# Towards a Determination of $R_K$ in Terms of the New LNE Calculable Cross Capacitor

O. Thevenot,[a] L. Lahousse,[a] C. Consejo,[a] J. M. David,[b] S. Leleu,[b] F. Piquemal [a]

[a] Laboratoire National de Métrologie et d'Essais
29 Avenue Roger Hennequin, F-78197, Trappes, Francia.
olivier.thevenot@lne.fr

[b] Ecole Nationale Supérieure d'Arts et Métiers
8 Boulevard Louis XIV, F-59046, Lille, France.

**ABSTRACT**

In the scope of a possible redefinition in the SI, the LNE has decided to develop a new Thompson Lampard calculable capacitor to decrease its uncertainty on the value of von Klitzing constant, $R_K$, to a level of one part in $10^8$. This paper gives an overview of the French experiment. The main characteristics of the new French standard are detailed. The fabrication of a new set of electrodes and results of cylindricity measurement obtained with a dedicated apparatus, are presented. Others developments of the impedance comparison are also discussed.

## 1. INTRODUCTION

The determination of the von Klitzing constant $R_K$ with the best uncertainties is a task which might greatly contribute to the future evolution of the International system of units (SI) [1]. Within the frame of a *mise en pratique* of a new definition of the ohm through the quantum Hall effect (QHE) with the value of $h/e^2$ fixed, the fact that there is no correction to the von Klitzing relation $R_K = h/e^2$ has to be checked experimentally with an uncertainty of one part in $10^8$.

The determination of $R_K$ based on the implementation of a Thompson Lampard calculable capacitor [2] is the only way to directly test this relation by comparing to the values of $h/e^2$ obtained by the most accurate measurements, such as the measurement of anomalous magnetic moment $a_e$ using quantum electrodynamics (QED) calculation and interferometric measurement of Planck constant to atomic mass ratio ($h/m$). The best determinations of $R_K$ so far have uncertainties between 2 and 6 parts in $10^8$, while the atomic physics measurements lead values of $h/e^2$ with uncertainties one or two orders better [3-5].

The aim in reducing the uncertainty on $R_K$ value in SI units down to one part in $10^8$, is a strong motivation for some NMIs such as NMIA, LNE to pursue their efforts in improving calculable capacitors and for other laboratories, as BIPM, to start the development of such a standard.

The last determination of $R_K$ performed by the LNE in 2000 has an uncertainty of 5.3 parts in $10^8$. The uncertainty attributed to the calculable capacitor is about 4 parts in $10^8$ and that of the measurement chain linking the calculable capacitor to dc resistance is about 2.5 parts in $10^8$ [6]. The LNE has decided to improve its experimental set-up. To achieve this goal, a new cross calculable capacitor is under development, and the different elements of the measurement chain are improved.

A Thompson-Lampard calculable capacitor [2] generates a calculable capacitance variation proportional to the displacement of a movable guard in its cross section, allowing the linking of the farad to the meter. This result comes from a *mise en pratique* of the theorem discovered by A. Thompson and D. Lampard in 1956 and giving rise to a calculable capacitor. This theorem stipulates that for a cylindrical system composed of four isolated electrodes of infinite length and placed in vacuum, the direct capacitances per unit of length $\gamma_{13}$ and $\gamma_{24}$ of two pairs of electrodes verify the relation:

$$\exp(-\pi\,\gamma_{13}/\varepsilon_0) + \exp(-\pi\,\gamma_{24}/\varepsilon_0) = 1, \qquad (1)$$

where $\varepsilon_0$ is the permittivity of vacuum. Moreover in case of perfect symmetry with identical capacitances per unit of length, it results:

$$\gamma_{13} = \gamma_{24} = \gamma = (\varepsilon_0 \ln 2)/\pi = 1.953\ 549\ 043\ ...\ \text{pF/m}; \qquad (2)$$

and then a value of the electrical capacitance can be directly linked to a length measurement.

The LNE calculable capacitor, which is composed of five cylindrical bars, originally differs from the others (with four bars). If one connects, successively, two of the adjacent bars (as shown on figure 1), a five bars system is equivalent to five different four-bar systems, and the Lampard theorem can be applied to each of this five systems.

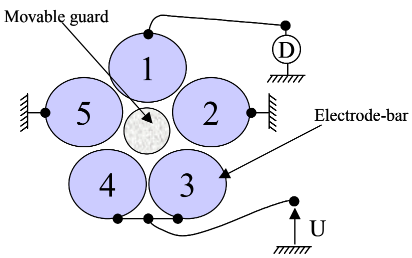



Fig. 1. A section of the LNE calculable capacitor.

The five cylindrical electrodes of the LNE calculable capacitor are 75.5 mm diameter, 450 mm long, in vertical position arranged at the vertices of a regular pentagon.

The metrological quality of a Thompson Lampard calculable capacitor resides, for a large a part in the control brought to the realization of the surfaces of electrodes and to their positioning. After briefly describing the measurement chain linking the farad to $R_K$, the article presents a progress report on the realisation of the new LNE calculable capacitor by detailing the fabrication of the electrodes and the mechanical design.

## 2. MEASUREMENT METHOD

The overall view of the successive measurements is shown on Fig. 2. The AC measurements are carried out at 3 angular frequencies $\omega$=2 500 rd/s, $\omega$=5 000 rd/s and $\omega$=10 000 rd/s [6].

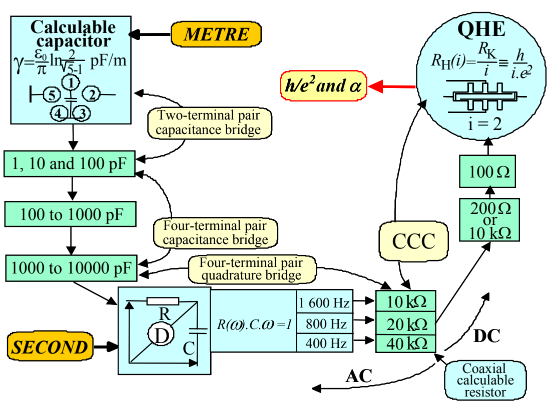



Fig. 2. Measurement method.

Firstly, a 1 pF capacitor ($C_{1pF}$) is compared to the Thompson-Lampard calculable capacitor with an 8:3 ratio bridge. Then two 10:1 ratio bridges are used successively to link two 10 000 pF capacitors (home made invar plates in vacuum capacitors) to the $C_{1pF}$ capacitor. The 1 000 pF transfer standard is nitrogen sealed capacitor placed in oil bath while the 100 pF and 10 pF transfer standards are thermoregulated fused silica capacitors (Andeen-Hagerling capacitors).

Next, a quadrature bridge allows one to compare with a very high accuracy the impedances of 10 000 pF capacitors ($C_1$ and $C_2$) to that of pair of resistors $R_1$ and $R_2$. Three couples of resistors are used with values of 40 kΩ, 20 kΩ and 10 kΩ, the bridge being balanced ($R_1R_2C_1C_2\omega^2$ = 1) at three angular frequencies $\omega$=2500 rd/s, $\omega$=5000 rd/s and $\omega$=10 000 rd/s, respectively. After correction of their frequency variations, by means of ac/dc calculable resistance standard, the resistances are compared to the quantum Hall resistance standard in DC. This involves the use of resistance bridge based on cryogenic current comparator (CCC).

The uncertainty of the whole chain measurement from 1 pF to $R_K$ (2.5 parts in $10^8$) will be reduced down to one part in $10^8$ or less by improving the capacitance ratio measurements (realization of new two-stage transformers) and the ac/dc resistance difference of the resistors used (fabrication of new Haddad type calculable resistor and improvement of the resistors involved in the quadrature bridge) [7].

## 3. FABRICATION OF A NEW ELECTRODES SET

The two major components in the uncertainty budget come from structural limitations of the cross capacitor standard. They consist in the geometrical defects of the gap between electrodes and the movable guard displacement control. In order to reach uncertainty of one part in $10^8$, the cylindricity defect of the gap must be lower than 100 nm. To reach this requirement, new electrodes with the same level of defects have to be fabricated.

### 3.1. Dedicated Measurement Device

Prior to the realization of the electrodes, a method of control enabling an evaluation of their geometry qualities has to be established. The significant parameters are the straightness and the parallelism of the generatrices of the cylinders.

Four capacitive sensors are used for measuring the surface of the cylinders. Their sensitive areas have a diameter of 5 mm. The use of capacitive sensors allows us to carry out measurement without contact, consequently avoiding any deterioration of the cylinder surface. The distance between the sensor and the cylinder was selected at about 300 µm to obtain the real distance most constant possible on the sensitive area without sacrificing too much the sensitivity of the measurement system. This choice leads to preserve a good linearity of measurement. The four sensors are installed on a rigid and directional ring. In this way, the machine will measure simultaneously four generatrices of the cylinder. By rotating the ring, another set of four generatrices can be measured. It is noteworthy that this search for another set of generatrices does not imply any displacement of the measured cylinder compared to the reference of the machine. The measurement of parallelism between two opposed generatrices has a level of quality corresponding to that of the sensors. Parallelism between two sets of four generatrices is evaluated with the same level of precision.

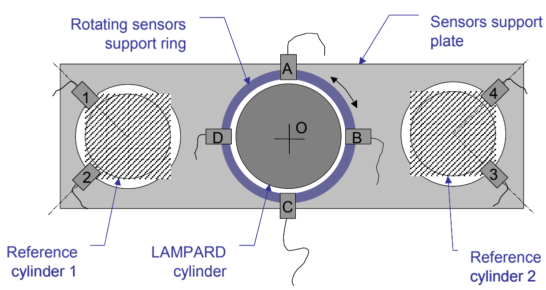



Fig. 3. Sketch of the sensors support plate.

The straightness measurement consists in comparing the trajectory of the sensors with the shape of the cylinder to be measured. In a traditional measuring machine, the location of position of the sensors in their movement is ensured by the guidance system. The location is often carried out by means of a chain of solids composed of mobile contacts. A set of crosshead guides materializes the form of the trajectory carried out by the mobile carriage. These crosshead guides present defects of form which can be measured and corrected if they are stable (Fig. 4a).

The link between surfaces materializing the cross head guides and the mobile carriage uses technologies corresponding to the establishment of contact between two solids with eventually a third body, *i.e.* element rolling or fluid flow. In spite of very elaborate techniques, it is impossible to obtain a quality of location equivalent to that given by transmitters. The use of transmitters leads results more suitable for calculating the mean value of the location from a very large number of measurements. The measurement uncertainty under the working conditions can approach the nanometer. This observation leads to locate a position by the means of transmitters rather than positioning effectively two solids, one compared to the other.

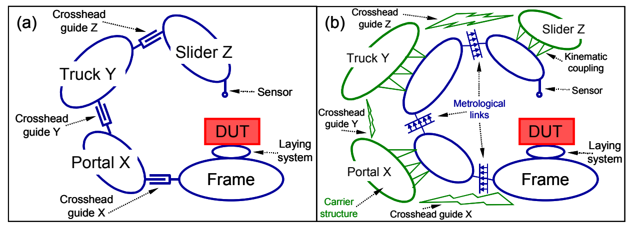



Fig. 4. (a) Metrological chain of a 3D measurement device of serial architecture: the final position depends on each element defects. Guiding defects and solids flexibility add entirely. (b) Application of the dissociated metrological structure to a 3D measurement device: the metrological chain is separated from the carrying and guiding chain- the link between them is made by a kinematic coupling.

We thus apply a concept called "dissociated metrological structure" in which the function "location" is provided by a measurement system, a parallel structure ensuring the role of "self-supporting" [8]. In the case of the realization of a machine of measurement, the application of this principle is simple, the geometrical level of quality requested from the parallel structure being simply limited to maintain the sensors in their effective range (Fig. 4b).

The sensors support plate is equipped with four additional sensors, which are used to locate it compared to two parallel cylindrical columns. These two columns and the four associated sensors constitute "the metrological crosshead guide" which is used as reference of straightness.

The measurement given by sensors 1, 2, 3 and 4 make it possible to locate the plate in its own plane. The definition of this position is redundant (4 measurements for 3 parameters). One can check the variation of the sum of these four measurements to control the stability of the reference. The metrological quality of the machine is thus followed in real-time. Moreover, this control can indicate any variation of thermal origin.

The average line of the centers of the sections of the two columns constitutes the straightness reference.

This is compared with the trajectory of the four sensors 5, 6, 7 and 8 that measure the straightness of the generatrices of the cylinder. This line is not perfectly right. In fact, the difference in form between this line and the generators of the cylinder is measured.

While swiveling the cylinder by a half-turn, the sign of the defects of the generatrices changes and the function of error of the machine is preserved. By simple difference between these measurements, the defect due to the straightness of the reference machine can be distinguished from the one coming from the generatrices of the cylinder. The measurement principle is illustrated on Fig. 5.


Fig. 5. Calibration of the machine principle.

It is noteworthy that this possibility of correction is valid only for the central point of the mobile plate. Because the axis of the sensors measuring the cylinders pass by this central point, it is possible to correct the measurement given by each sensor.

As shown on Fig. 6, the structure of the machine is in fact composed of two encased multi-stage structures. Each one of these structures consists of two plates connected by 8 oblique bars. The internal structure gathers the metrological elements; the external structure constitutes the “carrying” system. These two structures are connected one to the other by an isostatic connection.

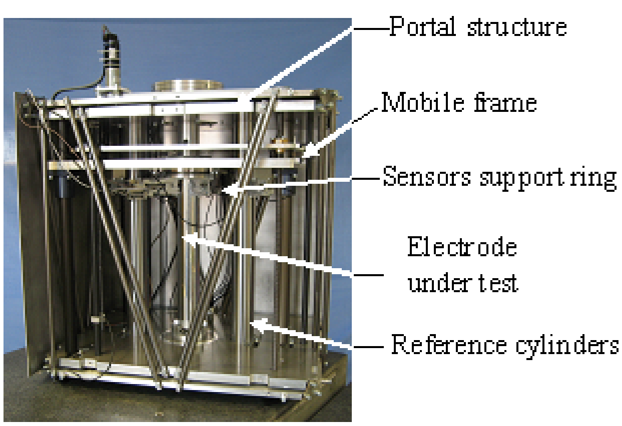



Fig. 6. Overview of the system.

The metrological structure carries the two reference columns, which are gripped in the horizontal plates. Three vertical columns are also pinched and are used to guide the sensors support plate. The guidance of this plate is organized in order to not impose any effort on the metrological structure.

The external structure comprises a guidance built on three columns on which goes up a carrying plate that supports the sensors support plate through an isostatic connection. The up and down movement is driven by a whole of three screws in parallel coupled by a synchronous belt.

A complete characterization of the machine has shown that it is possible to measure the cylindricity defects (straightness and diameter variation) on cylinder of 75.5 mm diameter and 450 mm length with an uncertainty (1$\sigma$) of 25 nm [9].

### 3.2 Fabrication of a new set of electrodes

The next set of electrodes is made of stainless steel grade 1.4539 which is an amagnetic material. After grinding, the cylinders have cylindricity defects in the order of 1.5 µm. Fig. 7 presents the results of the measurement carried out on one of these cylinders. The repeatability of this measurement is better than 10 nm.

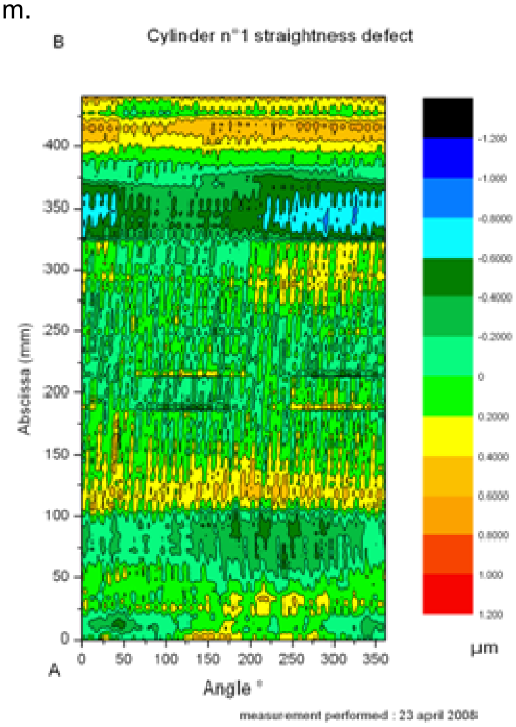
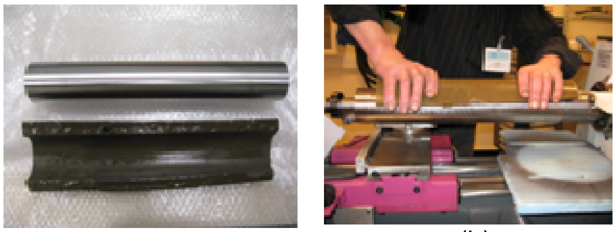



Fig. 7. Cylindricity measurement of the cylinder n°1 after grinding.

In order to reach a defect lower than 100 nm, a specific lapping-tool made with shellac and corundum has been built. The tool, shown on Fig. 8, is shaped to fit the bar.



(a) (b)

Fig. 8. (a) the cylinder and its grinding-tool (b) grinding-in operation.

After each operation the bar is measured with our dedicated device. This operation has begun in May 2008 and is planed to last six months.

## 4. NEW MECHANICAL DESIGN OF THE LNE CROSS CALCULABLE CAPACITOR

The new calculable capacitor will be composed of five electrodes, 75.5 mm diameter, 450 mm long, in vertical position. To guarantee that the cylindricity defect of the gap between electrodes is lower than 100 nm, a removable measurement device will be integrated to the standard (Fig. 9).

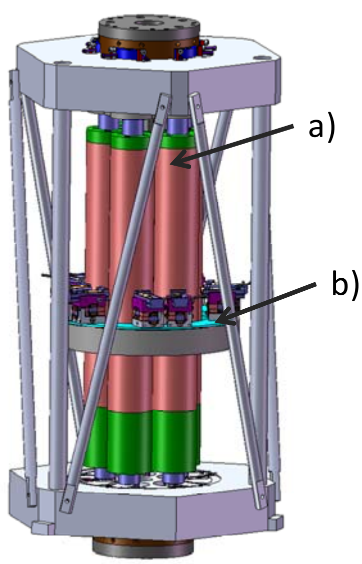



Fig. 9. Electrode and measurement device
a) electrode b) measurement device.

The measurement system is composed of a non-shrinking plate carrying ten capacitive sensors placed in front of the electrode surface (Fig. 10). Two sensors forming an angle of 90° measure each cylinder. A kinematic coupling supports this plate indexable in five positions. That allows each sensor to measure the five cylinders. In this way it is possible to check the assembly symmetry.

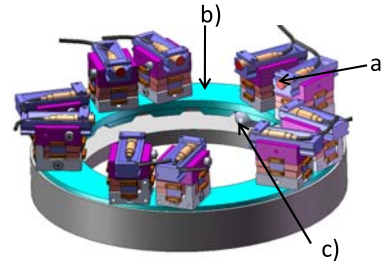



Fig. 10. Measurement device a) capacitive sensor b) non-shrinking plate c) kinematic coupling.

Thanks to a lifting system this plate can go up and down, that enables to measure straightness of the outside of the electrodes (Fig. 11).

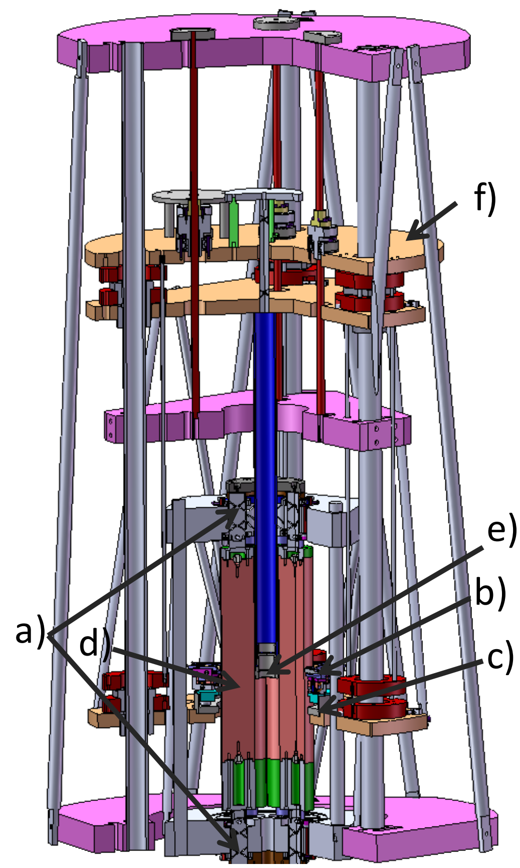



Fig. 11. Overview of the calculable capacitor;
a) bidimensional micrometric adjustement, b) capacitive sensor, c) non-shrinking plate, d) electrode, e) star shield, and f) lift system.

It is then possible to measure the cylindrical defect of the gap between electrodes by combining the knowledge on the electrodes geometry itself and information given by the integrated measurement system. A bidimensionnal micrometric system (Fig. 12) composed of flexure leaves located at each electrode ends, enables to adjust the gap position. A clamp designed to do not disturb the previous adjustment fixes the position of electrodes. This solution offers a better stability than the micrometric system.

The association of the removable measurement system and the bidimensionnal micrometric systems allows decreasing the uncertainty due to the cylindrical defect of the standard to a value close to 5 parts in $10^9$, instead of 2.4 parts in $10^8$ on the present standard.

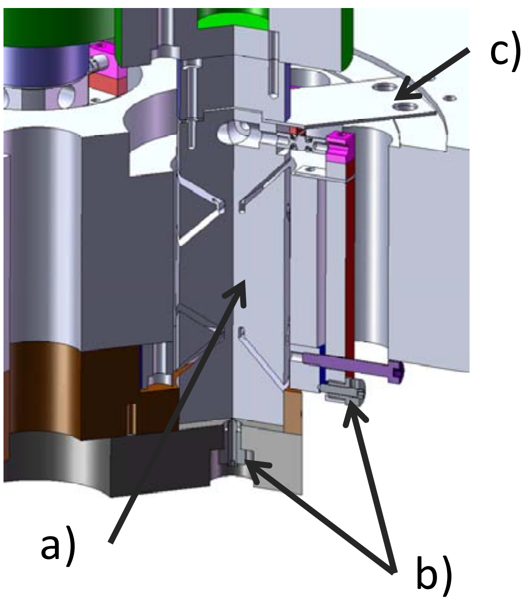



Fig. 12. Bidimensional micrometric system: a) Flexure leaves, b) micrometrics systems, and c) clamp.

The guidance of the movable guard will be realized in a reference system linked to the five electrodes. The cross section cylindricity quality will be used to guarantee the guard lateral position. A docking system composed of ten PTFE shoes is mounted on the star shield located at guard end. It fulfills a moderate controlled effort to optimize the positioning. At the upper part (see Fig. 11), a system based on a same principle with a fixed bearing will guarantee a proper translation. The lateral shifting of the movable guard is expected to be lower than 50 nm between the two measurement positions. The uncertainty due to this defect will be decreased to a level close to 5 parts in $10^9$, instead of 3 parts in $10^8$ on the present standard.

## 5. CONCLUSION

A novel apparatus for cylindricity measurements has been fabricated and characterized at LNE in the frame of development of a new Thompson Lampard calculable capacitor. Its performances make the measurement of the cylindricity defects on cylinder possible with an uncertainty ($1\sigma$) of 25 nm. This device is being used for the fabrication of new set of five electrodes to control the grinding-in operation in order to obtain cylindricity defects lower than 100 nm. The knowledge acquired during the development of the apparatus has allowed us to design the mechanical aspects of the new calculable capacitor (cylindricity defect of the gap between electrodes, guidance of the movable guard …). The calculable capacitor is designed to contribute to less than 1 part in $10^8$ in the uncertainty budget linked to the determination of $R_K$. In addition to this development, the on going improvement of the tricky parts of the impedance measurement chain from 1 pF capacitance to quantum Hall resistance standard should allow LNE to provide a new value of $R_K$ with an uncertainty of one part in $10^8$ in near future.

## REFERENCES


[1] BIPM 2004 *Proc.-Verb. Com. Int. Poids et Mesures,* **93** 219, http://www.bipm.org.

[2] A. M. Thompson, D. G. Lampard, *A new theorem in electrostatics and its application to calculable standards capacitance,* 1956, *Nature*, **177,** 888

[3] P.J. Mohr. and B.N. Taylor, *CODATA recommended values of the fundamental physical constants:2002,* 2005, *Rev. Mod. Phys.*, **77**, 1-107.

[4] G. Gabrielse *et al*, 2008, *New Measurement of the Electron Magnetic Moment and the Fine Structure Constant,* arXiv:0801.1134v1 [physics.atom-ph].

[5] P. Cladé *et al*, *A new determination of the fine structure constant based on Bloch oscillations of ultracold atoms in a vertical optical lattice,* 2006*, Phys. Rev.*, **A102**, 052109.

[6] G. Trapon *et al, Determination of the von Klitzing constant $R_K$ in terms of the BNM calculable capacitor-fifteen years of investigations,* 2003, *Metrologia,* **40**, 159-171

[7] C. Consédjo *et al*, *Toward a determination of* $R_K$ *at LNE with a new Thompson-Lampard*

*calculable capacitor, 2008,* in *Proceedings CPEM'08.*

[8] S. Leleu, *Vers une nouvelle référence angulaire nationale*, PhD Thesis, 2000, Ecole Nationale Supérieure d'Arts et Métiers, Metz, France. *In French*

[9] O. Thévenot *et al*, *A new apparatus for cylindricity measurement with uncertainty less than 25 nm*, 2007, in *Proceedings EUSPEN 2007*, **2**, 113-116.