

# VECTOR NETWORK ANALYSIS UNCERTAINTY EVALUATION FOR ONE-PORT MEASUREMENTS

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**Abstract:** Vector Network Analyzers (VNA) are frequently used in calibration laboratories or in production lines, for instance. Their versatility and expressive means of presenting the measurement results make them an ideal partner for numerous applications. But when it comes to qualify a measurement result the related measurement uncertainties are of great importance, if not mandatory. Finally traceability of the measurement needs to be achieved. However, determining the related measurement uncertainties can get out of hands quickly: the math behind the vector error correction is quite demanding and the number of input quantities high, sometimes overwhelming. Understanding the significant influences helps avoiding errors, to improve the measurement procedures and to keep the math "down-to-earth".

## 1. INTRODUCTION

Unlike most other T&M instruments VNAs have to be "calibrated"<sup>1</sup> [1] using a Calibration Kit before being used. Aim of this "calibration" is to correct for the systematic errors (e.g. loss, mismatch, electrical length) existing not only within the VNA itself, but also in the external measurement setup. This way the vector error correction captures also errors introduced by test cables, adapters, matching pads etc..

When performing measurements with a VNA it's important to understand it's capabilities – as well as it's limitations. Residual errors occur – even after the VNA has been "calibrated". These residual errors strongly depend on the calibration standards being used – and the way they have been used.

Aim of this paper is to provide guidance on how to estimate realistic uncertainties related to reflection measurements using a Vector Network Analyzer. The most significant error sources - and their impact on the measurand - will be discussed to help improving the procedures and to maintain high quality measurements.

The validity of the uncertainties has been verified using standards calibrated by the PTB (NMI of Germany).

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<sup>1</sup> The term "calibration" used in this context is not in line with the "VIM" (see References). However, this terminology has been chosen as it's commonly used in the VNA literature.

## 2. UNCERTAINTY EVALUATION FOR ONE-PORT MEASUREMENTS

### 2.1 RESIDUAL ERRORS AT A GLANCE

Measurement uncertainties of VNAs are a function of many parameters. Some of the most significant contributors are imperfections of the calibration standards used – but their effect on the measurand is not self-evident in most cases.

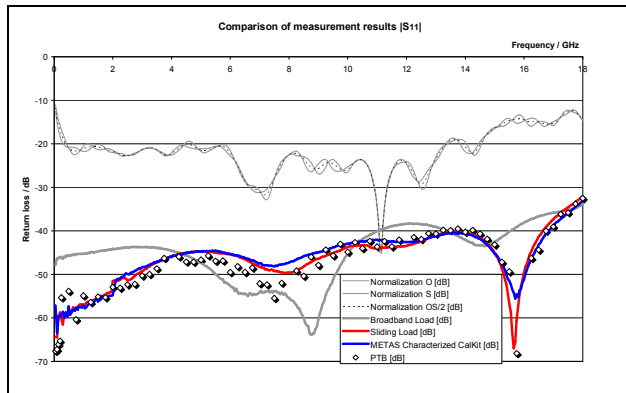
Examples are deviations of the high reflection standards OPEN and SHORT from their phase and attenuation model given in the standard definitions provided with the calibration kit: it's hard to predict the consequences of such errors on the measurand without exact knowledge of the error-correction algorithms used in the instrument's firmware.

In order to demonstrate the order in magnitude of "residual" errors of one-port measurements a coaxial precision termination (N50Ω type) has been measured 6 times. The results are presented graphically together with reference values provided by the PTB:

- Reflection-normalization only w/ OPEN standard (common practice with VSWR bridges).
- Reflection-normalization only w/ SHORT standard (common practice with VSWR bridges).
- Reflection-normalization only, but mean of OPEN and SHORT reading standard has been used (improved procedure, common practice with VSWR bridges).

- Full-one port OSM calibration, using a broadband load.
- Full-one port OSM calibration, using a low-band and a sliding load.
- Full-one port OSM calibration, using a broadband load.

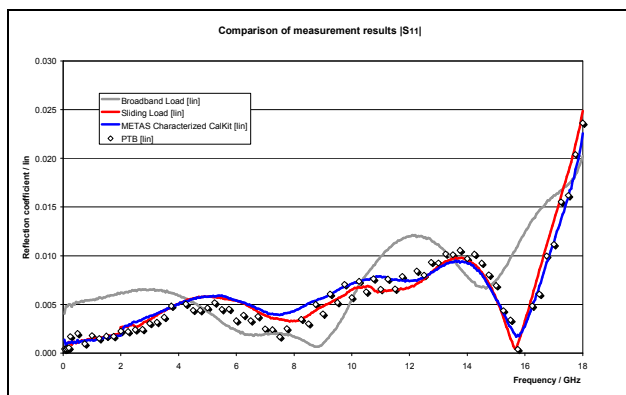
Calibration standards used have been calibrated by METAS (NMI of Switzerland). Calibration data have been used to characterize the standards and to improve the quality of the error correction.



**Fig. 1** – Comparison of measurement results. Different Calibration Kits and procedures used. *y* – axis logarithmic / dB

Goal of this figure is to demonstrate clearly:

- The differences between the measurements are SIGNIFICANT, not to say huge.
- The upper 3 traces show limitations due to the (raw, uncorrected) HW : without applying a vector error correction the raw directivity limits the dynamic range of the VNA.

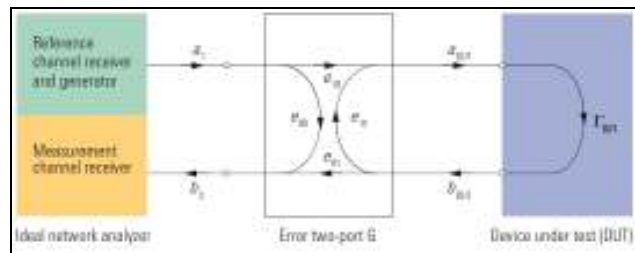


**Fig. 2** – Same results as in Fig. 1. Comparison of measurement results. Different Calibration Kits and Procedures used. *y* – axis linear

- In metrology “Return loss / dB” is not the “unit” recommended for comparisons and uncertainty calculations. The linear reflection coefficient is more suitable here.

### 2.2 BLACK-BOX APPROACH, ONE-PORT ERROR MODEL

The Vector Network Analyzer is considered as “ideal” whereas the systematic measurement errors are modeled using an error two-port. Using this approach we obtain a one-port device that can be termed an ideal network analyzer [2].



**Fig. 3** - One-Port Error Model

The following terms are introduced

- $e_{00}$  = “Raw” directivity
- $e_{01} e_{10}$  = “Raw” reflection tracking
- $e_{11}$  = “Raw” test port match
- $M_x$  = measured values

Using these terms, our “ideal” VNA measures:

$$\Gamma_E = \frac{b_3}{a_1} = e_{00} + \frac{e_{01}e_{10}\Gamma_{DUT}}{1 - e_{11}\Gamma_{DUT}} \quad (1)$$

or

$$M_{DUT} = \frac{b_3}{a_1} = e_{00} + \frac{e_{10}\Gamma_{DUT}}{1 - e_{11}\Gamma_{DUT}} \quad (2)$$

Note : “ $e_{01}$ ” is set to “1” – and captured in “ $e_{10}$ ” as well

### 2.3 SOLVING FOR $e_{xx}$

In order to solve for the 3 unknowns  $e_{00}$ ,  $e_{10}$  and  $e_{11}$  3 independent equations are needed, i.e.  $\Gamma_{DUT}$  of 3 standards must be known.

In most instances this requirement is solved by means of 3 “known”, “predictable” calibration standards: OPEN, SHORT and MATCH. The calibration procedure is also known as OSM calibration.

with

$\Gamma_O$  = complex valued reflection coefficient of OPEN standard.

A complex number consists of a magnitude and phase information.

Traditional standard definitions describe the phase response by means of a polynomial -the, so called, “fringing capacitances” plus the offset (electrical length) of the OPEN plane to the reference plane of the connector system.

The magnitude is modeled using the “definition” of the OPEN (reflection coefficient = +1) and the loss associated with an electrical length  $> 0$ .



Fig. 4 – Standard definition of OPEN calibration standard in VNA firmware.

$\Gamma_S$  = complex valued reflection coefficient of SHORT standard.  
Modelling : analogue to OPEN.

$\Gamma_M$  = complex valued reflection coefficient of MATCH standard.  
Modelling : reflection coefficient = 0.

Introducing  $\Gamma_O$ ,  $\Gamma_S$  and  $\Gamma_M$  in (2):

$$M_O = e_{00} + \frac{e_{10}\Gamma_O}{1 - e_{11}\Gamma_O} \quad (3)$$

$$M_S = e_{00} + \frac{e_{10}\Gamma_S}{1 - e_{11}\Gamma_S} \quad (4)$$

$$M_M = e_{00} + \frac{e_{10}\Gamma_M}{1 - e_{11}\Gamma_M} \quad (5)$$

with  $\Gamma_M = 0$  (ideal Match Standard), this equation becomes :

$$M_M = e_{00} + \frac{e_{10} \cdot 0}{1 - e_{11} \cdot 0} = e_{00} + \frac{0}{1} = e_{00} \quad (6)$$

Note : further improvement in accuracy can be achieved if  $\Gamma_M$  is not assumed to be equal = 0, but using the “true” complex valued data of  $M_M$  (e.g. calibration data provided by a NMI) instead.

After solving the 3 equations for  $e_{00}$ ,  $e_{10}$  and  $e_{11}$  we obtain the following results:

$$e_{00} = M_M \quad (7)$$

$$e_{10} = \frac{(\Gamma_O - \Gamma_S)(M_O - M_M)(M_S - M_M)}{\Gamma_O\Gamma_S(M_O - M_S)} \quad (8)$$

$$e_{11} = \frac{\Gamma_S(M_O - M_M) - \Gamma_O(M_S - M_M)}{\Gamma_O\Gamma_S(M_O - M_S)} \quad (9)$$

Solving equation (2) for  $\Gamma_{DUT}$  yields:

$$\Gamma_{DUT} = \frac{M_{DUT} - e_{00}}{e_{10} + e_{11}(M_{DUT} - e_{00})} \quad (10)$$

This equation is NOT an approximation i.e. if  $e_{00}$ ,  $e_{10}$  and  $e_{11}$  are perfectly known we can correct for the systematic errors applicable for one-port measurements.

Example: VNA internal error-box parameters after Full-One-Port “OSM” calibration

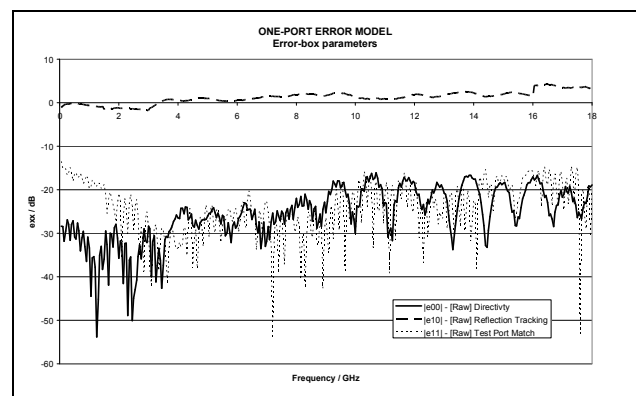


Fig. 5 –  $e_{00}$ ,  $e_{10}$  and  $e_{11}$ , after OSM calibration

## 2.4 PREVAILING CONDITIONS

Fractional equations, such as the ones for  $e_{10}$  (8) and  $e_{11}$  (9) can cause singularities, if the denominator is (or comes close to) zero. For

instance  $M_O = M_S$  would cause such unpredictable results.

$$e_{11} = \frac{\Gamma_S(M_O - M_M) - \Gamma_O(M_S - M_M)}{\Gamma_O \Gamma_S (M_O - M_S)}$$

For mechanical reasons most calibration standards show an electrical length  $> 0$ . This line section causes phase rotation with increasing frequency. As a first approximation (lossless) the reflection coefficient of a SHORT standard can be described as follows:

$$S_{11} \approx -1 \cdot e^{-j4\pi4\pi} \tag{11}$$

If our standard shows an electrical length of 15 mm, the phase shift @ 5 GHz is approx.  $180^\circ$  ! I.e. the "SHORT" turns into an "OPEN".

$$\lambda_{f=5\text{GHz}} = \frac{c}{f \cdot \sqrt{\epsilon_r}} \approx \frac{c}{f} \approx \frac{0.3 \cdot 10^9 \text{ m}}{5 \cdot 10^9 \frac{1}{\text{s}}} = 0.06\text{m} \tag{12}$$

$$S_{11, \text{SHORT}, 5\text{GHz}} \approx -1 \cdot e^{-j \frac{4\pi \cdot 0.015\text{m}}{0.06\text{m}}} = -1 \cdot e^{-j\pi} \tag{13}$$

$$\phi[S_{11, \text{SHORT}, 5\text{GHz}}] \approx 180^\circ \tag{14}$$

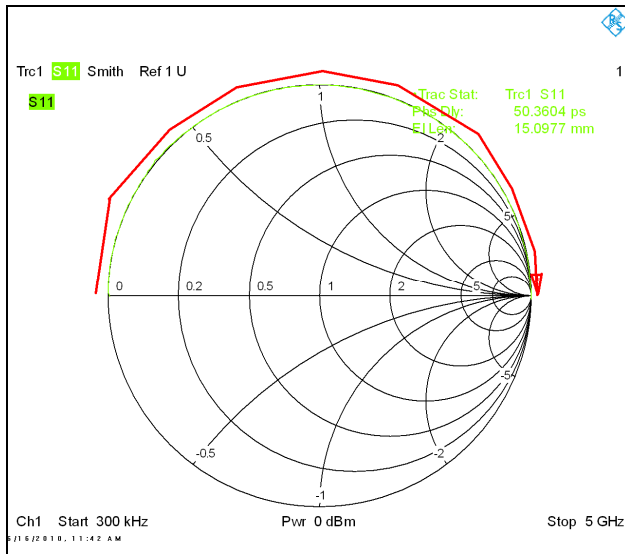


Fig. 6 – "SHORT" turns into "OPEN"

One of the design goals for OPEN & SHORT standards is to ensure approx.  $180^\circ$  phase difference between the standards over the complete frequency band the standards are specified to be used:

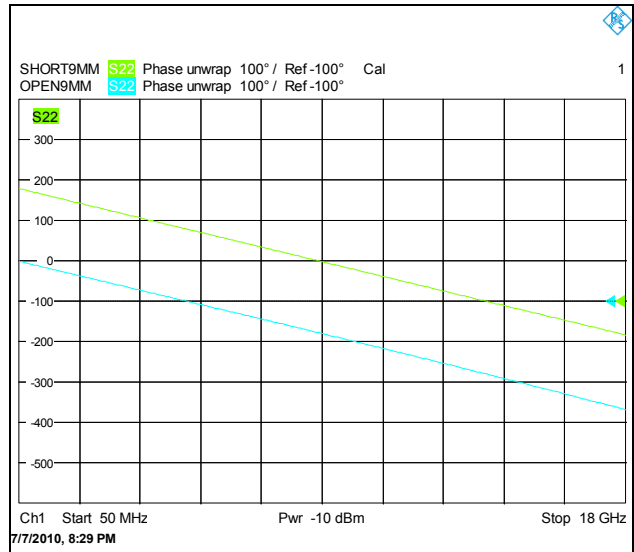


Fig. 7 – "Phase slope" of OPEN and SHORT

If the OPEN and SHORT calibration standards used are having enough difference in electrical length – the "SHORT" will take over the lead @ some frequency. If  $\Gamma_O = \Gamma_S$  the 3 equations are no longer independent, and the math behind the calibration get's corrupt:

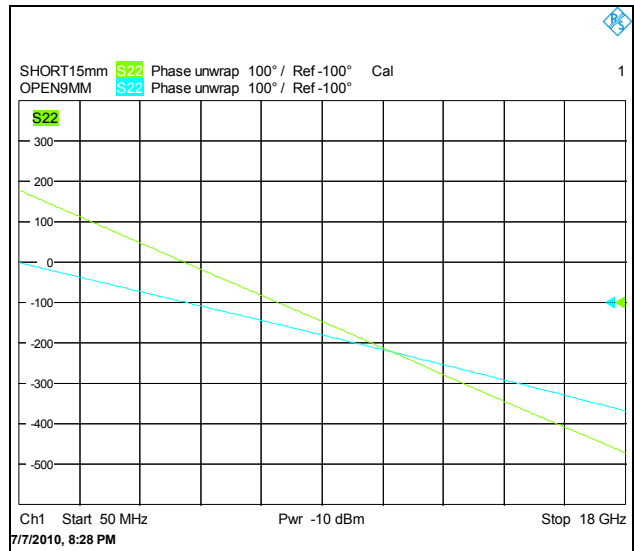
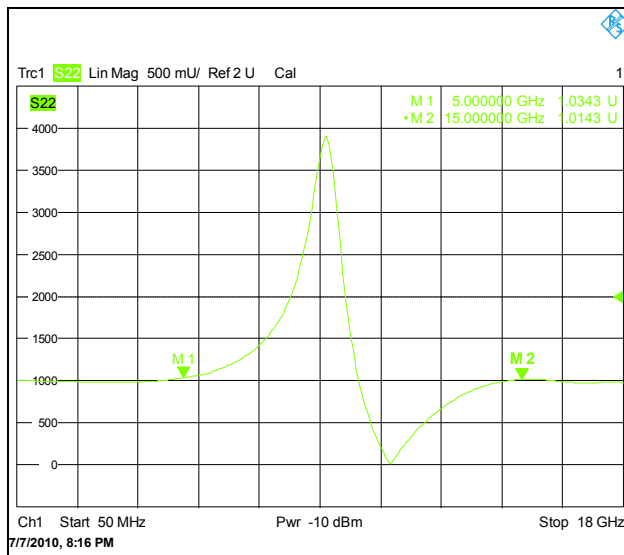


Fig. 8 – Phase response of OPEN and SHORT crossing due to unlucky combination of electrical lengths

This example is certainly an extreme one – but shows the limitations of the math applied within the VNA. However – deviations between the mathematical model, describing the calibration standard's response, and the "true" response of the calibration

standards) will lead to residual errors – contributing to the uncertainty of our measurement.



**Fig. 9 – Calibrated VNA shows huge resonance – due to division by zero !**

How will errors of the calibration standards contribute to the error terms  $e_{00}$ ,  $e_{10}$  and  $e_{11}$ ?

**2.5 EFFECTIVE SYSTEM DATA**

The user is usually not interested in the “system errors”  $e_{00}$ ,  $e_{10}$  and  $e_{11}$  itself, but in their (residual) errors! The corrected system performance after vector error correction is usually called “effective system performance”. The system errors are named analogue: effective system data.

Effective Directivity	$e_{00,eff}$
Effective Reflection Tracking	$e_{10,eff}$
Effective Source Match	$e_{11,eff}$

The same black-box model applies (Fig. 3) for the effective system data. The errors are just replaced by their “effective” counterpart.

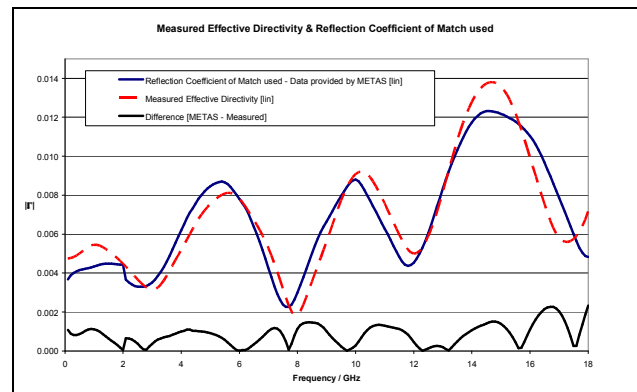
Effective directivity and - source match can be measured by means of an air line. The air line impedance is defined by it’s mechanical characteristics and provides traceability to the SI via dimensional calibration!

**2.5.1 EFFECTIVE DIRECTIVITY**

When solving for  $e_{00}$  the model of the MATCH (calibration) standard used is usually “0” reflection.

Deviations from the model will lead to residual errors in the computation of the error-box parameters which is introducing systematic errors later when performing the measurements. In case of  $e_{00}$  the residual errors correspond 1:1 with the deviations from the model (“0” in this case). The following graph shows

- The reflection coefficient of the MATCH used for the OSM calibration
- The measured effective directivity (having used this MATCH standard).



**Fig. 10 – Eff. Directivity reached with broadband match – compared with reflection coefficient of broadband load.**

**2.5.2 EFFECTIVE SOURCE MATCH**

In case of the effective source match things are getting a bit more complicated: the dominating “ingredients” are the eff. directivity and the phase errors of the reflection standards.

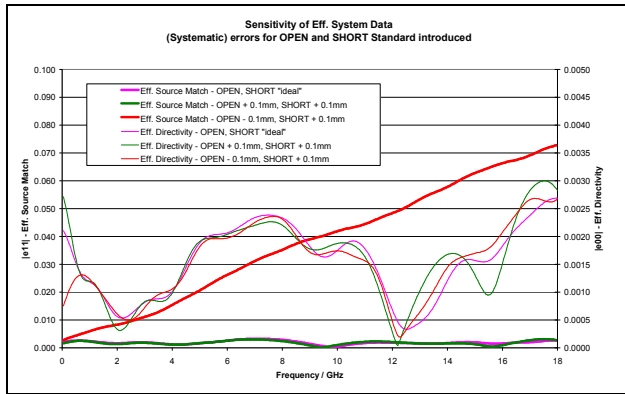
Rough estimate:

$$|e_{11,eff}| \approx |e_{00,eff}| + \left| \frac{\Delta\varphi_S - \Delta\varphi_O}{2} \right| \tag{15}$$

As the phase errors of the standards is not known in most cases (only the boundary limits – the specification), and as  $\sin(x) \approx x$  for small arguments  $|e_{11,eff}|$  can be approximated as follows:

$$|e_{11,eff}| \approx |e_{00,eff}| + \left| \frac{\Delta\varphi_{S,deg}}{2} \right| \cdot \frac{\pi}{180} + \left| \frac{\Delta\varphi_{O,deg}}{2} \right| \cdot \frac{\pi}{180} \tag{16}$$

Example: 0.1mm error introduced to the electrical length of both, OPEN and SHORT.



**Fig. 11 – Sensitivity of eff. Source Match and eff. Directivity on phase errors introduced in OPEN, SHORT standards.**

**Case 1 (pink):** OPEN and SHORT are “ideal”

**Case 2 (green):** both, OPEN and SHORT have an offset (0.1mm) in el. Length – but the same sign.

The effective directivity in all 3 cases remains unchanged (besides repeatability of this measurement).

The effective source match in case 1 &2 remains unchanged (but the Smith-Chart has been turned clockwise in Case 2, which seems having “no” effect!). As the error phasors are pointing in the same direction they will be canceled out!

**Case 3 (red):** both, OPEN and SHORT have an offset (0.1mm) in el. Length – but different signs.

Case 3 shows very poor eff. source match – the error is caused by the error phasors pointing in opposite directions!

### 3 UNCERTAINTIES

This chapter is loosely based on the “Guidelines on the Evaluation of Vector Network Analysers (VNA)” [4], resp. “Fundamentals of Vector Network Analysis” [2]. The equations have been adopted (in order to obtain conclusive terms in this document), but some changes in methods and probability distributions have been made.

When calculating measurement uncertainties the sensitivity coefficients for the individual contributors are required. In the case of one-port measurements the outcome of this exercise is very interesting and important (more details on that topic in [2]):

$$\Gamma_{DUT} = \frac{\Gamma_{DUT,M} - e_{00,eff}}{e_{10,eff} + e_{11,eff}(\Gamma_{DUT,M} - e_{00,eff})} \quad (17)$$

$$\frac{\partial \Gamma_{DUT}}{\partial e_{00,eff}} = -1 \quad (18)$$

$$\frac{\partial \Gamma_{DUT}}{\partial e_{10,eff}} = -\Gamma_{DUT,M} \quad (19)$$

$$\frac{\partial \Gamma_{DUT}}{\partial e_{11,eff}} = -\Gamma_{DUT,M}^2 \quad (20)$$

### 3.1 MODEL EQUATION (simplified [2])

$$U|\Gamma_{DUT}| = |e_{00,eff}| + |e_{10,eff}||\Gamma_{DUT,M}| + |e_{11,eff}||\Gamma_{DUT,M}|^2 + |A||\Gamma_{DUT,M}| + R_{VRC} \quad (21)$$

with

|A| Linearity in measurement range  
 R<sub>VRC</sub> Represents all random contributors.

#### 1<sup>st</sup> interim result:

obviously, U|Γ<sub>DUT</sub>| is not a fixed figure – but a function of the reflection coefficient Γ<sub>DUT,M</sub>.

#### 2<sup>nd</sup> interim result:

the effective directivity (e<sub>00,eff</sub>) contributes to the uncertainty U|Γ<sub>DUT</sub>| with a “fixed” amount.

#### 3<sup>rd</sup> interim result:

for small reflection coefficients (|Γ<sub>DUT</sub>| < 0.3) the effective directivity (e<sub>00,eff</sub>) is usually the dominating contributor to U|Γ<sub>DUT</sub>|.

### 3.2 ONE-PORT REFLECTION MEASUREMENT CALIBRATION KIT w/ BROADBAND LOAD

#### 3.2.1 MEASURED EFFECTIVE SYSTEM DATA

For better resolution and comparability, the residual errors – or “effective system performance” – after OSM calibration have been determined using the procedure according to [3]. Of course the procedure proposed in [4] can be used as well.



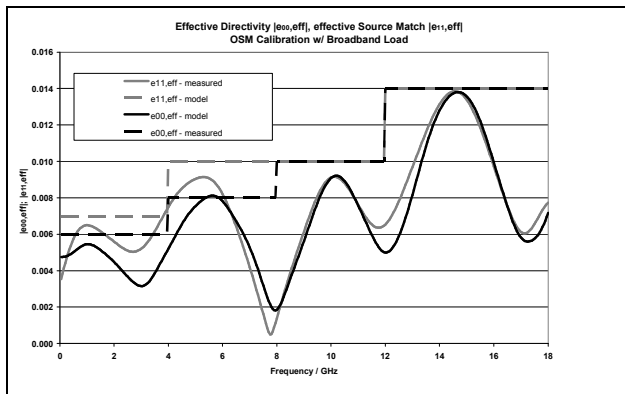


Fig. 12 – Eff. Directivity and eff. Source Match of OSM calibrated VNA, using a broadband load.

**Modeled eff. System Data**

For practical reasons it's recommended to divide the frequency range into some sub-ranges. 4 .. 6 sub-ranges are appropriate for most applications.

**3.2.2 UNCERTAINTY BUDGET > see ANNEX A**

**3.2.3 VALIDATION & TRACEABILITY**

Especially when performing this kind of uncertainty evaluation the first some times a validation of the results is strongly recommended.

Are the figures realistic or under- respectively over-estimated?

A validation of the uncertainties (proficiency testing using e.g. the  $E_N$  criteria) can be performed as comparison with other laboratories (preferably accredited labs or NMIs).

$$E_N = \frac{|X_{LAB} - X_{LAB}|}{\sqrt{U_{LAB}^2 - U_{REF}^2}} \quad (22), [5]$$

Recommended verification standards are match / mismatch standards covering the range of possible reflection coefficients (0 ..1). By means of such mismatch standards traceability of the measurement can be demonstrated.

However, when performing this kind of proficiency testing a boundary condition exists:

$$U_{REF} \leq U_{LAB} \quad (23)$$

**3.2.4 5 STEPS**

1. OSM calibration of VNA
2. Measurement of effective system data
3. Measurement of DUT
4. Uncertainty budget created
5. Measured data (incl. computed uncertainties) compared with reference data (incl. uncertainties in the calibration certificate)

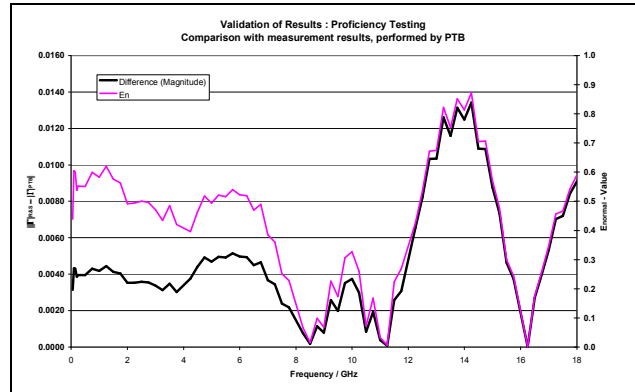


Fig. 13 – Validation of uncertainty budget

**3.2.5 UNCERTAINTY UNITS**

For reflection measurements it's strongly recommended to base all uncertainty calculations on "linear reflection coefficients". The final results can be converted to the target unit.

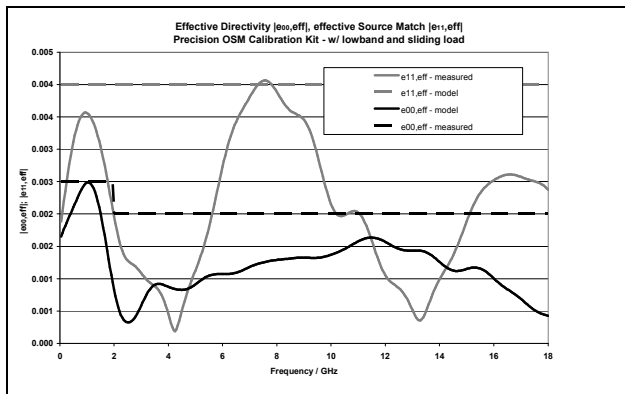
$$|S_{11}|_{linear} = 0.091 \pm 0.015 \rightarrow \frac{0.106_{max,linear}}{0.076_{min,linear}} \quad (24)$$

$$|S_{11}|_{dB} = -20.8dB \rightarrow \frac{0.106_{max,linear}}{0.076_{min,linear}} \rightarrow \frac{-19.5dB_{min}}{-22.4dB_{max}} \quad (25)$$

$$|S_{11}|_{SWR} = 1.20 \rightarrow \frac{0.106_{max,linear}}{0.076_{min,linear}} \rightarrow \frac{1.24_{SWR,max}}{1.16_{SWR,min}} \quad (26)$$

**3.3 ONE-PORT REFLECTION MEASUREMENT CALIBRATION KIT w/ LOWBAND and SLIDING LOAD**

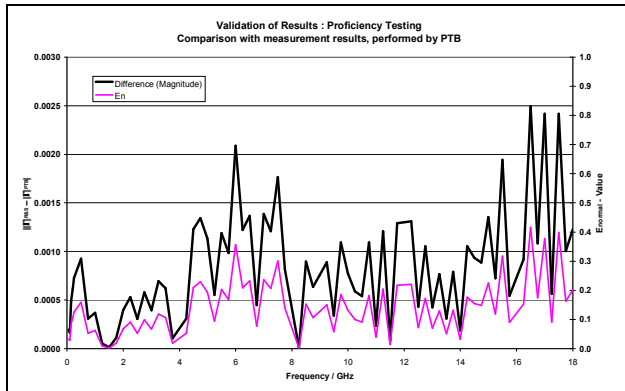
**3.3.1 MEASURED EFFECTIVE SYSTEM DATA**



**Fig. 14 – Eff. Directivity and eff. Source Match of OSM calibrated VNA, using a sliding load. Significant improvement of both residual errors : e.g.  $e_{00,eff}$  up to 18 GHz has been improved from  $0.014 > 0.002!$**

**3.3.2 UNCERTAINTY BUDGET > see ANNEX B**

**3.3.3 VALIDATION & TRACEABILITY**



**Fig. 15 – As the  $En$  value does not reach values  $> 0.4$  one of the uncertainties (reference or calculated) appears to be overstated.**

**3.3.4 UNCERTAINTIES – CASE 2**

$$|S_{11}|_{\text{linear}} = 0.025 \pm 0.005 \rightarrow \frac{0.030_{\text{max,linear}}}{0.020_{\text{min,linear}}} \quad (27)$$

$$|S_{11}|_{\text{dB}} = -32.0\text{dB} \rightarrow \frac{0.030_{\text{max,linear}}}{0.020_{\text{min,linear}}} \rightarrow \frac{-30.5\text{dB}_{\text{min}}}{-34.0\text{dB}_{\text{max}}} \quad (28)$$

$$|S_{11}|_{\text{SWR}} = 1.05 \rightarrow \frac{0.030_{\text{max,linear}}}{0.020_{\text{min,linear}}} \rightarrow \frac{1.062_{\text{SWR,max}}}{1.041_{\text{SWR,min}}} \quad (29)$$

**4 CONCLUSIONS**

The most significant contributors of a one-port VNA measurement have been identified and quantified. The measurement uncertainty has been derived and calculated. The calculated uncertainty figures validated by comparison with “known” standards calibrated from the PTB. By means of this comparison traceability has been achieved as well.

Performing the exercise of evaluating the measurement uncertainties not only resolves the mandatory part of a calibration – but also helps to understand the critical parameters and to keep control over the measurement process.

A set of unspectacular components – the calibration standards – turns out being responsible for the most significant uncertainty contributors in vector network analysis. Errors while entering e.g. correction values – or if the wrong connector gender is selected might lead to high uncertainties.

However, good measurement practice: clean, repeatable calibration standards (as well as the DUT itself), good thermal stability and mechanical precision of all components used is the foundation of excellent measurement results.

**ACKNOWLEDGEMENTS**

I like to take this opportunity to thank Thomas Reichel and Dr. Gerhard Rösel for the fruitful discussions and help during the last years resp. while developing the content of this paper.

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**ANNEX A - UNCERTAINTY BUDGET**

**ONE-PORT MEASUREMENT. CALIBRATION w/ BROADBAND LOAD,  
DUT : MISMATCH STANDARD, VSWR nominal = 1.2 (0.09091 linear).  
Budget covers the frequency range of 12 .. 18 GHz.**

Remark : a high numerical resolution has been chosen in the budget – to simplify reconstructing the calculations.

Contribution Input Quantity $X_i$	Parameter uncertainty limits $a_i$	Propability distribution	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i$	Contribution to the standard uncertainty $u_i(y)$
Eff. Directivity, measured, model	0.01400	Normal (2 $\sigma$ )	0.00700	1	0.00700
Airline – Z <sub>0</sub> reflection coefficient	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Airline – air gap reflection coefficient [6]	0.00200	Rectangular ( $\sqrt{3}\sigma$ )	0.00115	1	0.00115
<b>Eff. Directivity, incl. uncertainties</b>	-	<b>StdDev (<math>\sigma</math>)</b>	-		<b>0.00716</b>
-	-	-	-	-	-
Eff. Source Match, measured, model	0.01400	Normal (2 $\sigma$ )	0.00700	1	0.00700
Airline – Z <sub>0</sub> reflection coefficient	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Airline – air gap reflection coefficient [6]	0.00200	Rectangular ( $\sqrt{3}\sigma$ )	0.00115	1	0.00115
<b>Eff. Source Match, incl. uncertainties</b>	-	<b>StdDev (<math>\sigma</math>)</b>	-		<b>0.00716</b>
-	-	-	-	-	-
Eff. Directivity, incl. uncertainties	0.00716	StdDev ( $\sigma$ )	0.00716	1	0.00716
Eff. Transmission Tracking	0.00577 (0.05 dB)	Rectangular ( $\sqrt{3}\sigma$ )	0.00331	0.09091  S <sub>11,M</sub>	0.00030
Eff. Source Match, incl. uncertainties	0.00716	StdDev ( $\sigma$ )	0.00716	0.09091 <sup>2</sup>  S <sub>11,M</sub>   <sup>2</sup>	0.00006
Linearity	0.00577 (0.05 dB)	Rectangular ( $\sqrt{3}\sigma$ )	0.00331	0.09091  S <sub>11,M</sub>	0.00030
Ambient Conditions (Drift)	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
System repeatability (Noise)	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Connector repeatability [7],[8]	0.0010 (60 dB)	Normal (2 $\sigma$ )	0.00050	1	0.00050
<b>Combined Standard Uncertainty</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>0.00733</b>
<b>Expanded Uncertainty (k=2)</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>0.01466 0.015 (used)</b>

**ANNEX B - UNCERTAINTY BUDGET**

**ONE-PORT MEASUREMENT. CALIBRATION w/ LOWBAND & SLIDING LOAD,**

**DUT : MATCH STANDARD,  $|S_{11}|$  @ 18 GHz : 0.025 linear.**

**Budget covers the frequency range of 2 .. 18 GHz.**

Remark : a high numerical resolution has been chosen in the budget – to simplify reconstructing the calculations.

Contribution Input Quantity $X_i$	Parameter uncertainty limits $a_i$	Propability distribution	Standard uncertainty $u(x_i)$	Sensitivity coefficient $c_i$	Contribution to the standard uncertainty $u_i(y)$
Eff. Directivity, measured, model	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Airline – Z <sub>0</sub> reflection coefficient	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Airline – air gap reflection coefficient [6]	0.00200	Rectangular ( $\sqrt{3}\sigma$ )	0.00115	1	0.00115
<b>Eff. Directivity, incl. uncertainties</b>	-	<b>StdDev (<math>\sigma</math>)</b>	-		<b>0.00182</b>
-	-	-	-	-	-
Eff. Source Match, measured, model	0.00400	Normal (2 $\sigma$ )	0.00200	1	0.00200
Airline – Z <sub>0</sub> reflection coefficient	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Airline – air gap reflection coefficient [6]	0.00200	Rectangular ( $\sqrt{3}\sigma$ )	0.00115	1	0.00115
<b>Eff. Source Match, incl. uncertainties</b>	-	<b>StdDev (<math>\sigma</math>)</b>	-		<b>0.00251</b>
-	-	-	-	-	-
Eff. Directivity, incl. uncertainties	0.00182	StdDev ( $\sigma$ )	0.00182	1	0.00182
Eff. Transmission Tracking	0.00346 (0.03 dB)	Rectangular ( $\sqrt{3}\sigma$ )	0.00200	0.025 $ S_{11,M} $	0.00005
Eff. Source Match, incl. uncertainties	0.00251	StdDev ( $\sigma$ )	0.00251	0.025 <sup>2</sup> $ S_{11,M} ^2$	0.00000
Linearity	0.00577 (0.05 dB)	Rectangular ( $\sqrt{3}\sigma$ )	0.00331	0.025 $ S_{11,M} $	0.00014
Ambient Conditions (Drift)	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
System repeatability (Noise)	0.00200	Normal (2 $\sigma$ )	0.00100	1	0.00100
Connector repeatability [7],[8]	0.0010 (60 dB)	Normal (2 $\sigma$ )	0.00050	1	0.00050
<b>Combined Standard Uncertainty</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>0.00236</b>
<b>Expanded Uncertainty (k=2)</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>X</b>	<b>0.00473</b> <b>0.005 (used)</b>