A COMPARISON IN CONVENTIONAL MASS MEASUREMENTS (SIM.M.M-S3)

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Abstract: The present document reports the results of a comparison in the calibration of mass standards that was carried out between CESMEC (Chile), IBMETRO (Bolivia), and INTN (Paraguay); degrees of equivalence and levels of measurement agreement are reported. This comparison was carried out in the following nominal values: 100 mg, 20 g, and 1 kg.

1. INTRODUCTION

On November 2007 INTN, IBMETRO and CESMEC were under assessment by Deutscher Kalibrierdienst (DKD) and each laboratory had to participate in an independent comparison process with a laboratory recognized by the European co-operation for Accreditation (EA). The appointed laboratory by DKD was Mettler-Toledo AG [1], Switzerland, which has higher measurement capabilities than those of the laboratories under assessment.

The participant laboratories considered that activity as a good opportunity for executing a supplementary comparison between them for CIPM MRA Appendix B [2] purposes, so the resources needed for this activity are costly but necessary in order to obtain recognition for the CMCs according to the CIPM MRA [3].

Mettler-Toledo AG results are not to be reported to the CIPM MRA Appendix B and were not considered as reference values. Measurement results of Mettler-Toledo AG were used only to estimate the drift of the mass values during the process.

Correlations among measurement results are considered in this report since CESMEC calibrated INTN's and IBMETRO´s reference standards previously and Mettler-Toledo AG calibrated the test weights at the beginning and at the end of the comparison for the drift determination.

2. COMPARISON PROCESS

2.1. General Guidelines

A measurement protocol was previously agreed and reported to the SIM Mass & Related Quantities Metrology Working Group on November 2007. The SIM.M.M-S3 code was assigned [2].

Mettler-Toledo AG provided the weights to be measured by each laboratory. On the other hand, the DKD assessment team carried and delivered the weights to each laboratory.

The following relevant aspects were stated in the protocol:

- Measurements were done after the acclimatization time as specified for class E1 in [4].
- The participating laboratories measured the conventional mass of the artifacts according to [4].
- No washing was performed. Before measurements, dust particles were removed from the surface of the standard by a soft brush.
- All weightings were performed in air. Uncertainties were estimated and combined according to [5] the specific requirements of [4].
- The standards were transported by hand in order to assuring the conventional mass stability of the test objects.

2.2. Comparison Objects

The test objects have the nominal values and densities stated in Table 1.

***Table 1**. Density of the comparison objects.*

Nominal Value	Density	Expanded uncertainty of the density value (k=2)
100 mg	8000 kg/m^3	30 kg/m^3
20 g	8000 kg/m^3	30 kg/m^3
1 kg	8000 kg/m^3	30 kg/m^3

2.2 Comparison round

The comparison was performed in a round including an initial and a final measurement at Mettler-Toledo AG. See Table 2 for a listing of the standards used, the date of measurement for the comparison, and the laboratory that performed the calibrations of the standards.

***Table 2.** Participant laboratories and standards.*

Laboratory/ Country	Institute that calibrated the standard used for this comparison	Date of measurement
Mettler-Toledo AG/ Switzerland	Mettler-Toledo AG	2007-06-25 (100 mg) 2007-05-08 (20 g) 2007-05-08 (1 kg)
CESMEC (Pilot)/Chile	PTB	2007-11-23
INTN/Paraguay	CESMEC	2007-11-27
IBMETRO/Bolivia	CESMEC	2007-11-31
Mettler-Toledo AG/ Switzerland	Mettler-Toledo AG	2008-03-29 (100 mg) 2008-03-30 (20 g) 2008-04-01 (1 kg)

***Table 3.** Results as reported by each participant.*

Laboratory	Conventional mass value		Expanded Uncertainty of the conventional mass value (k=2)	Mean Air density	Expanded Uncertainty of the Mean Air density
Mettler-Toledo AG	100 mg 20 g 1 kg	+ 0.0047 mg + 0.013 mg + 0.17 mg	0.0027 mg 0.013 mg 0.27 mg	1.135 kg/m^3	0.002 kg/m^3
CESMEC (Pilot)	100 mg 20 g 1 kg	+ 0.006 mg + 0.010 mg + 0.0 mg	0.005 mg 0.025 mg 0.5 mg	1.125 kg/m^3	0.002 kg/m^3
INTN	100 mg 20 g 1 kg	+ 0.007 mg + 0.013 mg + 0.1 mg	0.016 mg 0.080 mg 1.6 mg	1.174 kg/m^3	0.002 kg/m^3
IBMETRO	100 mg 20 g 1 kg	+ 0.007 mg - 0.040 mg - 0.8 mg	0.016 mg 0.080 mg 1.6 mg	0.780 kg/m^3	0.003 kg/m^3
Mettler-Toledo AG	100 mg 20 g 1 kg	+ 0.0043 mg + 0.012 mg - 0.03mg	0.0027 mg 0.013 mg 0.27 mg	1.135 kg/m^3	0.002 kg/m^3

3. RESULTS

Results as reported by each laboratory for each nominal value and the corresponding expanded uncertainties (k = 2) are presented in Table 3.

The air density value was evaluated by IBMETRO according to equation (E.1-1) of [4]; INTN and CESMEC evaluated the air density according to (E.3-1) of [4].

4. DISCUSSION

The degrees of equivalence are given by the pair of values $\left(d_{ij}, U\left(d_{ij}\right)\right)$, where

$$d_{ij} = m_{ct}^{(i)} - m_{ct}^{(j)} \qquad (1)$$

$m_{ct}^{(i)}$ is the conventional mass value of the test weight determined by the laboratory i

- $i = b$ for IBMETRO, Bolivia;
- $i = p$ for INTN, Paraguay;
- $i = c$ for CESMEC, Chile and
- $i = m$ for Mettler-Toledo AG. In this case:

$$m_{ct}^{(m)} = \frac{m_{ct}^{(m_i)} + m_{ct}^{(m_f)}}{2} \qquad (2)$$

$i = m_i$ and $i = m_f$ for the initial and final measurements done by Mettler-Toledo AG, respectively.

In order to make the evaluations of d_{ij} we have to take into account that: a) The drift that the test weights may have suffered during transport. Mettler-Toledo AG measured them at the beginning and at the end of the comparison round in order to evaluate it. b) That CESMEC calibrated the reference standards that were used by IBMETRO and INTN, and that CESMEC participated in this comparison too.

Then all the results reported by the laboratories are correlated and correlations have to be considered in order to get a better evaluation of the degrees of equivalence. These calculations will be done in each of the following sections.

4.1 Uncertainty due to the drift of the test objects.

The drift, e, of the test object is evaluated by the difference between $m_{ct}^{(m_i)}$ and $m_{ct}^{(m_f)}$:

$$e = m_{ct}^{(m_f)} - m_{ct}^{(m_i)} \qquad (3)$$

And its standard uncertainty is given by the recommendations of reference [4]:

$$u(e) = \left[\begin{array}{l} \left(\frac{\partial e}{\partial m_{ct}^{(m_f)}}\right)^2 u^2\left(m_{ct}^{(m_f)}\right) + \\ + \left(\frac{\partial e}{\partial m_{ct}^{(m_i)}}\right)^2 u^2\left(m_{ct}^{(m_i)}\right) + \\ + 2 \frac{\partial e}{\partial m_{ct}^{(m_f)}} \frac{\partial e}{\partial m_{ct}^{(m_i)}} u\left(m_{ct}^{(m_f)}, m_{ct}^{(m_i)}\right) \end{array} \right]^{1/2} \qquad (4),$$

or

$$u(e) = \left[u^2\left(m_{ct}^{(m_f)}\right) + u^2\left(m_{ct}^{(m_i)}\right) - 2u^2\left(m_{cr}^{(m)}\right)\right]^{1/2} \qquad (5)$$

In Appendix further details are given about the evaluation of $u\left(m_{ct}^{(m_f)}, m_{ct}^{(m_i)}\right)$. $m_{cr}^{(m)}$ is the conventional mass value of the reference standard of Mettler-Toledo AG.

To simplify the evaluations, the drift value, e, won't be used to correct the results. We will assume a drift value equal to cero, $e^* = 0$, and its standard uncertainty will be evaluated according to reference [6]:

$$u(e^*) = \sqrt{(e - e^*)^2 + u^2(e)} \qquad (6)$$

Table 5. *Drift value assigned to the test weights, e^*, and its standard uncertainty $u(e^*)$.*

Nominal Value	e^* / mg	$e - e^*$ / mg	$u(e)$ / mg	$u(e^*)$ / mg
100 mg	0.000 0	- 0.0004	0.0015	0.0015
20 g	0.000	- 0.001	0.007	0.007
1 kg	0.00	- 0.20	0.15	0.25

A similar analysis to that explained in [7, 8] shows that the amount of drift is not statistically significant;

anyway it will be included in the uncertainty evaluations.

4.2 Degrees of equivalence between participant laboratories.

4.2.1 Degrees of equivalence between participants i, j (with $i = c$ and $j = b, p$) are given by $\left(d_{ij}, U\left(d_{ij}\right)\right)$, where

$$d_{cj} = m_{ct}^{(c)} - m_{ct}^{(j)} + e^* \quad (7)$$

$$U\left(d_{cj}\right) = 2 \cdot \sqrt{\begin{array}{l}\left(\frac{\partial d_{cj}}{\partial m_{ct}^{(c)}}\right)^2 u^2\left(m_{ct}^{(c)}\right) + \\ + \left(\frac{\partial d_{cj}}{\partial m_{ct}^{(j)}}\right)^2 u^2\left(m_{ct}^{(j)}\right) + \\ + 2\frac{\partial d_{cj}}{\partial m_{ct}^{(c)}}\frac{\partial d_{cj}}{\partial m_{ct}^{(j)}} u\left(m_{ct}^{(c)}, m_{ct}^{(j)}\right) + \\ + \left(\frac{\partial d_{ij}}{\partial e^*}\right)^2 u^2\left(e^*\right)\end{array}} \quad (8)$$

$$U\left(d_{cj}\right) = 2 \cdot \sqrt{\begin{array}{l}u^2\left(m_{ct}^{(c)}\right) + u^2\left(m_{ct}^{(j)}\right) + \\ - 2u^2\left(m_{cr}^{(c)}\right) + u^2\left(e^*\right)\end{array}} \quad (9)$$

See in A.2 the evaluation of $u\left(m_{ct}^{(c)}, m_{ct}^{(j)}\right)$ for $j = b, p$.

4.2.2 Degrees of equivalence between IBMETRO and INTN, $\left(d_{ij}, U\left(d_{ij}\right)\right)$.

In this case: $i = b$ and $j = p$

$$d_{bp} = m_{ct}^{(b)} - m_{ct}^{(p)} + e^* \quad (10)$$

$$U\left(d_{bp}\right) = 2 \cdot \sqrt{\begin{array}{l}\left(\frac{\partial d_{bp}}{\partial m_{ct}^{(b)}}\right)^2 u^2\left(m_{ct}^{(b)}\right) + \\ + \left(\frac{\partial d_{bp}}{\partial m_{ct}^{p}}\right)^2 u^2\left(m_{ct}^{(p)}\right) + \\ + 2\frac{\partial d_{bp}}{\partial m_{ct}^{(b)}}\frac{\partial d_{bp}}{\partial m_{ct}^{p}} u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right) + \\ + \left(\frac{\partial d_{bp}}{\partial e^*}\right)^2 u^2\left(e^*\right)\end{array}} \quad (11)$$

$$U\left(d_{bp}\right) = 2 \cdot \sqrt{\begin{array}{l}u^2\left(m_{ct}^{(c)}\right) + u^2\left(m_{ct}^{(p)}\right) + \\ - 2u^2\left(m_{cr}^{(c)}\right) + u^2\left(e^*\right)\end{array}} \quad (12)$$

$u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right) \approx u^2\left(m_{cr}^{(c)}\right)$, see in A.2 the evaluation of $u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right)$ for $i = b, c, p$.

4.2.3 Degrees of equivalence between each participant and Mettler-Toledo AG, $\left(d_{ij}, U\left(d_{ij}\right)\right)$.

In this case: $i = c, b, p$ and $j = m$

$$d_{im} = m_{ct}^{(i)} - \frac{m_{ct}^{(m_i)} + m_{ct}^{(m_f)}}{2} \quad (13)$$

$$U\left(d_{im}\right) = 2 \cdot \sqrt{\begin{array}{l}\left(\frac{\partial d_{im}}{\partial m_{ct}^{(i)}}\right)^2 u^2\left(m_{ct}^{(i)}\right) + \\ + \left(\frac{\partial d_{im}}{\partial m_{ct}^{(m_i)}}\right)^2 u^2\left(m_{ct}^{(m_i)}\right) + \\ + \left(\frac{\partial d_{im}}{\partial m_{ct}^{(m_f)}}\right)^2 u^2\left(m_{ct}^{(m_f)}\right) + \\ + 2\frac{\partial d_{im}}{\partial m_{ct}^{(m_i)}}\frac{\partial d_{im}}{m_{ct}^{(m_j)}} \cdot u\left(m_{ct}^{(m_i)}, m_{ct}^{(m_j)}\right)\end{array}} \quad (14)$$

$$U\left(d_{im}\right) = 2 \cdot \sqrt{\begin{array}{l}u^2\left(m_{ct}^{(i)}\right) + \frac{u^2\left(m_{ct}^{(m_i)}\right)}{4} + \\ + \frac{u^2\left(m_{ct}^{(m_j)}\right)}{4} + \frac{1}{2} \cdot u^2\left(m_{cr}^{(m)}\right)\end{array}} \quad (15)$$

$u\left(m_{ct}^{(m_i)}, m_{ct}^{(m_j)}\right) \approx u^2\left(m_{cr}^{(m)}\right)$, as in 4.1

Table 6. *Degrees of equivalence among participants.* $\left(d_{ij}, U\left(d_{ij}\right)\right)$

Nominal Value	Institute	IBMETRO / mg	INTN / mg
100 mg	CESMEC	(-0.001, 0.016)	(-0.001, 0.016)
	IBMETRO		(0.000, 0.022)
20 g	CESMEC	(0.050, 0.085)	(-0.003, 0.085)
	IBMETRO		(-0.053, 0.114)
1 kg	CESMEC	(0.800, 1.749)	(-0.100, 1.749)
	IBMETRO		(-0.900, 2.317)

Table 7. *Levels of measurement agreement among participants (normalized errors)* $\left(d_{ij}/U\left(d_{ij}\right)\right)$

Nominal Value	Institute	IBMETRO	INTN
100 mg	CESMEC	-0.064	-0.064
	IBMETRO		0.000
20 g	CESMEC	0.588	-0.035
	IBMETRO		-0.465
1 kg	CESMEC	0.457	-0.057
	IBMETRO		-0.388

From previous equations we get the values summarized in Tables 6 and 7:

In Table 7 is shown that all levels of measurement agreement are less than one. This implies that measurements performed by the laboratories are equivalent within the reported uncertainties according to normalized error criteria.

5. CONCLUSIONS

Degrees of equivalence and levels of measurement agreement are stated in Tables 6 and 7. Good levels of agreement were found between participants.

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APPENDIX

The symbols used in Appendix are presented as follows. "(n)" is an index only, it is not an exponent when used in symbols of the form $x_j^{(n)}$.

For the variables considered during the measurements of the test weight:

- $m_{cr}^{(i)}$ conventional mass of the reference weight of each participant laboratory i.
- $\rho_r^{(i)}$ density of the reference weight of each participant laboratory i.
- $\rho_a^{(i)}$ air density during the measurements done at each participant laboratory i.
- $\Delta m^{(i)}$ the conventional mass difference between the test weight and $m_{cr}^{(i)}$, measured at each laboratory i.
- ρ_o the conventional density of the air.
- ρ_t density of the test weight,

Obs.: For Mettler-Toledo AG $i = m_i = m_f \equiv m$

For the variables considered during the calibrations of the reference standards of IBMETRO and INTN at CESMEC:

- ρ_a' air density during the determination of $m_{cr}^{(p)}$ at CESMEC
- ρ_a'' air density during the determination of $m_{cr}^{(b)}$ at CESMEC.
- $\Delta m'$ difference between the balance readings for the reference standard of INTN, Paraguay, and the reference standard of CESMEC.
- $\Delta m''$ difference between the balance readings for the reference standard of IBMETRO, Bolivia, and the reference standard of CESMEC.

Then, the following relations apply:

$$m_{ct}^{(i)} = m_{cr}^{(i)}\left(1 + \left(\rho_a^{(i)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(i)}}\right)\right) + \Delta m^{(i)} \quad \text{for}$$

$i = b, c, p, m_i, m_j$ (see Section 4)

(A1)

$$m_{cr}^{(p)} = m_{cr}^{(c)}\left(1 + \left(\rho_a' - \rho_o\right)\left(\frac{1}{\rho_r^{(p)}} - \frac{1}{\rho_r^{(c)}}\right)\right) + \Delta m'$$

(A2)

$$m_{cr}^{(b)} = m_{cr}^{(c)}\left(1 + \left(\rho_a'' - \rho_o\right)\left(\frac{1}{\rho_r^{(b)}} - \frac{1}{\rho_r^{(c)}}\right)\right) + \Delta m''$$

(A3)

For each laboratory, equation (A1) relates the reference standard conventional mass value and the conventional mass value determined for the test weight.

Equations (2) and (3) were used by CESMEC when performed the calibrations of the reference standards belonging to IBMETRO and INTN.

On the other hand, we also define the next auxiliary relations:

$$\rho_a^{(i)} = \overline{\rho_a}^{(i)} + c_{\rho_a^{(i)}} \qquad \text{(A4)}$$

$$\Delta m^{(i)} = \overline{\Delta m}^{(i)} + c_{\Delta m^{(i)}} \qquad \text{(A5)}$$

$$\rho_a' = \overline{\rho_a}' + c_{\rho_a^{(c)}} \qquad \text{(A6)}$$

$$\rho_a'' = \overline{\rho_a}'' + c_{\rho_a^{(c)}} \qquad \text{(A7)}$$

$$\Delta m' = \overline{\Delta m}' + c_{\Delta m^{(c)}} \qquad \text{(A8)}$$

$$\Delta m'' = \overline{\Delta m}'' + c_{\Delta m^{(c)}} \qquad \text{(A9)}$$

Where,

- $\overline{\rho_a}^{(i)}$ the mean of the air density values measured at laboratory i
- $c_{\rho_a^{(i)}}$ correction for the air density values measured at laboratory i
- $\overline{\Delta m}^{(i)}$ mean of the weighing differences measured at laboratory i
- $c_{\Delta m^{(i)}}$ correction for the weighing differences measured at laboratory i

$\overline{\rho_a}'$ the mean f the air density values measured at CESMEC during the calibration of the mass standard of IBMETRO.

$\overline{\rho_a}''$ the mean of the air density values measured at CESMEC during the calibration of the mass standard of INTN.

$\overline{\Delta m}'$ mean of the weighing differences measured at CESMEC during the calibration of the mass standard of IBMETRO

$\overline{\Delta m}''$ mean of the weighing differences measured at CESMEC during the calibration of the mass standard of INTN

A.1 Evaluation of $u\left(m_{ct}^{(m_f)}, m_{ct}^{(m_i)}\right)$.

This is the covariance between the measurements performed by Mettler-Toledo AG.

$u\left(m_{ct}^{(m_f)}, m_{ct}^{(m_i)}\right)$ is evaluated using equation (F.2) of reference [5]:

$$u\left(m_{ct}^{(m_f)}, m_{ct}^{(m_i)}\right) = \frac{\partial m_{ct}^{(m_f)}}{\partial m_{cr}^{(m)}} \frac{\partial m_{ct}^{(m_i)}}{\partial m_{cr}^{(m)}} u^2\left(m_{cr}^{(m)}\right) + \frac{\partial m_{ct}^{(m_f)}}{\partial \rho_t} \frac{\partial m_{ct}^{(m_i)}}{\partial \rho_t} u^2\left(\partial \rho_t\right) + \frac{\partial m_{ct}^{(m_f)}}{\partial \rho_r^{(m)}} \frac{\partial m_{ct}^{(m_i)}}{\partial \rho_r^{(m)}} u^2\left(\rho_r^{(m)}\right) + \frac{\partial m_{ct}^{(m_f)}}{\partial c_{\rho_a^{(m)}}} \frac{\partial m_{ct}^{(m_i)}}{\partial c_{\rho_a^{(m)}}} u^2\left(c_{\rho_a^{(m)}}\right) + \frac{\partial m_{ct}^{(m_f)}}{\partial c_{\Delta m^{(m)}}} \frac{\partial m_{ct}^{(m_i)}}{\partial c_{\Delta m^{(m)}}} u^2\left(c_{\Delta m^{(m)}}\right) \quad \text{(A10)}$$

From (A1), (A4) and (A5):

$$u\left(m_{ct}^{(m_f)}, m_{ct}^{(m_i)}\right) = \left(1 + \left(\rho_a^{(m_f)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(m)}}\right)\right) \left(1 + \left(\rho_a^{(m_i)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(m)}}\right)\right) u^2\left(m_{cr}^{(m)}\right) + \left(m_{cr}^{(m)}\right)^2 \left(\rho_a^{(m_f)} - \rho_o\right)\left(\rho_a^{(m_i)} - \rho_o\right) \left[\frac{u^2(\rho_t)}{\rho_t^4} + \frac{u^2\left(\rho_r^{(m)}\right)}{\left(\rho_r^{(m)}\right)^4}\right] + \left(m_{cr}^{(m)}\right)^2 \left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(m)}}\right)^2 u^2\left(c_{\rho_a^{(m)}}\right) + u^2\left(c_{\Delta m^{(m)}}\right) \quad \text{(A11)}$$

If $\rho_a^{(m_i)} \approx \rho_a^{(m_f)} \equiv \rho_a^{(m)} = 1.135$ kg/m^3, $\rho_r^{(m)} \approx \rho_t$, $u\left(c_{\Delta m^{(m)}}\right) <$ 0.1 µg, $u^2\left(c_{\rho_a^{(m)}}\right) <$ 0.002 kg/m^3 and $\frac{u^2\left(\rho_r^{(m)}\right)}{\left(\rho_r^{(m)}\right)^4} < \frac{u^2(\rho_t)}{\rho_t^4} << 1$, then:

$$u(e) = \left[u^2\left(m_{ct}^{(m_f)}\right) + u^2\left(m_{ct}^{(m_i)}\right) - 2u^2\left(m_{cr}^{(m)}\right)\right]^{1/2}$$

A.2 Evaluation of $u\left(m_{ct}^{(c)}, m_{ct}^{(j)}\right)$, with $j = b, p$

For $j = p$ and from (A1), (A2), (A6) and (A8):

$$u\left(m_{ct}^{(c)}, m_{ct}^{(p)}\right) = \frac{\partial m_{ct}^{(c)}}{\partial m_{cr}^{(c)}} \frac{\partial m_{ct}^{(p)}}{\partial m_{cr}^{(c)}} u^2\left(m_{cr}^{(c)}\right) + \frac{\partial m_{ct}^{(c)}}{\partial \rho_r^{(c)}} \frac{\partial m_{ct}^{(p)}}{\partial \rho_r^{(c)}} u^2\left(\rho_r^{(c)}\right) + \frac{\partial m_{ct}^{(c)}}{\partial \rho_t} \frac{\partial m_{ct}^{(p)}}{\partial \rho_t} u^2\left(\rho_t\right) + \frac{\partial m_{ct}^{(c)}}{\partial c_{\rho_a^{(c)}}} \frac{\partial m_{ct}^{(p)}}{\partial c_{\rho_a^{(c)}}} u^2\left(c_{\rho_a^{(c)}}\right) + \frac{\partial m_{ct}^{(c)}}{\partial c_{\Delta m^{(m)}}} \frac{\partial m_{ct}^{(p)}}{\partial c_{\Delta m^{(m)}}} u^2\left(c_{\Delta m^{(c)}}\right)$$

$$u\left(m_{ct}^{(c)}, m_{ct}^{(p)}\right) =$$
$$\left(1+\left(\rho_a^{(c)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(c)}}\right)\right)$$
$$\left(1+\left(\rho_a^{,}-\rho_o\right)\left(\frac{1}{\rho_r^{(p)}}-\frac{1}{\rho_r^{(c)}}\right)\right)$$
$$\left(1+\left(\rho_a^{(p)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(p)}}\right)\right)u^2\left(m_{cr}^{(c)}\right)+$$
$$+\left(m_{cr}^{(c)}\right)^2\left(\rho_a^{(c)}-\rho_o\right)\left(\rho_a^{,}-\rho_o\right)$$
$$\left(1+\left(\rho_a^{(p)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(p)}}\right)\right)\frac{1}{\left(\rho_r^{(c)}\right)^4}u^2\left(\rho_r^{(c)}\right)+$$
$$+m_{cr}^{(c)}\left(\rho_a^{(c)}-\rho_o\right)\left(\rho_a^{(p)}-\rho_o\right)$$
$$\left[m_{cr}^{(c)}\left(1+\left(\rho_a^{,}-\rho_o\right)\left(\frac{1}{\rho_r^{(p)}}-\frac{1}{\rho_r^{(c)}}\right)\right)+\Delta m^{,}\right]$$
$$\frac{1}{\rho_t^4}u^2(\rho_t)$$
$$+\left(m_{cr}^{(c)}\right)^2\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(c)}}\right)\left(\frac{1}{\rho_r^{(p)}}-\frac{1}{\rho_r^{(c)}}\right)$$
$$\left(1+\left(\rho_a^{(p)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(p)}}\right)\right)u^2\left(c_{\rho_a^{(c)}}\right)$$
$$+u^2\left(c_{\Delta m^{(c)}}\right)$$

If $\rho_r^{(c)} \approx \rho_r^{(p)} \approx \rho_t$, $\rho_a^{,} \approx \rho_a^{(c)} = 1.125$ kg/m^3 , $\frac{u^2\left(\rho_r^{(c)}\right)}{\left(\rho_r^{(c)}\right)^4} < \frac{u^2(\rho_t)}{\rho_t^4} << 1$, $u\left(c_{\Delta m^{(c)}}\right)$< 1 µg, $u^2\left(c_{\rho_a^{(c)}}\right)$ < 0.002 kg/m^3 and $\frac{\Delta m^{,}}{m_{cr}^{(c)}} << 1$, then:
$$u\left(m_{ct}^{(c)}, m_{ct}^{(p)}\right) \approx u^2\left(m_{cr}^{(c)}\right)$$

For $j = b$ is the same but using equation (A3):

$$u\left(m_{ct}^{(c)}, m_{ct}^{(b)}\right) =$$
$$\left(1+\left(\rho_a^{(c)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(c)}}\right)\right)$$
$$\left(1+\left(\rho_a^{,,}-\rho_o\right)\left(\frac{1}{\rho_r^{(b)}}-\frac{1}{\rho_r^{(c)}}\right)\right)$$
$$\left(1+\left(\rho_a^{(b)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(b)}}\right)\right)u^2\left(m_{cr}^{(c)}\right)+$$
$$+\left(m_{cr}^{(c)}\right)^2\left(\rho_a^{(c)}-\rho_o\right)\left(\rho_a^{,,}-\rho_o\right)$$
$$\left(1+\left(\rho_a^{(b)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(b)}}\right)\right)\frac{1}{\left(\rho_r^{(c)}\right)^4}u^2\left(\rho_r^{(c)}\right)+$$
$$+m_{cr}^{(c)}\left(\rho_a^{(c)}-\rho_o\right)\left(\rho_a^{(b)}-\rho_o\right)$$
$$\left[m_{cr}^{(c)}\left(1+\left(\rho_a^{,,}-\rho_o\right)\left(\frac{1}{\rho_r^{(b)}}-\frac{1}{\rho_r^{(c)}}\right)\right)+\Delta m^{,,}\right]$$
$$\frac{1}{\rho_t^4}u^2(\rho_t)+\left(m_{cr}^{(c)}\right)^2\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(c)}}\right)\left(\frac{1}{\rho_r^{(b)}}-\frac{1}{\rho_r^{(c)}}\right)$$
$$\left(1+\left(\rho_a^{(b)}-\rho_o\right)\left(\frac{1}{\rho_t}-\frac{1}{\rho_r^{(b)}}\right)\right)u^2\left(c_{\rho_a^{(c)}}\right)+u^2\left(c_{\Delta m^{(c)}}\right)$$

In this case $\rho_a^{,,} \approx \rho_a^{(c)}, \frac{\Delta m^{,,}}{m_{cr}^{(c)}} << 1$ and $\frac{u^2\left(\rho_r^{(c)}\right)}{\left(\rho_r^{(c)}\right)^4} < \frac{u^2(\rho_t)}{\rho_t^4} << 1$ also, $u\left(c_{\Delta m^{(c)}}\right)$< 1 µg, $u^2\left(c_{\rho_a^{(c)}}\right)$ < 0.002 kg/m^3, but the difference $\left(\rho_a^{(b)}-\rho_o\right)$ is not so small. If we consider that $\rho_r^{(b)} \approx \rho_r^{(c)} \approx \rho_t$ $u\left(m_{ct}^{(c)}, m_{ct}^{(b)}\right) \approx u^2\left(m_{cr}^{(c)}\right)$

A.3 Evaluation of $u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right)$

For this evaluation, $m_{ct}^{(b)}$ and $m_{ct}^{(p)}$ are re-written using relations (A1, A2, A3,A6, A7, A8 and A9). Then
$$u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right) = \frac{\partial m_{ct}^{(b)}}{\partial m_{cr}^{(m)}}\frac{\partial m_{ct}^{(p)}}{\partial m_{cr}^{(m)}}u^2\left(m_{cr}^{(m)}\right)+$$
$$+\frac{\partial m_{ct}^{(b)}}{\partial c_{\rho_a^{(c)}}}\frac{\partial m_{ct}^{(p)}}{\partial c_{\rho_a^{(c)}}}u^2\left(c_{\rho_a^{(c)}}\right)+$$
$$+\frac{\partial m_{ct}^{(b)}}{\partial \rho_r^{(c)}}\frac{\partial m_{ct}^{(p)}}{\partial \rho_r^{(c)}}u^2\left(\rho_r^{(c)}\right)+\frac{\partial m_{ct}^{(b)}}{\partial c_{\Delta m^{(c)}}}\frac{\partial m_{ct}^{(p)}}{\partial c_{\Delta m^{(c)}}}u^2\left(c_{\Delta m^{(c)}}\right)$$

$$u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right) = \left(1 + \left(\overline{\rho_a}'' + c_{\rho_a^{(c)}} - \rho_o\right)\left(\frac{1}{\rho_r^{(b)}} - \frac{1}{\rho_r^{(c)}}\right)\right)$$

$$\left(1 + \left(\rho_a^{(b)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(b)}}\right)\right)$$

$$\left(1 + \left(\overline{\rho_a}' + c_{\rho_a^{(c)}} - \rho_o\right)\left(\frac{1}{\rho_r^{(p)}} - \frac{1}{\rho_r^{(c)}}\right)\right)$$

$$\left(1 + \left(\rho_a^{(p)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(p)}}\right)\right)u^2\left(m_{cr}^{(m)}\right) +$$

$$+\left(m_{cr}^{(c)}\right)^2\left(1 + \left(\rho_a^{(b)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(b)}}\right)\right)\left(\frac{1}{\rho_r^{(b)}} - \frac{1}{\rho_r^{(c)}}\right)$$

$$\left(1 + \left(\rho_a^{(p)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(p)}}\right)\right)\left(\frac{1}{\rho_r^{(p)}} - \frac{1}{\rho_r^{(c)}}\right)u^2\left(c_{\rho_a^{(c)}}\right) +$$

$$+\left(m_{cr}^{(c)}\right)^2\left(1 + \left(\rho_a^{(b)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(b)}}\right)\right)\left(\overline{\rho_a}'' + c_{\rho_a^{(c)}} - \rho_o\right)$$

$$\left(1 + \left(\rho_a^{(p)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(p)}}\right)\right)\left(\overline{\rho_a}' + c_{\rho_a^{(c)}} - \rho_o\right)$$

$$\frac{1}{\left(\rho_r^{(c)}\right)^4}u^2\left(\rho_r^{(c)}\right) + \left(1 + \left(\rho_a^{(b)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(b)}}\right)\right)$$

$$\left(1 + \left(\rho_a^{(p)} - \rho_o\right)\left(\frac{1}{\rho_t} - \frac{1}{\rho_r^{(p)}}\right)\right)u^2\left(c_{\Delta m^{(c)}}\right)$$

If $\rho_r^{(b)} \approx \rho_r^{(c)} \approx \rho_t$,

$$u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right) = u^2\left(m_{cr}^{(m)}\right) + \left(m_{cr}^{(c)}\right)^2\left(\overline{\rho_a}'' + c_{\rho_a^{(c)}} - \rho_o\right)$$

$$\left(\overline{\rho_a}' + c_{\rho_a^{(c)}} - \rho_o\right)\frac{u^2\left(\rho_r^{(c)}\right)}{\left(\rho_r^{(c)}\right)^4} + u^2\left(c_{\Delta m^{(c)}}\right)$$

From the previous considerations about $u^2\left(c_{\Delta m^{(c)}}\right)$ and $\dfrac{u^2\left(\rho_r^{(c)}\right)}{\left(\rho_r^{(c)}\right)^4}$:

$$u\left(m_{ct}^{(b)}, m_{ct}^{(p)}\right) \approx u^2\left(m_{cr}^{(m)}\right)$$

The low air density at IBMETRO does not have a significant effect in the evaluation of the covariances if the density of the weights are almost the same and are calibrated in conventional mass.

In the case of a comparison in mass, factors like $\left(\rho_a^{(i)} - \rho_o\right)$ are absent and covariances would have to be considered for higher accuracy evaluations.