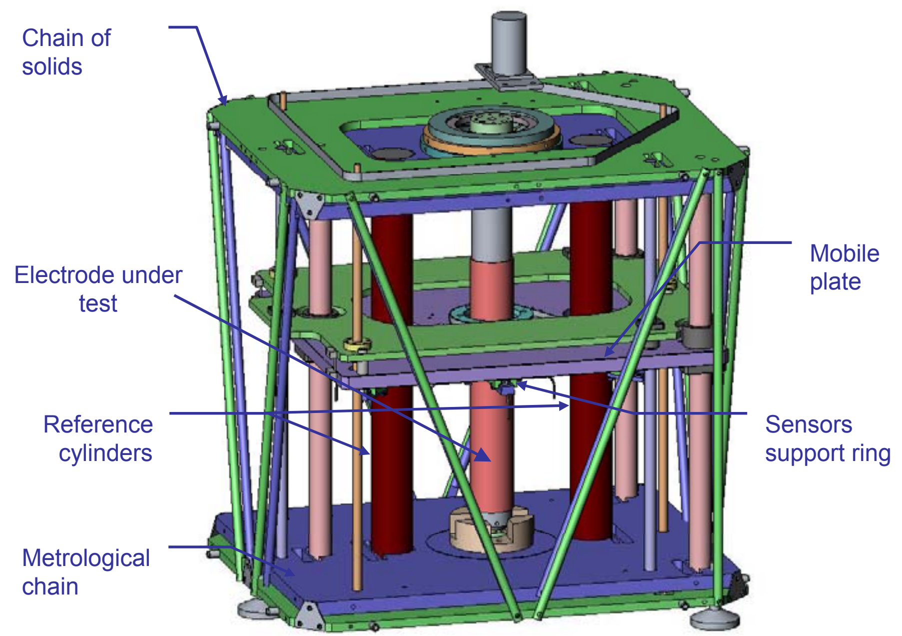
Fabrication of a new set of electrodes

➢ Dedicated Measurement Device

Objective : Measurement of the straightness and the parallelism of the generating lines of the future electrodes with an uncertainty lower than 25 nm

Key elements of the device :

- based on the dissociated metrological principle
- designed to respect the **Abbe principle**
- **two vertical references** (in red) are the straightness reference
- **a mobile plate** (in purple) with 8 capacitive sensors : 4 sensors measure the reference cylinders ➔ information of position, 4 sensors measure the electrode
- **a second mobile plate** (in green) supports the first one without transmitting any deformation, this plate can go up and down along the electrode.



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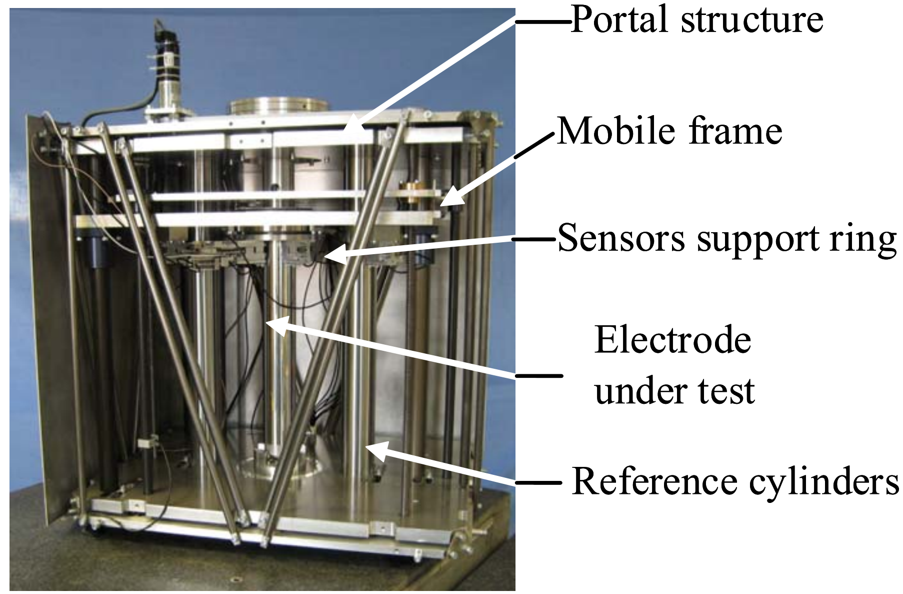

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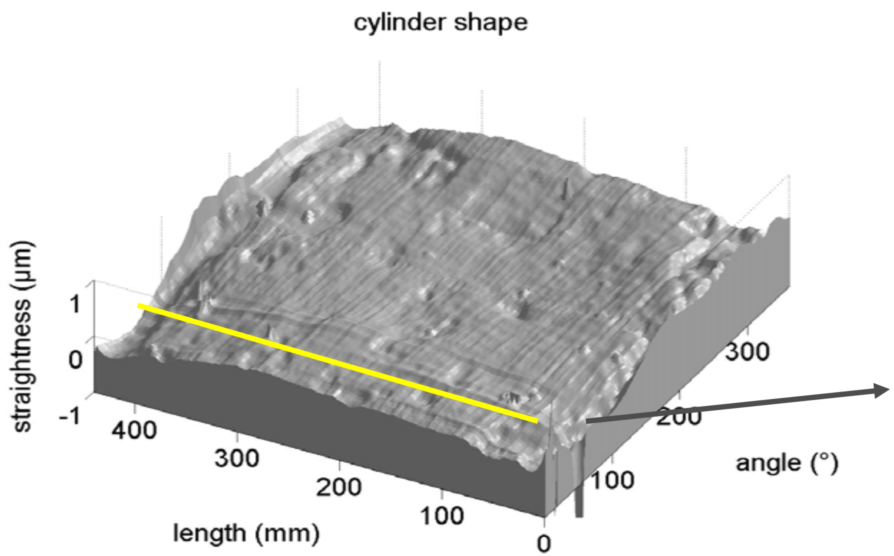
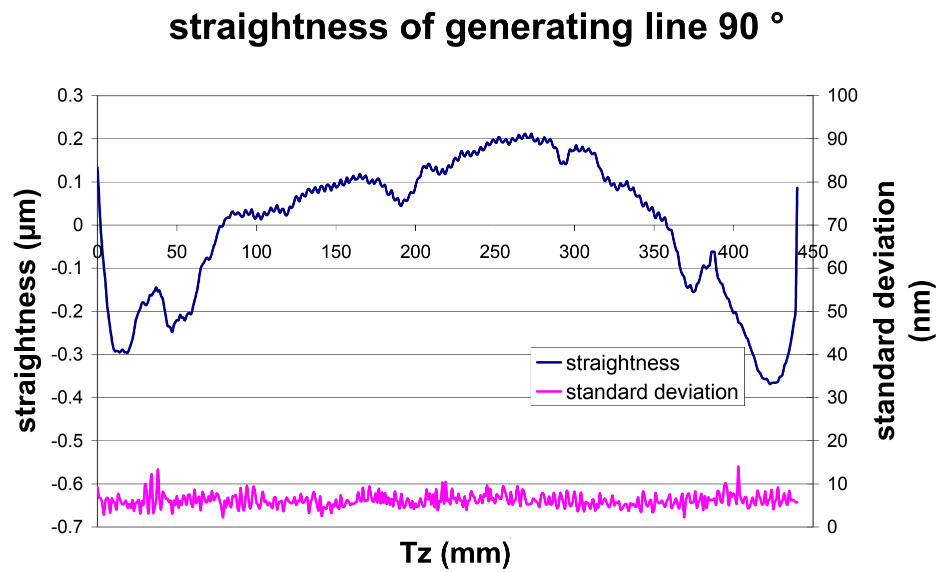
Cylindricality dedicated measurement device

Straightness measurement of the present standard reserve electrode





Cylindricality measurement (without conicity defect) with a 2,5 mm and 2,5 ° steps. The measurement was performed during 30 hours (4 forth and back runs)

Measurement of the same generating line during 12 hours with a 1 mm step (8 forth and back runs)

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Fabrication of a new set of electrodes

➢Fabrication of a new set of electrodes

The new set of electrodes is made of amagnetic stainless steel (grade 1.4539)

Dimensions : diameter 75.5 mm, length : 450 mm

After grinding the cylindricality defect is about 1,5 µm

To reduce this defect to a value close to 100 nm, the cylinders are manually lapped and polished

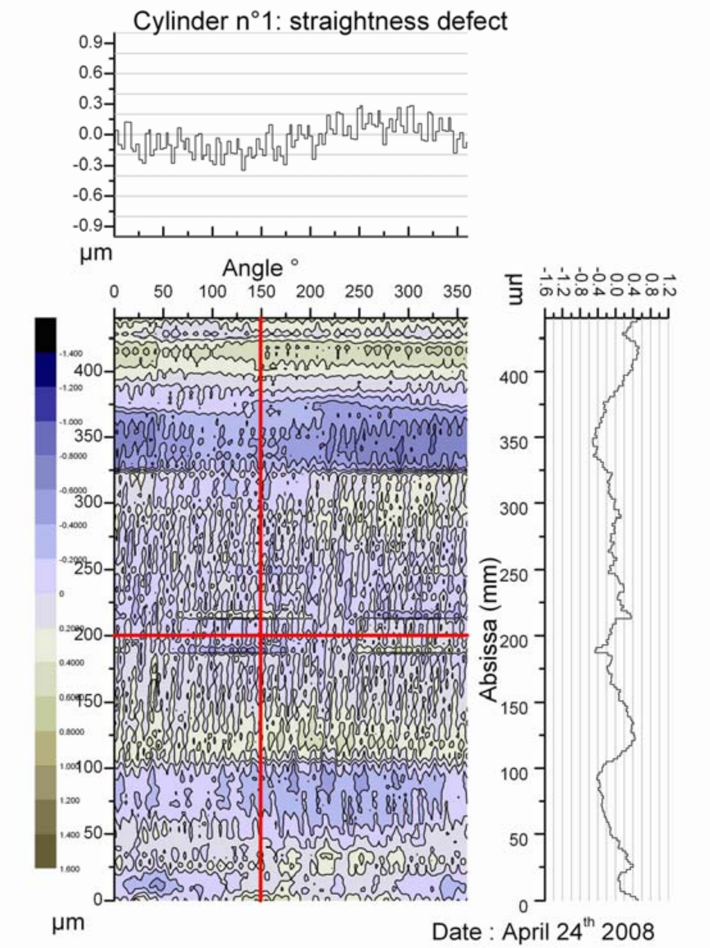
Fabrication of specific lapping-tool

The best results were obtained with grinding-tool made of corundum (abrasive) with shellac and rosin for bonding. The tool is shaped to fit the cylinders.

Fabrication of a new set of electrodes

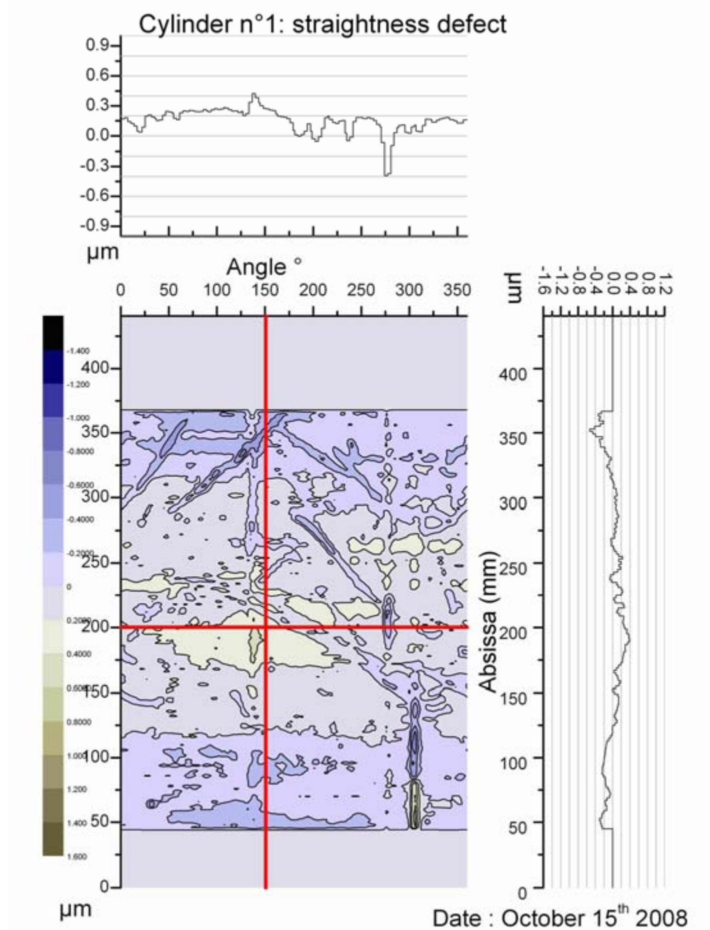
➢ **First results**

After the grinding stage



Straightness defect ~+/- 0,8 µm

After 30 hours of lapping



Straightness defect ~+/- 0.15 µm

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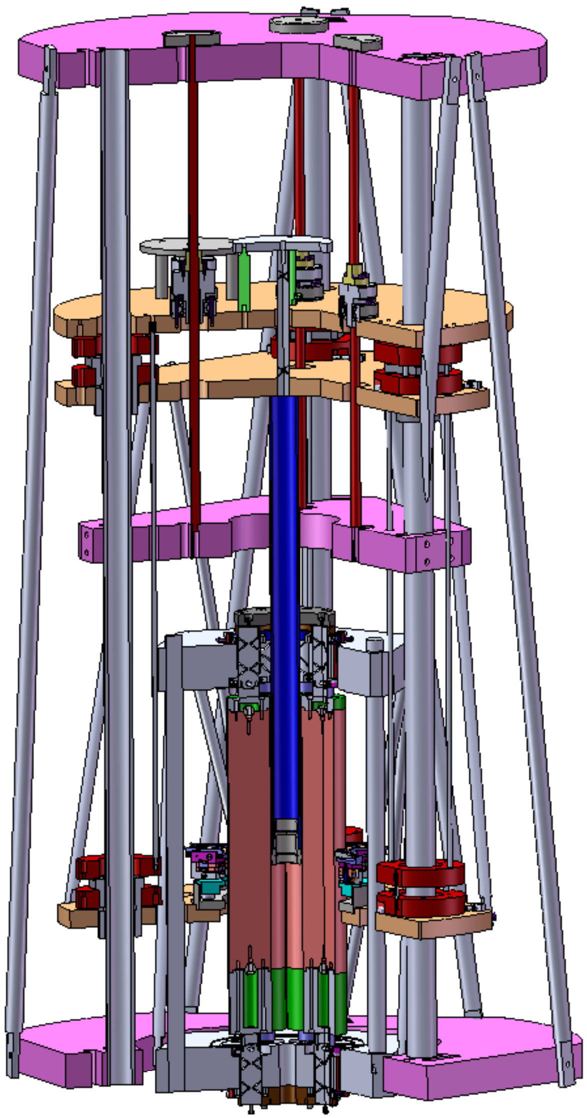
New mechanical design of the LNE cross calculable capacitor



Main characteristics :

- 5 electrodes in vertical position, 75.5 mm diameter, 450 mm long, with straightness defects < 0.1 µm (polishing process in progress), amagnetic stainless steel

- Integrated device to control electrodes relative position

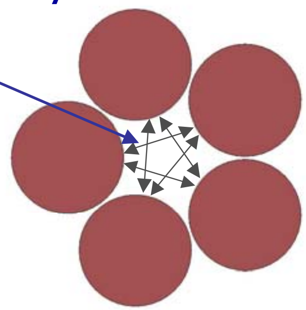
- Independent micrometric adjustment of the electrodes position coupled with a maintain system

- Cross section cylindricality quality is used to guarantee the movable guard lateral position

New mechanical design of the LNE cross calculable capacitor

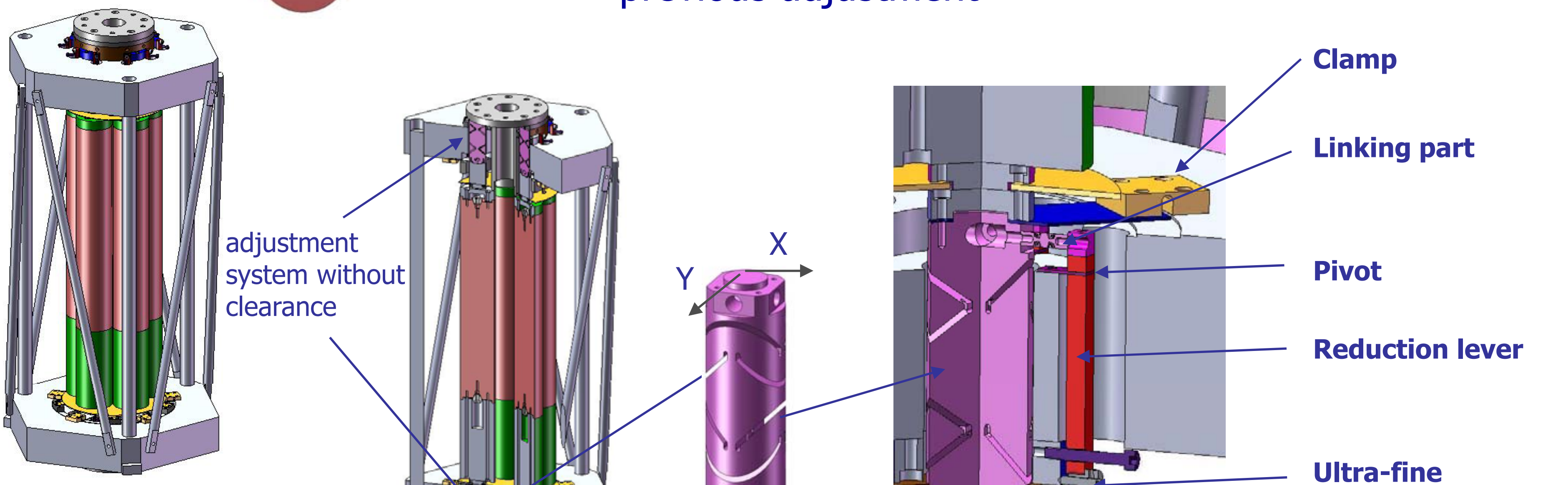
➢ Positionning the electrodes

cylindricality defects < 0.1 µm



Bidimensional micrometric adjustment with flexure leaves

Clamp fixe position, designed to not disturb previous adjustment



Lampard structure

Flexure stage

Bidimensional positioning system

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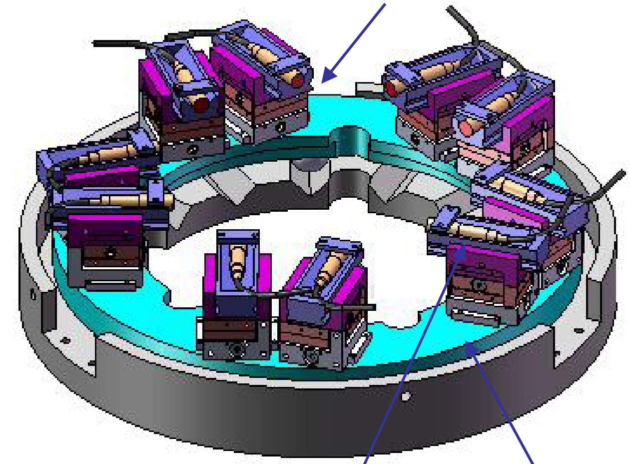
New mechanical design of the LNE cross calculable capacitor

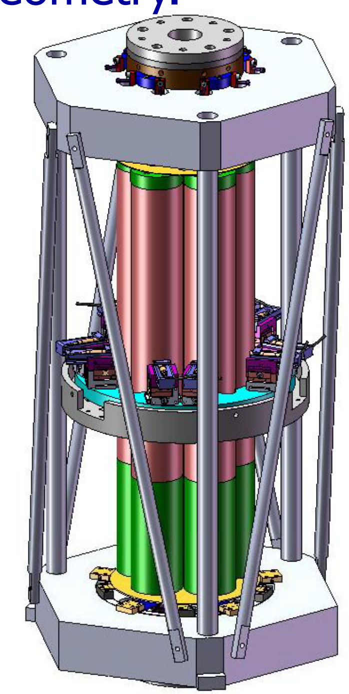
➢ **Control the distance inter-electrodes**

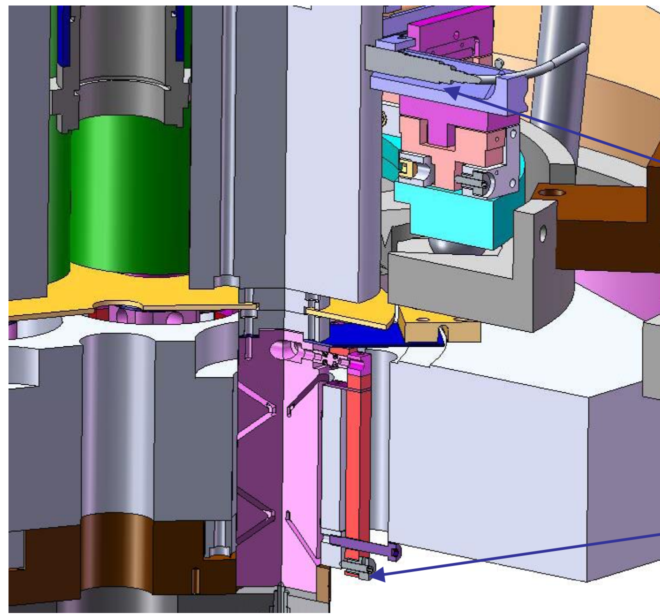
Integrated and removable measurement system : ten capacitive sensors on a non-shrinking plate indexable in 5 positions ➔ each sensor can measure the 5 cylinders.

This plate measure straightness of outside electrodes. This information combined to the geometry knowledge coming from the dedicated measurement apparatus allows to know the cross section geometry.



Lampard structure





Capacitive sensor in the same vertical plane of the micrometric screw

New mechanical design of the LNE cross calculable capacitor

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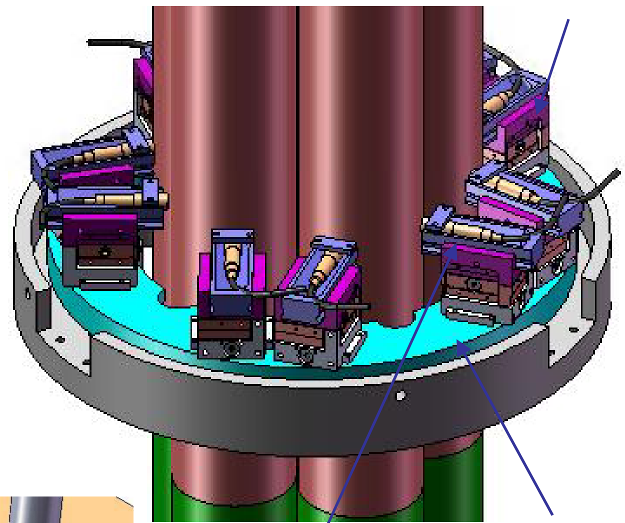
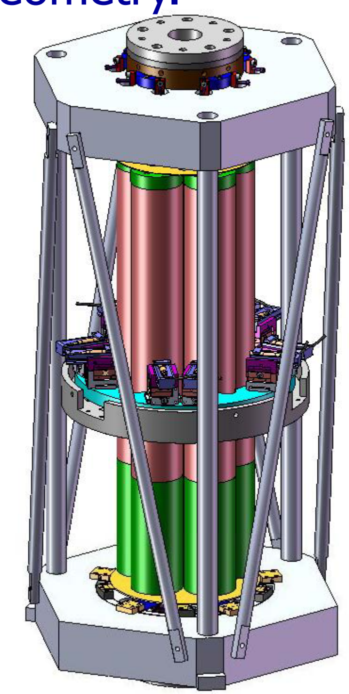
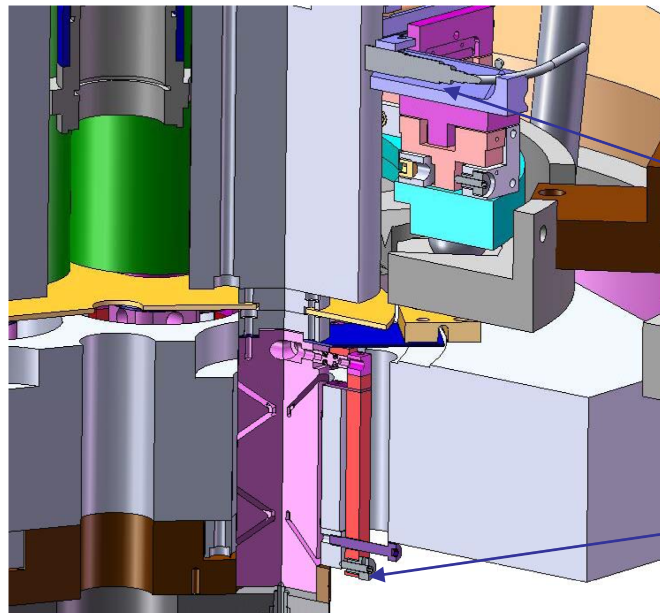


Lampard structure



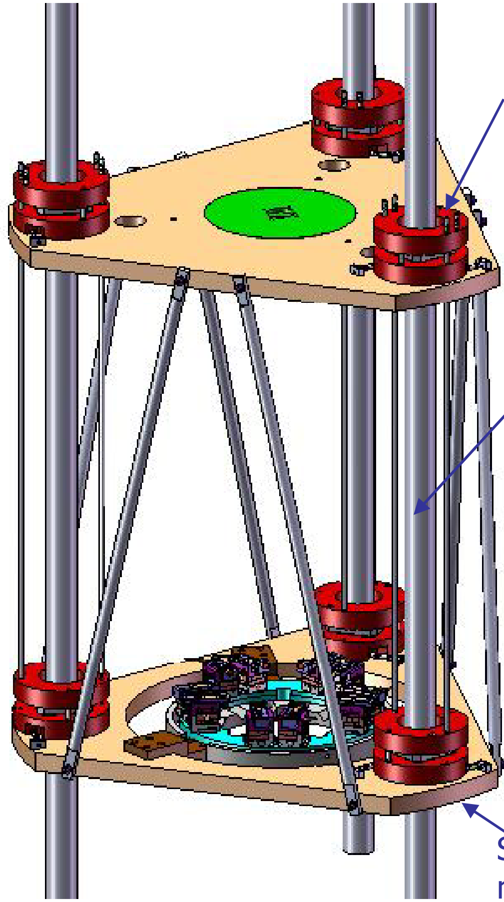


Capacitive sensor in the same vertical plane of the micrometric screw

New mechanical design of the LNE cross calculable capacitor

➢The movable guard

Metrological frame
non-shrinking structure



Sleeve bearing mounted in a double cardan

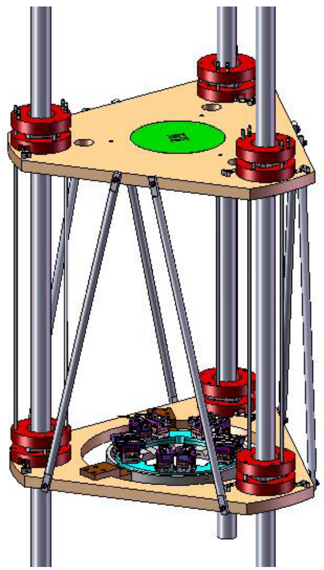
Guidance bars

Support plate of the measurement device

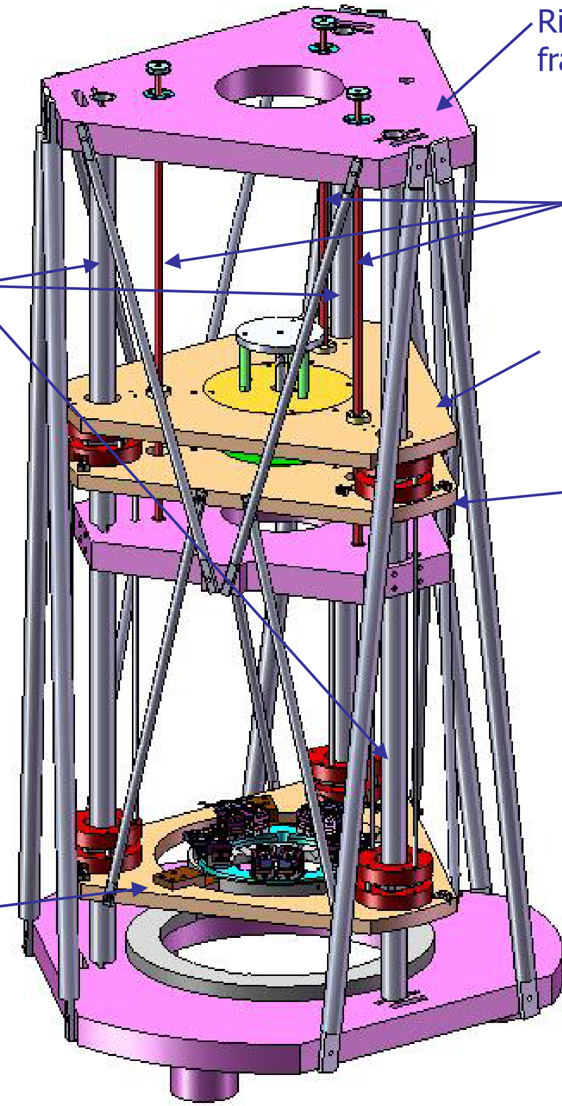
Metrological frame
non-shrinking structure
+ guidance bars

New mechanical design of the LNE cross calculable capacitor

➢The movable guard



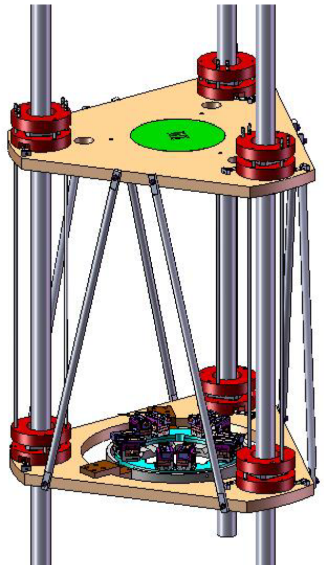
Metrological frame
non-shrinking structure



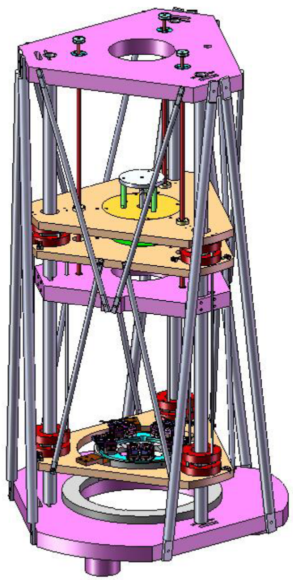
Metrological frame implented
in the rigid structure frame

New mechanical design of the LNE cross calculable capacitor

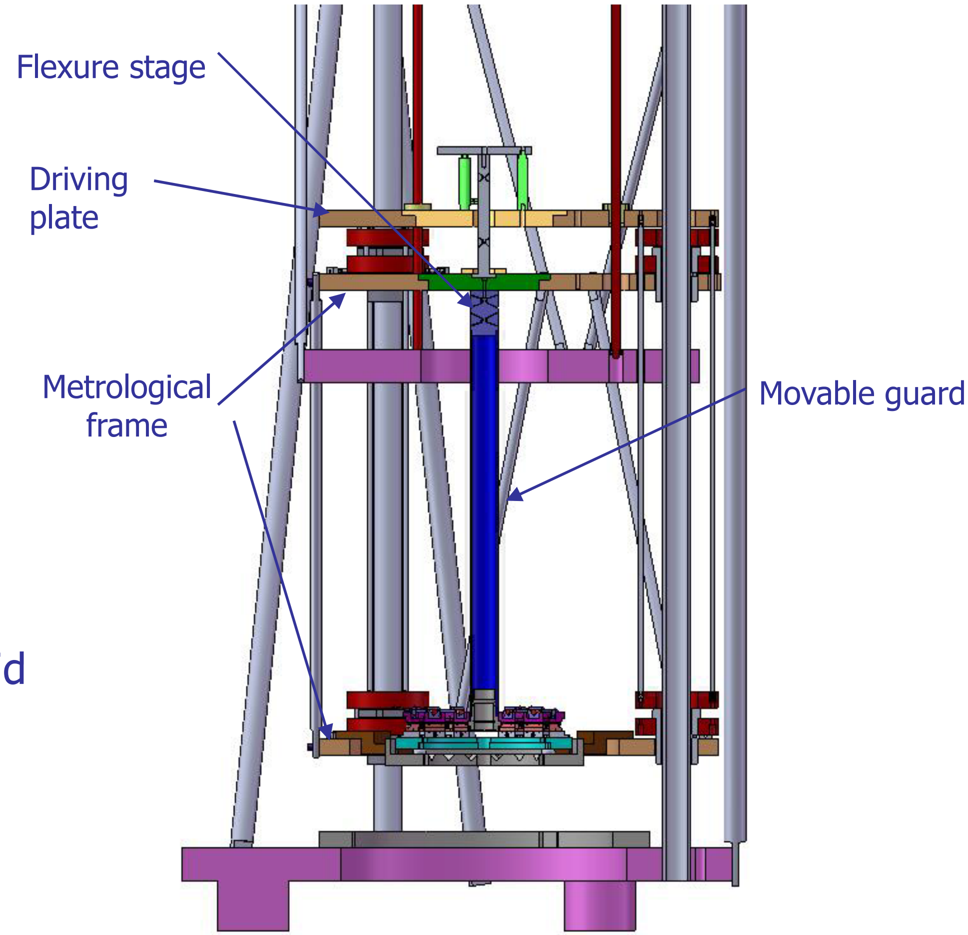
➢The movable guard



Metrological frame
non-shrinking structure



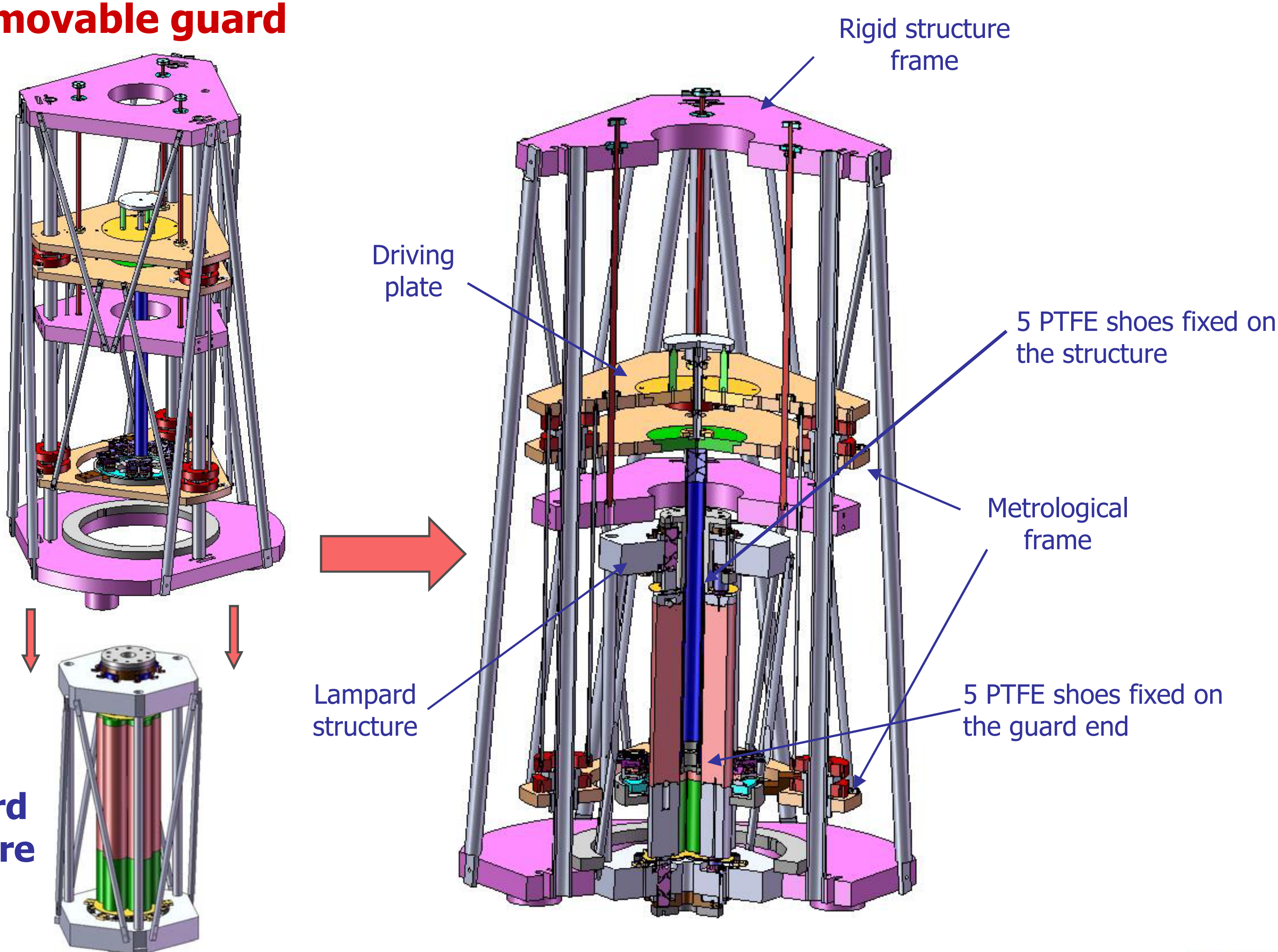
Metrological frame
implented in the rigid
structure frame



Integration of the movable
guard in the metrological frame

New mechanical design of the LNE cross calculable capacitor

➢The movable guard



Lampard structure

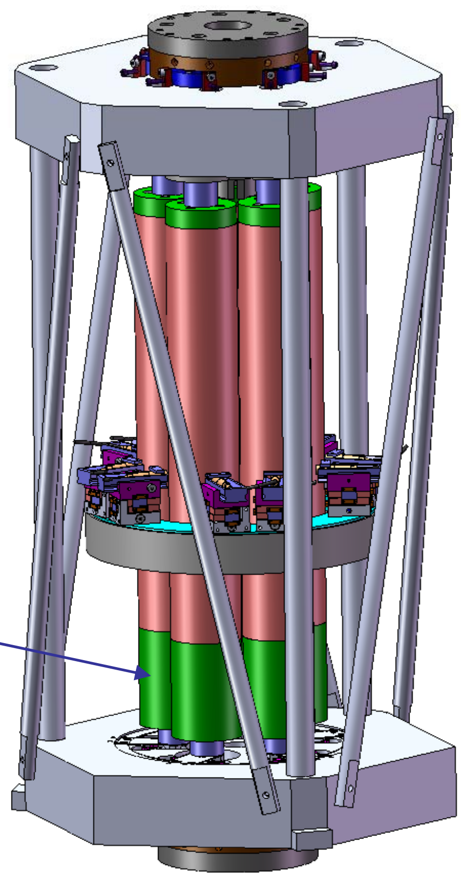
➔ Expected quality of guide better than 100 nm

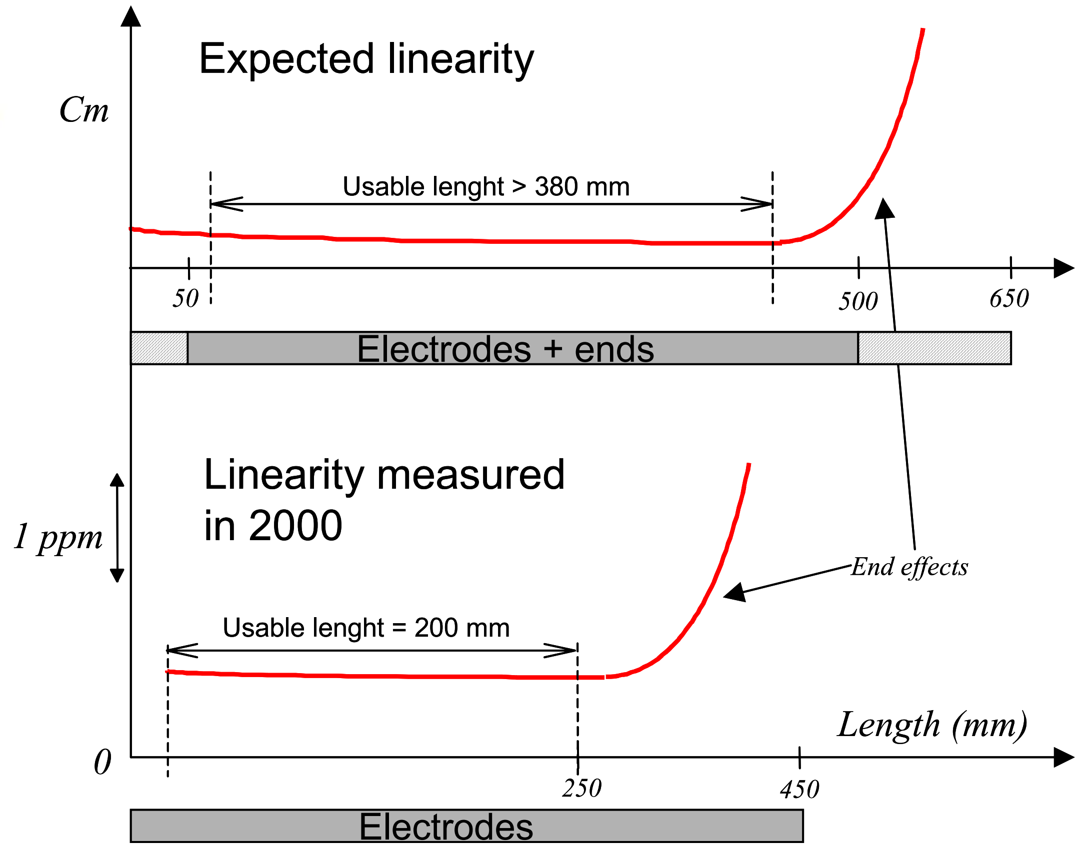
New mechanical design of the LNE cross calculable capacitor

Prolongations of the electrodes with same diameter but with larger cylindricity defects are expected to extend the usable lenght from 200 mm to 400 mm

➔ Capacitance variation between 0.3 and 1 pF

Electrodes prolongation





Summary

Introduction

- The SI electrical base unit - Ampere definition
- Filiation of the electrical units

Part 1 - Thompson Lampard calculable capacitor

- Presentation
- Mise en pratique at LNE
- Last results in 2000

Part 2 – Present developments

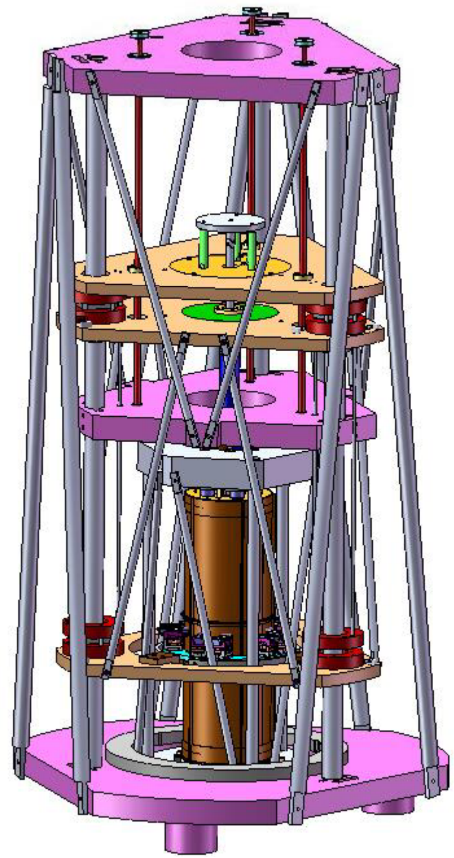
- Dedicated measurement apparatus
- Fabrication of a new set of electrodes
- Design of the new LNE Thompson Lampard calculable capacitor

Conclusion

Conclusion

➤ The new LNE cross calculable capacitor has the objective to make a direct determination of R_K in the SI in the next two years with an uncertainty of about 1~2 10^{-8}

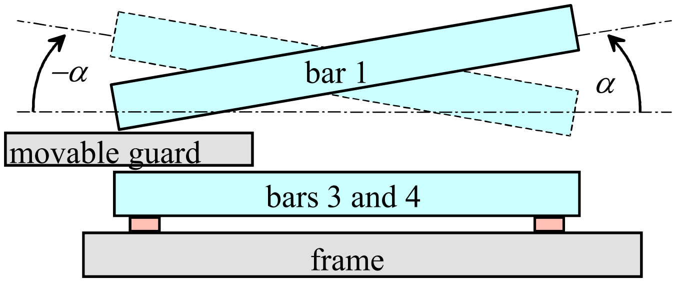
➤ This experiment will contribute to the knowledge of constants, in particular to test the relation $R_K = h/e^2$, and take part to the discussion on revising the SI based on fundamental constants.



➤ If *h*, *e*… are fixed in a « near » future, this measurement chain will contribute to the determination of constants that might become floating like μ_0, ε_0, Z_0

Thank you

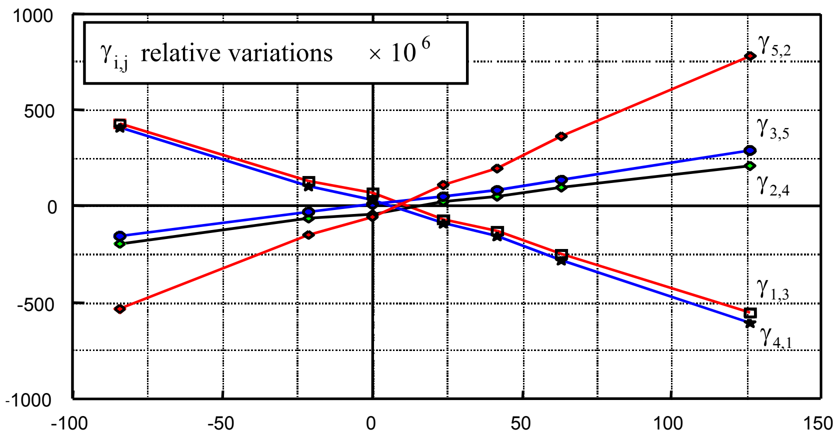
Effect of the tilt of one electrode

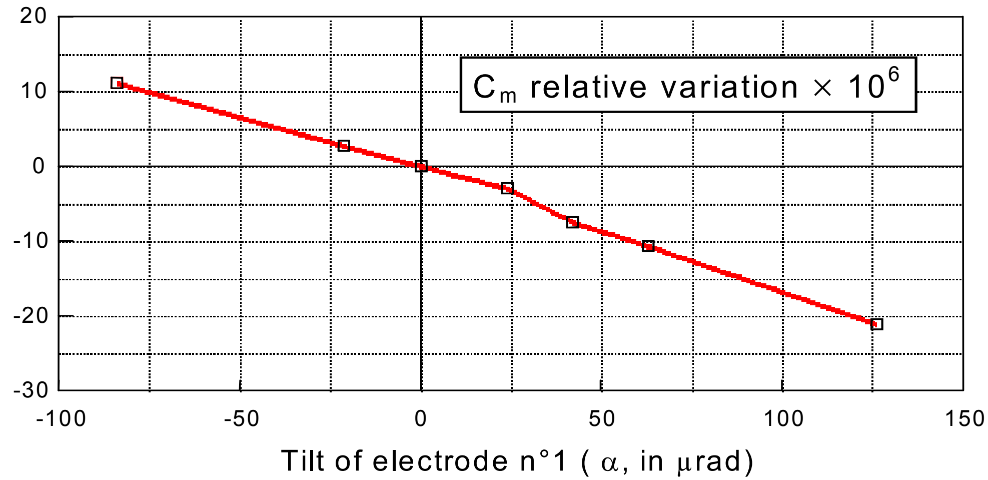


Influence coefficient : -0.16 µF.F^{-1}.µrad^{-1}

Residual cylindricity defect : 3.5 µrad

Corresponding relative error : -59 10^{-8}

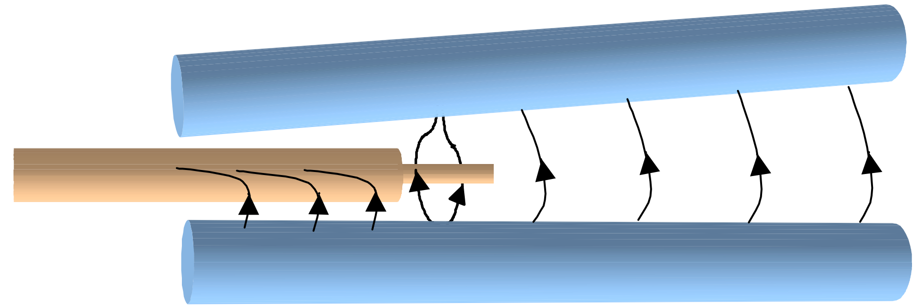


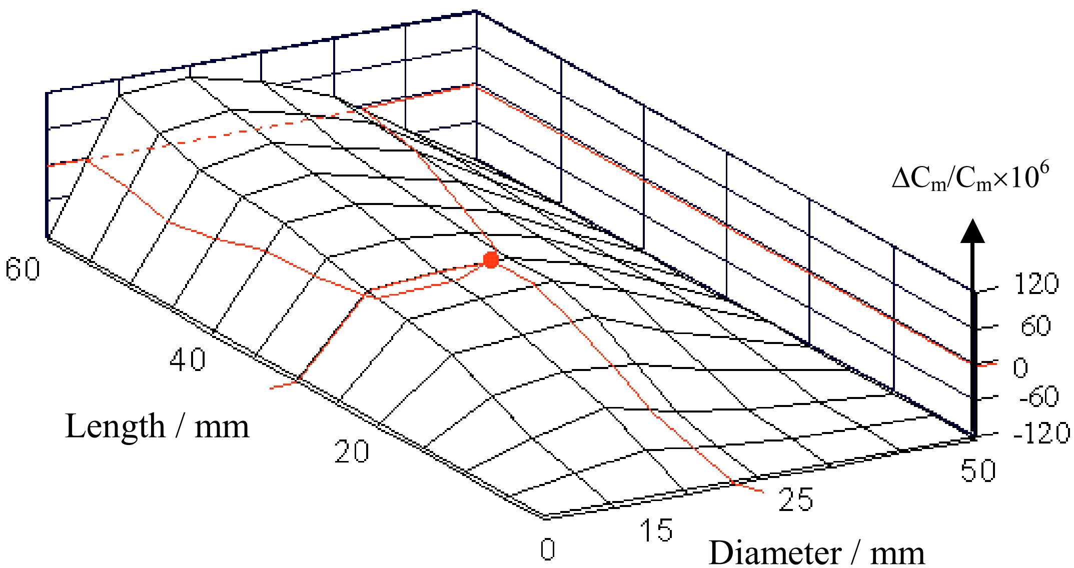


The French Thompson Lampard calculable capacitor

➢ **Compensation for the residual cylindricality defect**

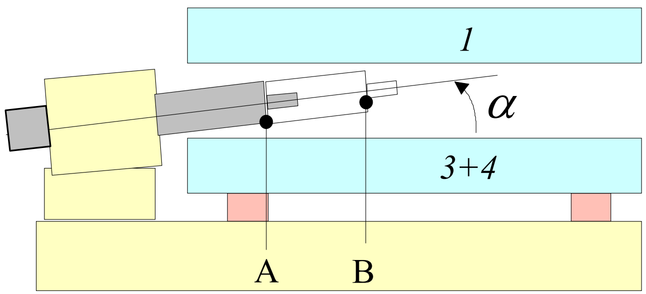
Influence of the "spike" length and diameter on ΔCm/Cm, for 750 µrd tilting of electrode n°1

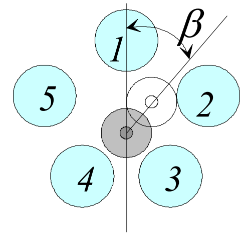




The influence coefficient (about 2 10^{-7}/µrd) is reduced by a factor ≥ 25. The effect of the residual cylindricality imperfections (about 3 µrad) is now taken into account as a 2.4 10^{-8} uncertainty (1σ) with the spike in place.

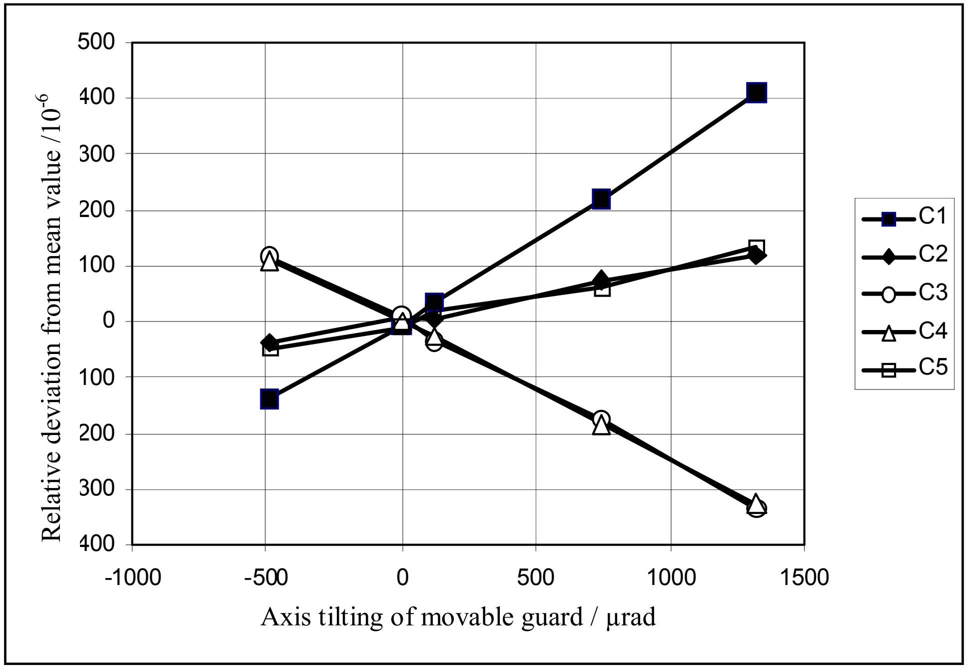
Movable guard displacement





➔ Variation of the extremity effect between the two measurements

$$\frac{C_i - C_m}{C_m} = -0.3\alpha\cos\left[\beta + (i-1)\frac{2\pi}{5}\right]$$



➔ A lateral displacement of 5 µm leads to a defect of 6 10^{-7}

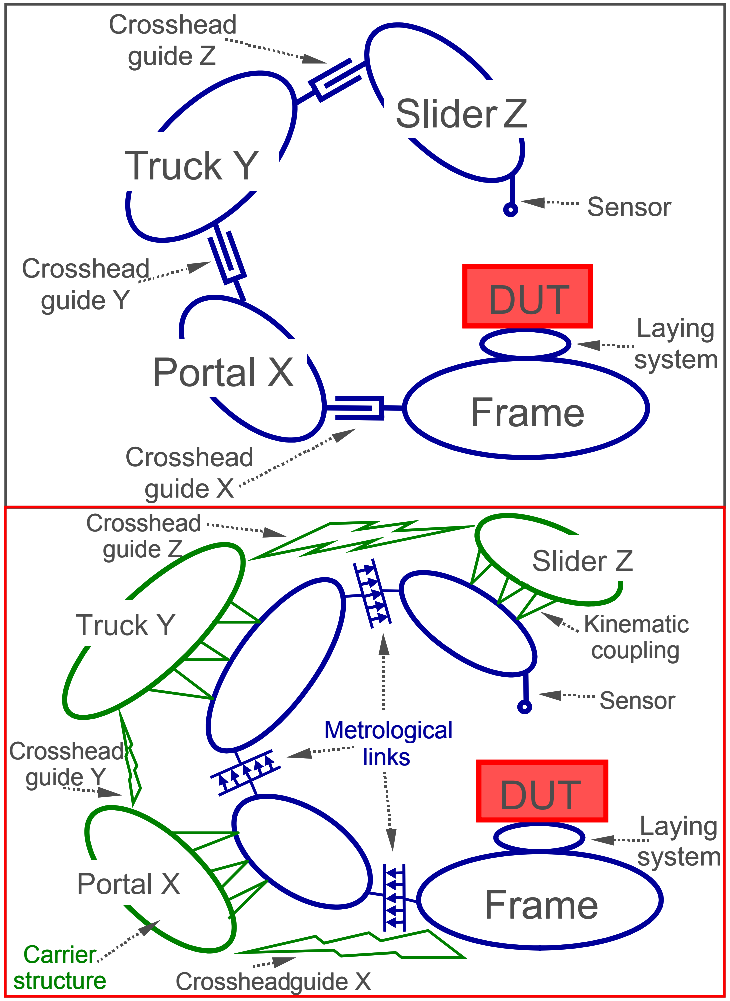
PTFE shoes guidance system

Lateral displacement < 0.3 µm

Corresponding uncertainty : 3 10^{-8}

Fabrication of a new set of electrodes

Objective : Measurement of the straightness and the parallelism of the generating lines of the future electrodes with an uncertainty lower than 25 nm



Classical structure

Metrological chain of a 3D measurement device of serial architecture : the final precision depends on each element defects. Guiding defects and solids flexibility add entirely

➔ **This kind of structure is not appropriate with the target uncertainty**

The dissociated metrological structure

The « dissociated metrological structure » consists in separating the metrological chain, wich assumes the metrological reference, from the solids chain, wich assumes displacement, positioning and support of the sample. The link between them is made by a kinematic coupling

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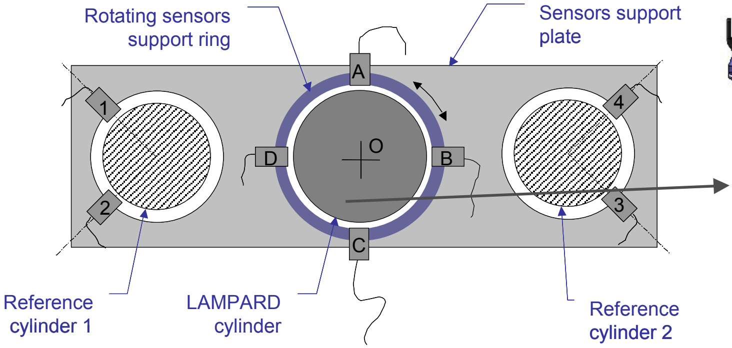
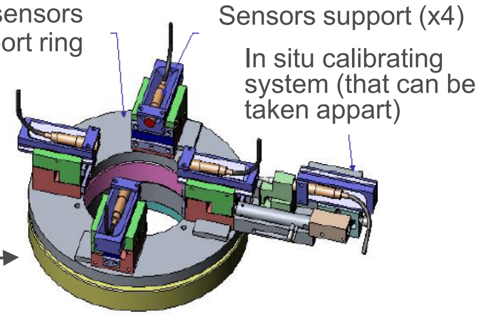
Cylindricality dedicated measurement device

The metrological mobile plate

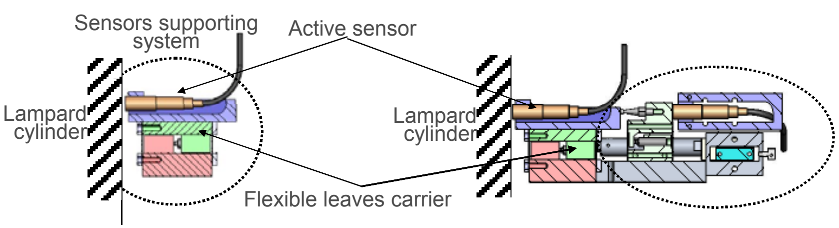




Rotating ring with sensors A,B,C,D

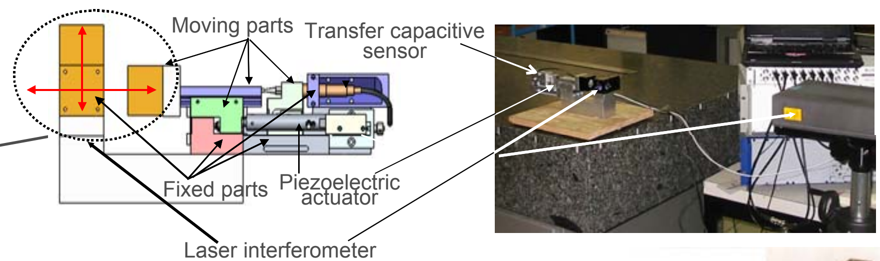
- 8 capacitive sensors
- Sensors 1,2,3,4 ➔ mobile plate position in the plane (reference)
- Sensors A,B,C,D on the rotating support ➔ electrodes generating lines measurement

Sensors calibration



sensor working situation

In-situ sensor calibration



Calibration of the calibrating device

The sensors are calibrated on a 40 µm range, they are located at 300 µm from the electrode surface, the uncertainty on their calibration is ~10 nm

Capacitive sensors

The sensors made by FOGALEnanotech :

Type MCC10EC

Active surface area : diam 5.5 mm

Measuring range : 200 µm, bandwitch 0-10 kHz

Thermal drift : < 0.005% - Resolution ~ nm

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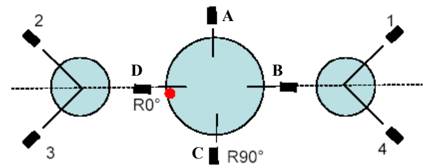
Cylindricality dedicated measurement device

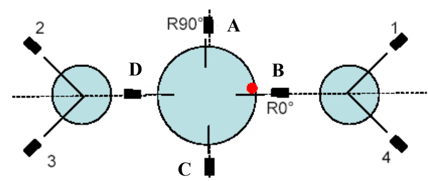
Calibration of the machine straightness

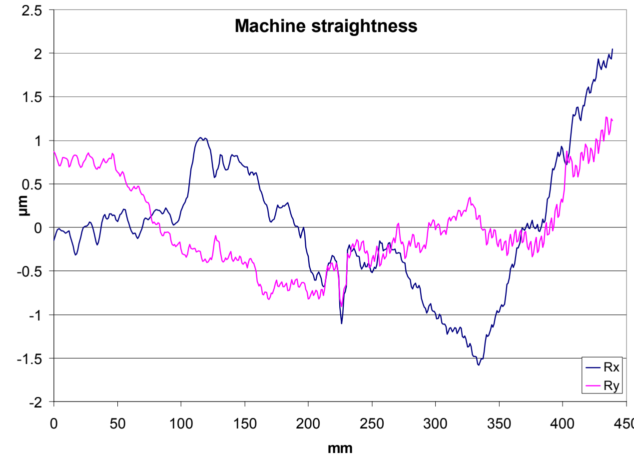
Defaults elimination by reversal

Before reversal

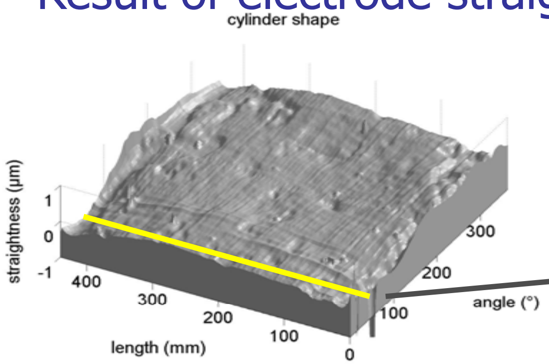


After reversal

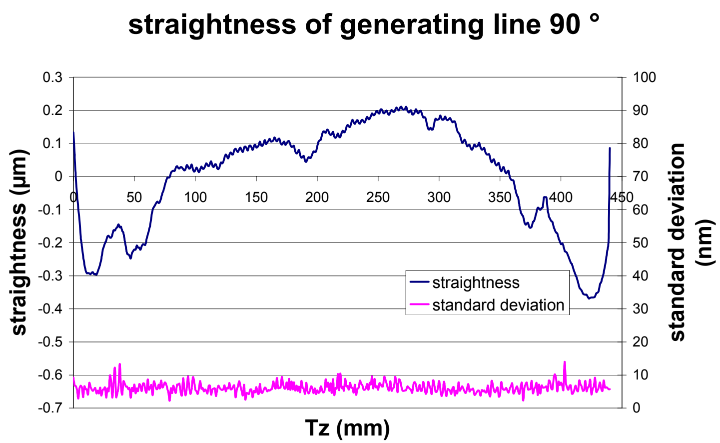




Result of electrode straightness



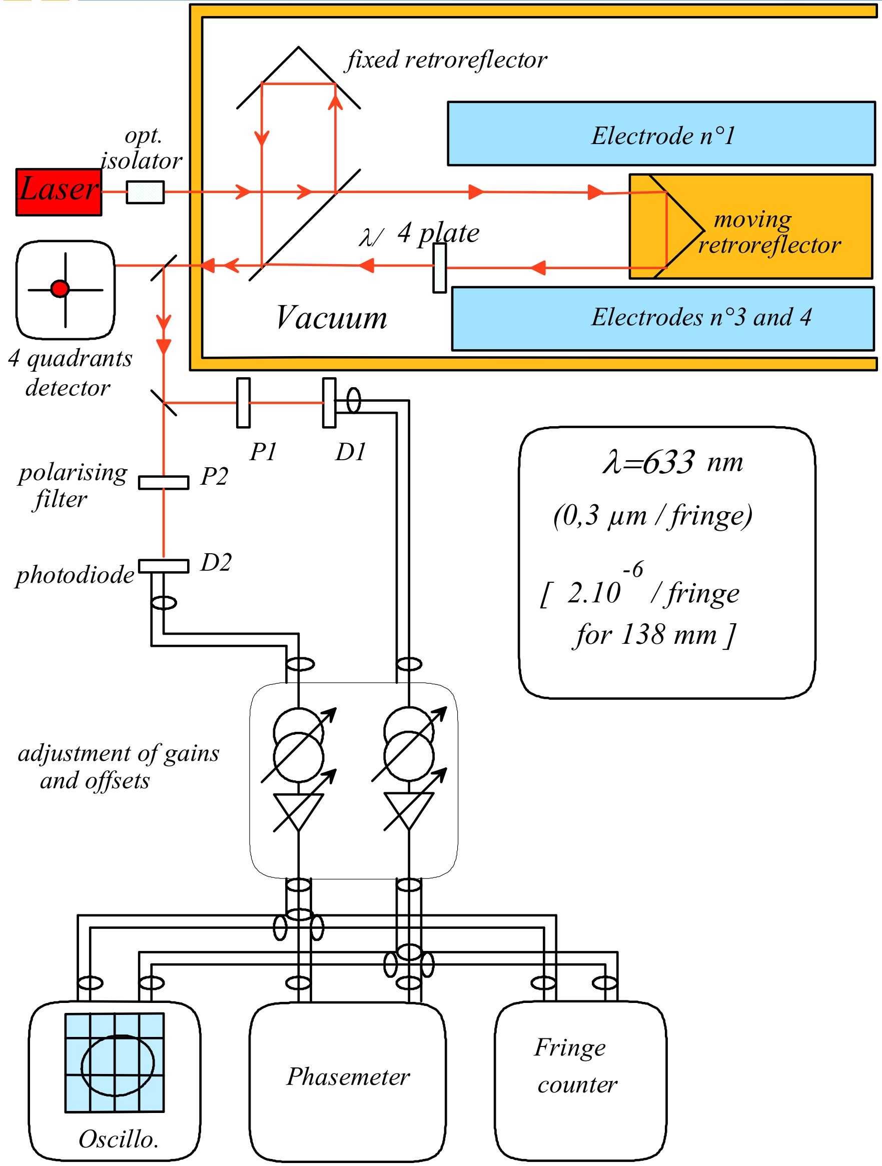
Cylindricity measurement (without conicity defect) with a 2,5 mm and 2,5 ° steps. The measurement was performed during 30 hours (4 forth and back runs)



Measurement of the same generating line during 12 hours with a 1 mm step (8 forth and back runs)

Measurement of one electrode of the present standard

Length measurement of the movable guard displacement



Michelson interferometer

Iodine stabilized He-Ne laser

Uncertainty components :

- Laser wavelength : **$3\ 10^{-11}$**
- Laser alignment : 20 µm over 230 mm : **$9\ 10^{-10}$**
- Air refractive index : pressure in the cross section of the standard < 0,05 Pa : **$1.3\ 10^{-10}$**
- Counting : type A

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