

# HYDROMETER CALIBRATION: LINKING SIM NMIs TO THE EURAMET KEY COMPARISON REFERENCE VALUE OF EURAMET.M.D-K4

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## 1. INTRODUCTION

Calibration of hydrometers is an important activity of metrology due to importance of density in quality of products, (sugar concentration, percentage of alcohol per volume, etc..) as well as part of measurements for quantity (flow and volume), among others.

Due to the above reasons it is important for laboratories to demonstrate their capability in density meters calibration. A possibility is to participate in comparisons.

Within the EURAMET region a key comparison on hydrometer calibration was carried out from October 2003 to May 2004 under the identification of EURAMET.M.D-K4. Another key comparison between SIM NMIs on hydrometer calibration was carried out from April 2007 to October 2008 identified as SIM.M.D-K4. INRIM-Italy and CENAM-Mexico were the pilot laboratories for the two key comparisons respectively.

With the intention of linking these two key comparisons, a bilateral comparison between CENAM-Mexico and INRIM-Italy was carried out from June to October 2007; this bilateral comparison was identified as SIM.M.D-S1.

In the present paper the methodology is presented to link the results of the participant NMIs of the SIM.M.D-K4 comparison to EURAMET.M.D-K4 key comparison reference value ( $KCRV_{EU}$ ), trough the bilateral comparison SIM.M.D-S1.

## 2. PARTICIPANTS AND TRAVELLING STANDARDS OF COMPARISONS

### 2.1. EURAMET.M.D-K4

Participants  
 IMGC – Italy (now INRIM)  
 OMH – Hungary  
 PTB – Germany  
 BNM-LNE – France  
 IPQ – Portugal  
 MIKES – Finland  
 BEV – Austria  
 UME – Turkey  
 GUM – Poland  
 SMU – Slovakia  
 VNIIM – Russia

Travelling standards  
 For the comparison two sets of hydrometers were used within the following ranges

**Table 1** Set 1 for EURAMET.M.D-K4

Range	Scale division
0.610 0 g/cm <sup>3</sup> – 0.620 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
0.810 0 g/cm <sup>3</sup> – 0.820 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
1.000 0 g/cm <sup>3</sup> – 1.010 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
1.290 0 g/cm <sup>3</sup> – 1.300 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>

**Table 2** Set 2 for EURAMET.M.D-K4

Range	Scale division
0.600 0 g/cm <sup>3</sup> – 0.610 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
0.800 0 g/cm <sup>3</sup> – 0.810 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
0.990 0 g/cm <sup>3</sup> – 1.000 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
1.290 0 g/cm <sup>3</sup> – 1.300 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>

### 2.2. SIM.M.D-K4

Participants  
 CENAM-Mexico  
 BSJ – Jamaica  
 CENAMEP – Panama  
 CESMEC – Chile  
 IBMETRO – Bolivia  
 INDECOPI – Peru

INEN – Ecuador  
 INMETRO – Brazil  
 INTI – Argentina  
 LACOMET – Costa Rica  
 LATU – Uruguay  
 NIST – United States  
 NRC – Canada  
 SIC – Colombia

Travelling standards

For the comparison, two similar sets of hydrometers were used within the following ranges,

**Table 3** Set for SIM.M.D-K4

Range	Scale division
0.610 0 g/cm <sup>3</sup> – 0.620 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
0.800 0 g/cm <sup>3</sup> – 0.810 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
0.990 0 g/cm <sup>3</sup> – 1.000 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>
1.290 0 g/cm <sup>3</sup> – 1.300 0 g/cm <sup>3</sup>	0.000 1 g/cm <sup>3</sup>

**2.3. SIM.M.D-S1**

Participants  
 INRIM – Italy  
 CENAM – Mexico

Travelling standards

For this supplementary comparison two hydrometers were used with the following characteristics,

**Table 4** Travelling standards for the SIM.M.D-S1

Range	Scale division
0.800 0 g/cm <sup>3</sup> – 0.820 0 g/cm <sup>3</sup>	0.000 2 g/cm <sup>3</sup>
1.180 0 g/cm <sup>3</sup> – 1.200 0 g/cm <sup>3</sup>	0.000 2 g/cm <sup>3</sup>

**3. ANALYSIS OF RESULTS OF THE THREE COMPARISONS**

From the results of participants of EURAMET.M.D-K4, a  $KCRV_{EU}$  for each nominal value was calculated. These  $KCRV_{EU}$  were calculated either as the weighted mean or as the median from results of participants, according to [1].

Degrees of equivalence  $DoEs$  between the values reported by participants and the corresponding  $KCRV_{EU}$ ,  $D_{i(EURAMET)}$  were calculated as well as the degrees of equivalence  $doEs$  for each pair of participants of EURAMET.M.D-K4 [2].

As concerns SIM.M.D-K4 a similar analysis was done [3]. A  $KCRV_{SIM}$  for each nominal value was calculated as the weighted mean of results of participants.

$DoEs$  between participating NMIs and the  $KCRV_{SIM}$  were calculated as well as  $doEs$  among participants of SIM.M.D-K4.

Finally, in the analysis of SIM.M.D-S1 [4] like this was a bilateral comparison there was not need to calculate  $RV$  but  $doEs$  between both participants were calculated.

In order to link the results of the SIM NMIs to the  $KCRV$  of EURAMET, the following analysis was done:

-  $DoEs$  between results reported by INRIM and the  $KCRV_{EU}$  in EURAMET.M.D-K4,

$$D_{INRIM} = INRIM_{EU} - KCRV_{EU} \quad (1)$$

-  $doEs$  between results reported by NMI  $i$  and CENAM in SIM.M.D-K4,

$$d_{i(SIM)} = X_{i(SIM)} - CENAM_{SIM} \quad (2)$$

-  $doEs$  between results reported by CENAM and INRIM in SIM.M.D-S1

$$d_{BIL} = CENAM_{BIL} - INRIM_{BIL} \quad (3)$$

in order to calculate the  $DoEs$  between the SIM NMI  $i$  and the  $KCRV_{EU}$  we have to calculated as follow,

$$D'_i = d_{i(SIM)} + d_{BIL} + D_{INRIM}$$

$$D'_i = X_{i(SIM)} - KCRV_{EU} \quad (4)$$

$D'_i$   $DoEs$  between results reported by SIM NMI  $i$  and the  $KCRV_{EU}$   
 $X_{i(SIM)}$  Value reported by SIM NMI  $i$

In order to calculate (4), it is important to assume that results reported by CENAM in both comparison SIM.M.D-K4 and SIM.M.D-S1 are strongly correlated. The same is assumed for INRIM, results reported by INRIM in both comparisons EURAMET.M.D-K4 and SIM.M.D-S1 are strongly correlated too.

The nominal values measured in the different comparisons are not the same, and due to this

reason it is not possible to calculate (4) for each individual value.

The linking procedure used is described in [5], it consists in calculating a regression curve for the corresponding *DoEs*. According to the specific purpose of this work a straight lines (first order curves) were calculated for  $D_{INRIM}$ ,  $d_{BIL}$ ,  $d_{i(SIM)}$ .

The straight line function is the first option for the fitting curves, where *DoEs* are considered proportional to the nominal density.

In order to introduce all set of values a matrix equation takes the form,

$$\mathbf{D} = \boldsymbol{\rho}\boldsymbol{\beta} - \boldsymbol{\varepsilon} \quad (5)$$

where  $\mathbf{D}$  is the column vector of degrees of equivalence,  $\boldsymbol{\rho}$  is the matrix of nominal values of density,  $\boldsymbol{\varepsilon}$  is the column vector of measurement errors and  $\boldsymbol{\beta}$  is the column vector which contains the slope  $m$  and the intercept  $b$  of the proposed function, straight line,

$$\mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix}, \boldsymbol{\rho} = \begin{bmatrix} \rho_1 & 1 \\ \rho_2 & 1 \\ \vdots & \vdots \\ \rho_{n-1} & 1 \\ \rho_n & 1 \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} m \\ b \end{bmatrix}, \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{n-1} \\ \varepsilon_n \end{bmatrix}$$

The weighted least squares method (WLS) was used to calculate the regression curves for the corresponding *DoEs*.

The solution of (5) by WLS is [5]

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{\rho}^T \boldsymbol{\Psi}_D^{-1} \boldsymbol{\rho})^{-1} \boldsymbol{\rho}^T \boldsymbol{\Psi}_D^{-1} \mathbf{D} \quad (6)$$

The weighing matrices  $\boldsymbol{\Psi}_D^{-1}$ , were formed by the variance (and covariance) of the corresponding *DoEs*, [5, 6].

Variances were calculated as the square of the uncertainties of *DoEs* reported in each comparison, and the covariances were calculated with pairs of the uncertainties of *DoEs* and the corresponding correlation coefficient.

For the correlation coefficients it was assumed 0.9 for *DoEs* corresponding to the calibration of same hydrometer and 0.3 for *DoEs* corresponding to the calibration of different hydrometers. These correlation coefficients were assumed based on the experience of the authors.

The calculated curves take the following form,

$$D(\rho_i) = m\rho_i + b - \varepsilon_i \quad (7)$$

where:

- $D(\rho_i)$  *DoEs* calculated for the corresponding comparison, g/cm<sup>3</sup>
- $m$  slope of the straight line calculated, dimensionless
- $b$  intercept of the straight line calculated, g/cm<sup>3</sup>
- $\varepsilon_i$  adjustment error for the proposed function, g/cm<sup>3</sup>

In order to check the agreement between predicted and observed values the chi-square test  $\chi^2$  was used.

### 3.1. Calculated continuous functions of *DoEs*

The calculated curve coefficients for  $D_{INRIM}(\rho)$  in the dominium from 0.6005 g/cm<sup>3</sup> to 1.2995 g/cm<sup>3</sup> is,

$$b = 5.81029E-06 \text{ g/cm}^3$$

$$m = -1.84023E-05$$

The calculated curve coefficients for  $d_{BIL}(\rho)$  in the dominium<sup>1</sup> from 0.802 g/cm<sup>3</sup> to 1.198 g/cm<sup>3</sup> is,

$$b' = -2.31066E-05 \text{ g/cm}^3$$

$$m' = 1.71067E-05$$

both approximations were checked by  $\chi^2$  test.

For SIM NMIs the following coefficients for curves to describe  $d_{i(SIM)}(\rho)$  were calculated in the dominium from 0.601 g/cm<sup>3</sup> to 1.299 g/cm<sup>3</sup>, except for  $d_{NRC(SIM)}$  which is the dominium from 0.801 g/cm<sup>3</sup> to 1.299 g/cm<sup>3</sup> because NRC did not participate in the lower range in SIM comparison.

<sup>1</sup> The dominium of each function is fixed by the nominal values measured in the corresponding comparison.

$d_{i(SIM)}(\rho)$  describes the difference (2) as a continuous function.

**Table 5.** Fitting coefficients for the continuous functions  $d_{i(SIM)}(\rho)$

$d_{i(SIM)}(\rho)$	$b''$ , g/cm <sup>3</sup>	$m''$
$d_{SIC(SIM)}(\rho)$	1.539E-05	-1.921E-05
$d_{LATU(SIM)}(\rho)$	6.071E-05	-7.332E-05
$d_{INDECOPI(SIM)}(\rho)$	6.463E-05	-4.714E-05
$d_{INTI(SIM)}(\rho)$	4.655E-05	-9.329E-05
$d_{NIST(SIM)}(\rho)$	1.463E-05	-8.212E-06
$d_{NRC(SIM)}(\rho)$	1.070E-04	-1.486E-04
$d_{CENAMEP(SIM)}(\rho)$	1.397E-04	-9.877E-05

The other six participants of SIM.M.D-K4, did not pass the  $\chi^2$  test for the proposed function.

**3.2. Calculation of DoEs of SIM NMIs respect to the KCRV<sub>EU</sub>**

The continuous functions of DoEs of SIM NMIs respect to KCRV<sub>EU</sub>, as continuous functions  $D'_i(\rho)$ , were calculated by the following formula [5],

$$D'_i(\rho) = d_{i(SIM)}(\rho) + d_{BIL}(\rho) + D_{INRIM}(\rho) \quad (8)$$

this takes the following form,

$$D'_i(\rho) = (m''+m'+m)\rho_i + (b''+b'+b) - \varepsilon'_i \quad (9a)$$

or

$$D'_i(\rho) = (m^*)\rho_i + (b^*) - \varepsilon'_i \quad (9b)$$

where  $\varepsilon'_i$  is the adjustment error for the resulting function in g/cm<sup>3</sup>,  $m^* = m'' + m' + m$  and  $b^* = b'' + b' + b$ .

For CENAM's DoEs with respect to KCRV<sub>EU</sub>,  $D'_{CENAM}(\rho)$ , the continuous function was calculated for the following formula,

$$D'_{CENAM}(\rho) = d_{BIL}(\rho) + D_{INRIM}(\rho) \quad (10)$$

or presented as the addition of the coefficients,

$$D'_{CENAM}(\rho) = (m'+m)\rho_i + (b'+b) - \varepsilon_i \quad (11)$$

The calculated coefficients for the continuous functions of  $D'_i(\rho)$  are listed in table 6.

**Table 6.** Fitting coefficients for the continuous functions  $D'_i(\rho)$

$D'_i(\rho)$	$b^*$ , g/cm <sup>3</sup>	$m^*$
$D'_{CENAM}(\rho)$	-1.730E-05	-1.296E-06
$D'_{SIC}(\rho)$	-1.911E-06	-2.051E-05
$D'_{LATU}(\rho)$	4.341E-05	-7.462E-05
$D'_{INDECOPI}(\rho)$	4.733E-05	-4.844E-05
$D'_{INTI}(\rho)$	2.925E-05	-9.458E-05
$D'_{NIST}(\rho)$	-2.664E-06	-9.507E-06
$D'_{NRC}(\rho)$	8.971E-05	-1.499E-04
$D'_{CENAMEP}(\rho)$	1.224E-04	-1.000E-04

The dominium of  $D'_i(\rho)$  is from 0.802 g/cm<sup>3</sup> to 1.198 g/cm<sup>3</sup>, due to the fact that the linking comparison SIM.M.D-S1 is within the same dominium. The calculated  $D'_i(\rho)$  are shown in Fig. 1.

**3.3. Uncertainty of DoEs of SIM NMIs respect to the KCRV<sub>EU</sub>**

In order to calculate the uncertainty of  $D'_i(\rho)$ , numerical simulations by Monte Carlo method were done for each  $D'_i(\rho)$  [6,7], see Fig. 2.

For the numerical simulations, DoEs and their corresponding standard uncertainties were considered as the mean value and the standard deviation of normal probability density distributions of the input quantities. For the numerical simulation 100 000 trials were done for each  $D'_i(\rho)$ .

The normalized errors to an approximated 95% of confidence level were calculated with the following expression,

$$E_n = \frac{D'_i(\rho)}{U(D'_i(\rho))} \quad (12)$$

In Fig. 3 are shown the curves of the normalized errors of  $D'_i(\rho)$ , the accepted values are  $\leq 1$ .

In table 7 are shown the values of  $D'_i(\rho)$ ,  $U(D'_i(\rho))$ , and  $E_n$  calculated for selected densities within the dominium of the functions.

## 5. CONCLUSIONS

In the present paper were presented the *DoEs* of SIM NMIs to the  $KCRV_{EU}$ ,  $D'_i(\rho)$ .

From fourteen participants of SIM.M.D-K4, only eight participants were linked to the  $KCRV$  of the EURAMET.M.D-K4. The other six participants failed the  $\chi^2$  test for the proposed function.

All participants that passed the  $\chi^2$  test had normalized errors smaller than 1.

The straight line function is the first option for the fitting curves, where *DoEs* are considered proportional to the nominal density.

Indeed it is possible to calculate other functions that could fit better than straight line (e.g. by a second order polynomial function), but the straight line is one of the simplest assumption.

In the straight line function, the systematic effects could be divided into two groups, the constant effects which could be included in the intercept ( $b$ ) and the relative effects which could be included in the slope ( $m$ ), see formula (7).

In Figure 4 are shown the *DoEs* between the SIM NMIs considered for this work and the  $KCRV_{SIM}$  in SIM.M.D-K4 [3].

In Figure 4, it could be observed the systematic effects besides to the random effects. In Figure 5 is shown the same information but presented in a different way. In Figure 5, it could be noted the dispersion of the calculated *DoEs* for SIM NMIs in SIM.M.D-K4, which could be reasonably be attributed to random effects arising from measurement systems of the SIM NMIs. These dispersions are not included in the linking functions, but do are included in the uncertainty of these functions.

The uncertainty of the  $D'_i(\rho)$  was dominated by the uncertainties of the *DoEs* of the linking comparison SIM.M.D-S1 due to the scale division of the

travelling standards used in such bilateral comparison.

A similar analysis could be done for linking SIM.M.D-K4 to the CCM key comparison CCM.D-K4 which will be started in 2010, activity waiting to be done, once results from CCM.D-K4 be available.

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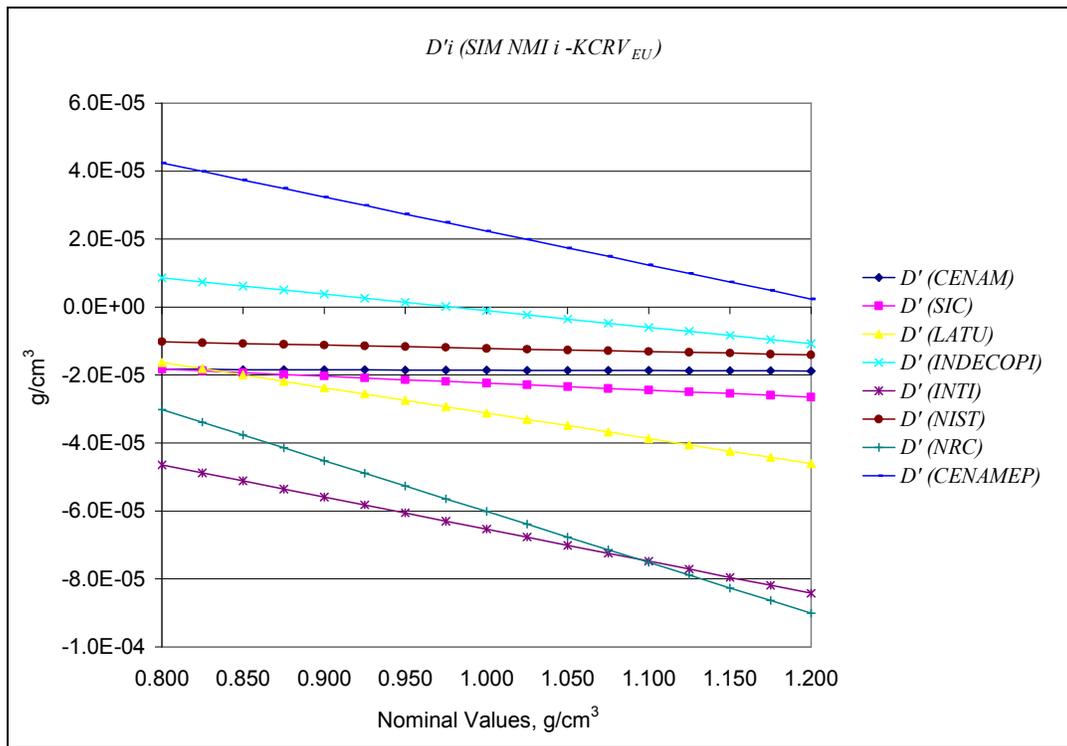


Fig. 1. DoEs between SIM NMIs and  $KCRV_{EU}$ ,  $D'_i(\rho)$ .

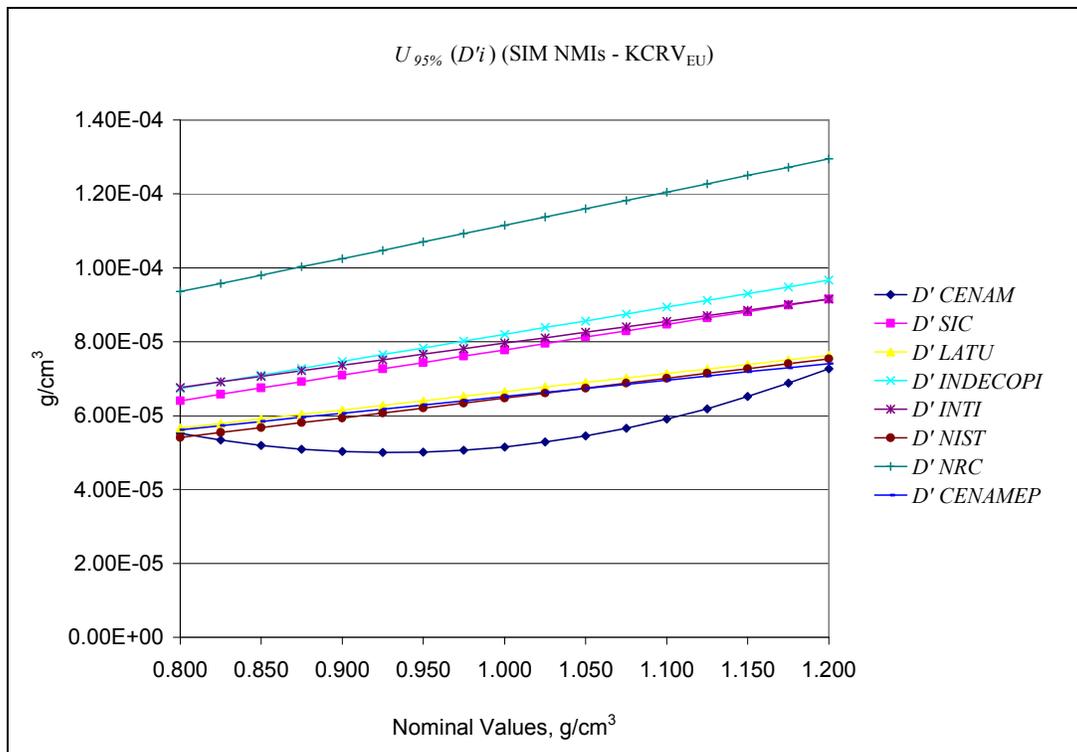
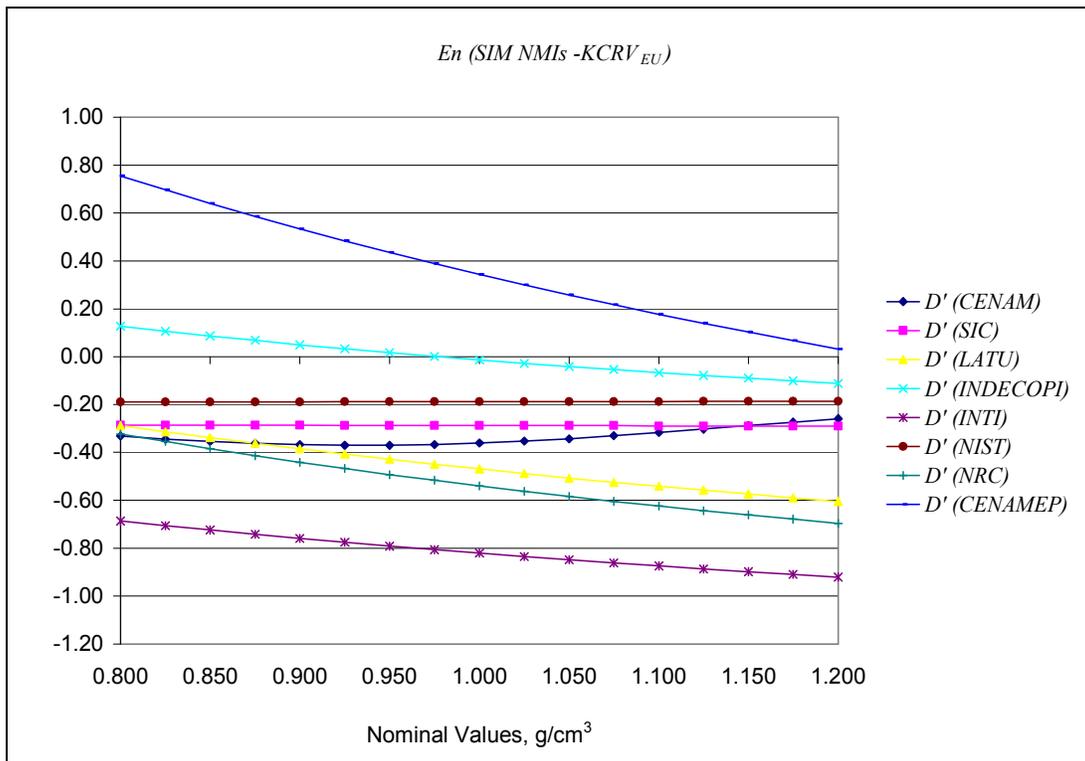
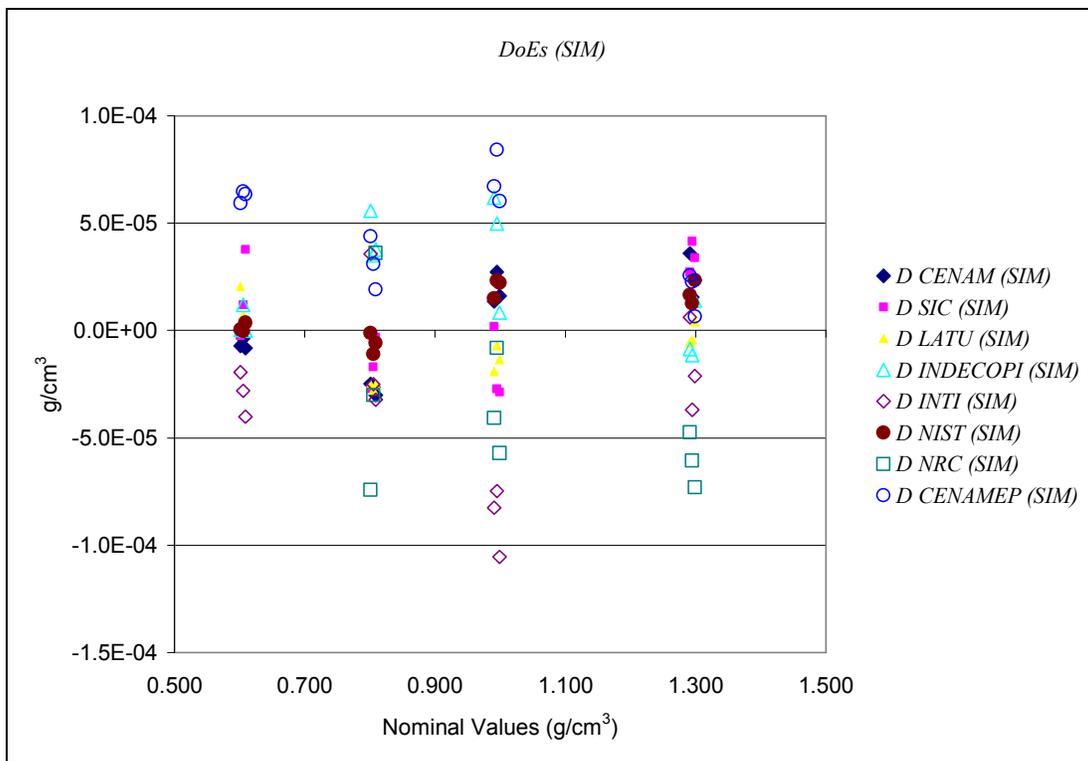


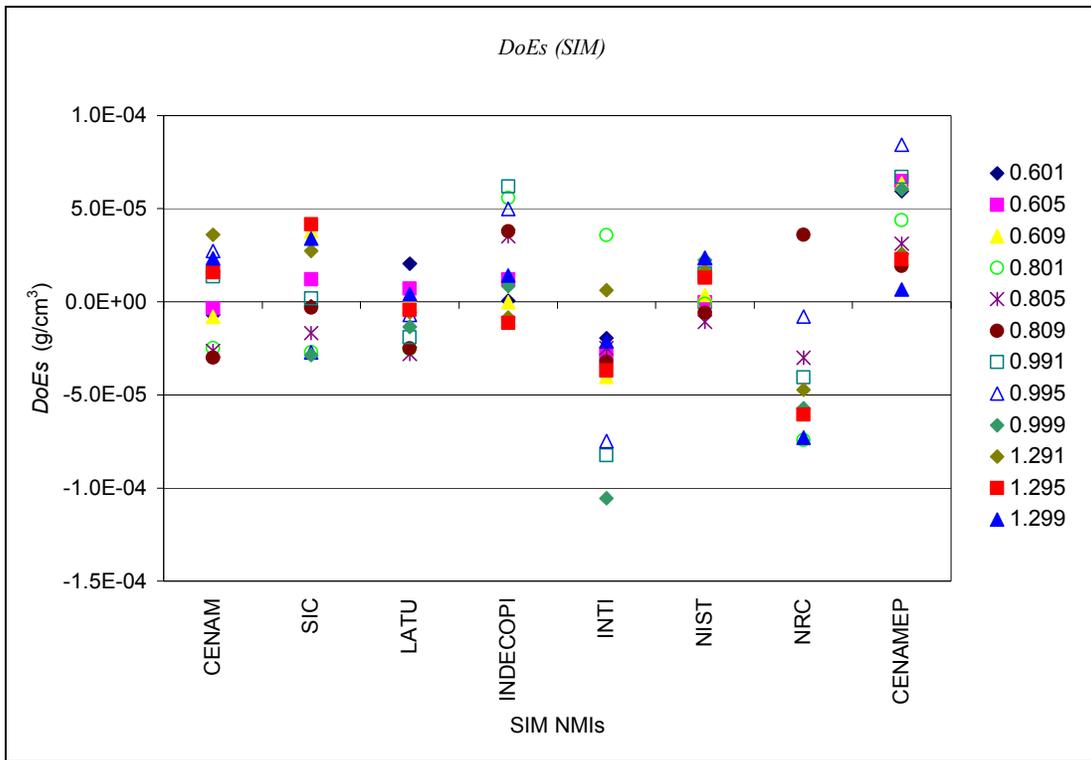
Fig. 2. Expanded uncertainty (approx. 95%) of DoEs between SIM NMIs and  $KCRV_{EU}$ .



**Fig. 3.** Normalized errors between SIM NMIs and KCRV<sub>EU</sub>



**Fig. 4.** DoEs between selected SIM NMIs and KCRV<sub>SIM</sub> as calculated in SIM.M.D-K4.



**Fig. 5.** DoEs between selected SIM NMIs and  $KCRV_{SIM}$  as calculated in SIM.M.D-K4.

**Table 7.** Calculated values of DoEs of SIM NMI's regards to the  $KCRV_{EU}$ , their expanded uncertainties associated and the calculated normalized errors, for selected values of density within the dominium of the functions.

$\rho$	$D' CENAM$	$U (95\%)$	$En$	$D' SIC$	$U (95\%)$	$En$	$D' LATU$	$U (95\%)$	$En$	$D' INDECOPI$	$U (95\%)$	$En$
$g/cm^3$	$g/cm^3$	$g/cm^3$		$g/cm^3$	$g/cm^3$		$g/cm^3$	$g/cm^3$		$g/cm^3$	$g/cm^3$	
0.80	-1.83E-05	5.52E-05	-0.33	-1.83E-05	6.40E-05	-0.29	-1.63E-05	5.67E-05	-0.29	8.58E-06	6.73E-05	0.13
0.83	-1.84E-05	5.34E-05	-0.34	-1.88E-05	6.58E-05	-0.29	-1.81E-05	5.79E-05	-0.31	7.37E-06	6.91E-05	0.11
0.85	-1.84E-05	5.19E-05	-0.35	-1.93E-05	6.75E-05	-0.29	-2.00E-05	5.91E-05	-0.34	6.16E-06	7.10E-05	0.09
0.88	-1.84E-05	5.09E-05	-0.36	-1.99E-05	6.92E-05	-0.29	-2.19E-05	6.03E-05	-0.36	4.95E-06	7.28E-05	0.07
0.90	-1.85E-05	5.03E-05	-0.37	-2.04E-05	7.09E-05	-0.29	-2.37E-05	6.16E-05	-0.39	3.73E-06	7.46E-05	0.05
0.93	-1.85E-05	5.00E-05	-0.37	-2.09E-05	7.26E-05	-0.29	-2.56E-05	6.28E-05	-0.41	2.52E-06	7.65E-05	0.03
0.95	-1.85E-05	5.01E-05	-0.37	-2.14E-05	7.44E-05	-0.29	-2.75E-05	6.40E-05	-0.43	1.31E-06	7.83E-05	0.02
0.98	-1.86E-05	5.06E-05	-0.37	-2.19E-05	7.61E-05	-0.29	-2.93E-05	6.52E-05	-0.45	1.01E-07	8.02E-05	0.00
1.00	-1.86E-05	5.15E-05	-0.36	-2.24E-05	7.78E-05	-0.29	-3.12E-05	6.65E-05	-0.47	-1.11E-06	8.20E-05	-0.01
1.03	-1.86E-05	5.28E-05	-0.35	-2.29E-05	7.95E-05	-0.29	-3.31E-05	6.77E-05	-0.49	-2.32E-06	8.38E-05	-0.03
1.05	-1.87E-05	5.45E-05	-0.34	-2.34E-05	8.12E-05	-0.29	-3.49E-05	6.89E-05	-0.51	-3.53E-06	8.57E-05	-0.04
1.08	-1.87E-05	5.66E-05	-0.33	-2.40E-05	8.30E-05	-0.29	-3.68E-05	7.02E-05	-0.52	-4.74E-06	8.75E-05	-0.05
1.10	-1.87E-05	5.90E-05	-0.32	-2.45E-05	8.47E-05	-0.29	-3.87E-05	7.14E-05	-0.54	-5.95E-06	8.93E-05	-0.07
1.13	-1.88E-05	6.19E-05	-0.30	-2.50E-05	8.64E-05	-0.29	-4.05E-05	7.26E-05	-0.56	-7.16E-06	9.12E-05	-0.08
1.15	-1.88E-05	6.51E-05	-0.29	-2.55E-05	8.81E-05	-0.29	-4.24E-05	7.38E-05	-0.57	-8.38E-06	9.30E-05	-0.09
1.18	-1.88E-05	6.87E-05	-0.27	-2.60E-05	8.98E-05	-0.29	-4.43E-05	7.51E-05	-0.59	-9.59E-06	9.48E-05	-0.10
1.20	-1.89E-05	7.27E-05	-0.26	-2.65E-05	9.15E-05	-0.29	-4.61E-05	7.63E-05	-0.60	-1.08E-05	9.67E-05	-0.11
$\rho$	$D' INTI$	$U (95\%)$	$En$	$D' NIST$	$U (95\%)$	$En$	$D' NRC$	$U (95\%)$	$En$	$D' CENAMEP$	$U (95\%)$	$En$
$g/cm^3$	$g/cm^3$	$g/cm^3$		$g/cm^3$	$g/cm^3$		$g/cm^3$	$g/cm^3$		$g/cm^3$	$g/cm^3$	
0.80	-4.64E-05	6.77E-05	-0.69	-1.03E-05	5.41E-05	-0.19	-3.02E-05	9.35E-05	-0.32	4.24E-05	5.62E-05	0.75
0.83	-4.88E-05	6.91E-05	-0.71	-1.05E-05	5.54E-05	-0.19	-3.39E-05	9.58E-05	-0.35	3.99E-05	5.73E-05	0.70
0.85	-5.11E-05	7.06E-05	-0.72	-1.07E-05	5.67E-05	-0.19	-3.77E-05	9.80E-05	-0.38	3.74E-05	5.84E-05	0.64
0.88	-5.35E-05	7.21E-05	-0.74	-1.10E-05	5.81E-05	-0.19	-4.14E-05	1.00E-04	-0.41	3.49E-05	5.95E-05	0.59
0.90	-5.59E-05	7.36E-05	-0.76	-1.12E-05	5.94E-05	-0.19	-4.52E-05	1.03E-04	-0.44	3.24E-05	6.06E-05	0.53
0.93	-5.82E-05	7.51E-05	-0.78	-1.15E-05	6.07E-05	-0.19	-4.89E-05	1.05E-04	-0.47	2.99E-05	6.18E-05	0.48
0.95	-6.06E-05	7.66E-05	-0.79	-1.17E-05	6.21E-05	-0.19	-5.27E-05	1.07E-04	-0.49	2.74E-05	6.29E-05	0.44
0.98	-6.30E-05	7.81E-05	-0.81	-1.19E-05	6.34E-05	-0.19	-5.64E-05	1.09E-04	-0.52	2.49E-05	6.40E-05	0.39
1.00	-6.53E-05	7.96E-05	-0.82	-1.22E-05	6.47E-05	-0.19	-6.02E-05	1.11E-04	-0.54	2.23E-05	6.51E-05	0.34
1.03	-6.77E-05	8.11E-05	-0.83	-1.24E-05	6.61E-05	-0.19	-6.39E-05	1.14E-04	-0.56	1.98E-05	6.62E-05	0.30
1.05	-7.01E-05	8.26E-05	-0.85	-1.26E-05	6.74E-05	-0.19	-6.77E-05	1.16E-04	-0.58	1.73E-05	6.74E-05	0.26
1.08	-7.24E-05	8.41E-05	-0.86	-1.29E-05	6.87E-05	-0.19	-7.14E-05	1.18E-04	-0.60	1.48E-05	6.85E-05	0.22
1.10	-7.48E-05	8.56E-05	-0.87	-1.31E-05	7.01E-05	-0.19	-7.51E-05	1.20E-04	-0.62	1.23E-05	6.96E-05	0.18
1.13	-7.72E-05	8.71E-05	-0.89	-1.34E-05	7.14E-05	-0.19	-7.89E-05	1.23E-04	-0.64	9.84E-06	7.07E-05	0.14
1.15	-7.95E-05	8.85E-05	-0.90	-1.36E-05	7.27E-05	-0.19	-8.26E-05	1.25E-04	-0.66	7.34E-06	7.18E-05	0.10
1.18	-8.19E-05	9.00E-05	-0.91	-1.38E-05	7.41E-05	-0.19	-8.64E-05	1.27E-04	-0.68	4.84E-06	7.29E-05	0.07
1.20	-8.42E-05	9.15E-05	-0.92	-1.41E-05	7.54E-05	-0.19	-9.01E-05	1.29E-04	-0.70	2.34E-06	7.41E-05	0.03